Radio Interferometry

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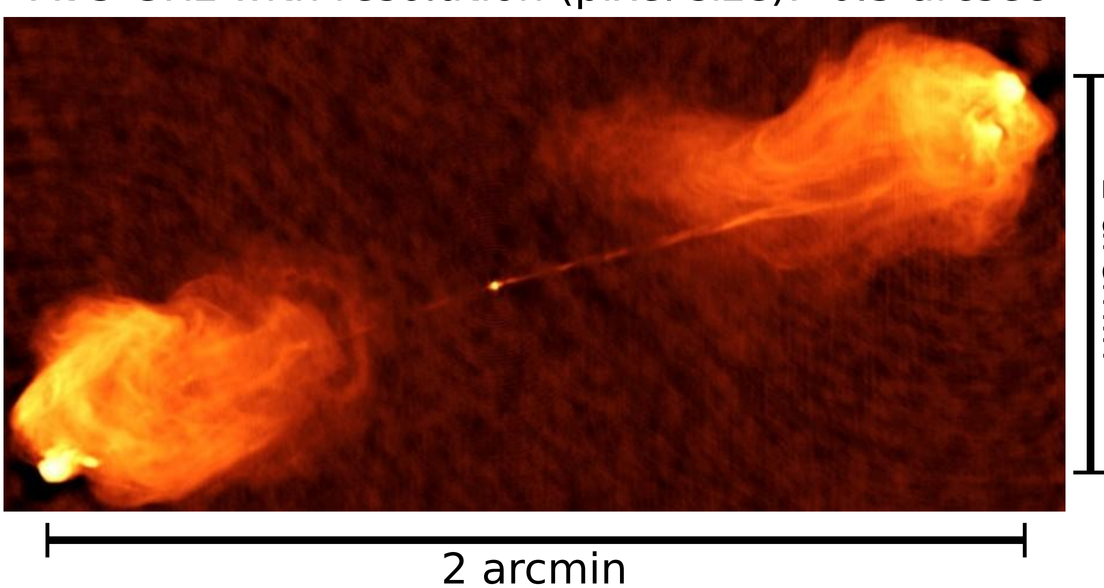
Very Large Array in New Mexico
Each Dish Diameter: d=25m

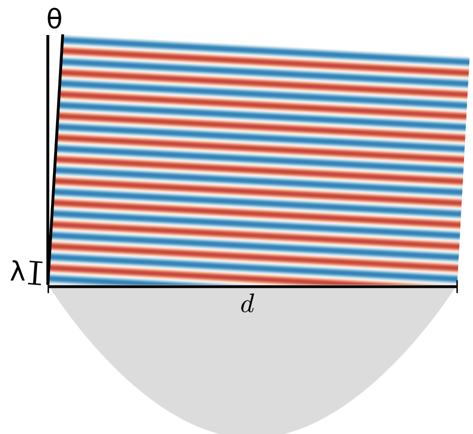
At 5 GHz with resolution (pixel size): 0.5 arcsec

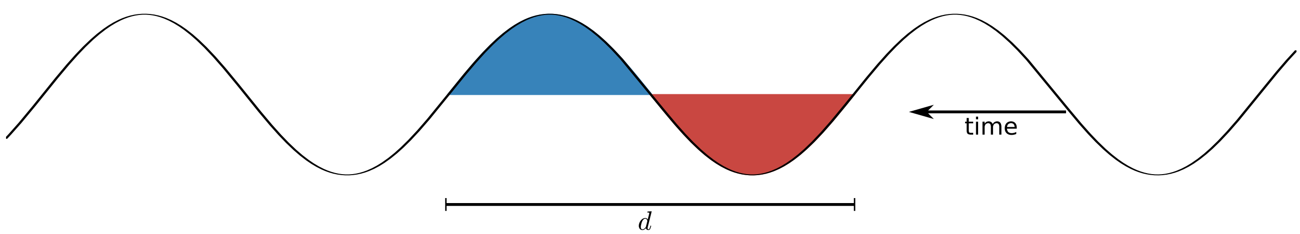


Cygnus A, a galaxy some 600 million light-years away with a central black hole emitting electroncs that get trapped in magnetic fields
Image courtesy of NRAO/AUI: http://images.nrao.edu/110

Telescope Resolution

As the source's angle θ is increases, the intensity (time average of the square of the total amplitude) initially decreases. The first (smallest) angle where the sum across the aperture is zero (independent of time) sets the resolution.





In the small angle approximation, $\theta_r =$

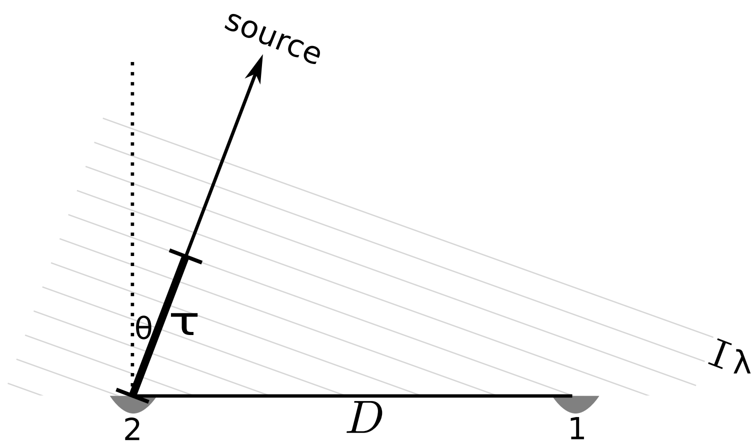
Examples

	Wavelength λ	Aperture Diameter d	Resolution θ_r
Human Eye	500 nm	2 mm	______ arcmin
Single VLA Dish, 5 GHz	______ cm	25 m	______ arcmin
CygA Image, 5 GHz	______ cm	______ km	0.5 arcsec

$$\begin{aligned} 1\,\text{rad} &= \frac{180}{\pi} \cdot 60 \text{ arcmin} &&\approx 3 \cdot 10^3\,\text{arcmin} \\ &= \frac{180}{\pi} \cdot 60 \cdot 60 \text{ arcsec} &&\approx 2 \cdot 10^5\,\text{arcsec} \end{aligned}$$

Geometric Delay and Correlation

Source at *very small* angle θ (exaggerated in diagram), infinitely far away:



At the speed of light c, what is the delay time τ between the wavefront hitting antennas 1 and 2?

$$\tau =$$

The signal (voltage) at antenna 1 is oscillating at frequency f (5 GHz in our example) with amplitude V.

$$v_1(t) = V \sin(2\pi f t)$$

At 2, it's the same amplitude, but delayed by τ. (Don't substitute for τ yet.)

$$v_2(t) = v_1(t + \tau) =$$

A radio interferometer *correlates* pairs of signals by doing two things:

- Multiplies the signals from a pair of antennas. Use $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$.

$$v_1(t)v_2(t) =$$

- Averages this product for a 'long' time (milliseconds — millions of cycles at 5 GHz). Use $\langle \sin \rangle = 0$, and $\langle \sin^2 \rangle = 1/2$.

$$\text{correlation} = \langle v_1(t)v_2(t) \rangle =$$

Now use your expression for τ, $f = c/\lambda$, and the intensity of the source $I = V^2/2$ (up to antenna area and reflectivity).

$$\langle v_1(t)v_2(t) \rangle =$$

For small θ, you should get a function of separation $\delta = D/\lambda$ that oscillates at a "spatial frequency" of θ:

$$\langle v_1(t)v_2(t) \rangle = I \cos(2\pi\theta\delta)$$