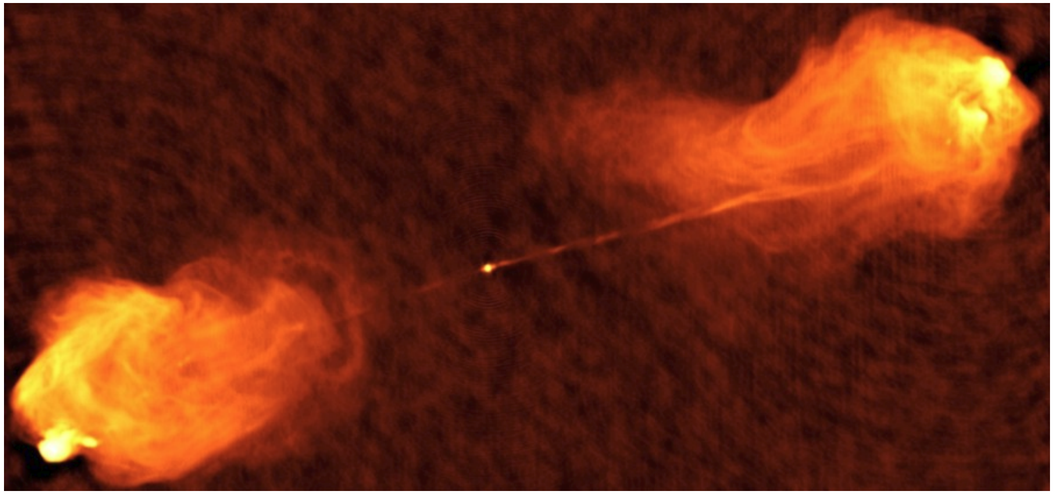
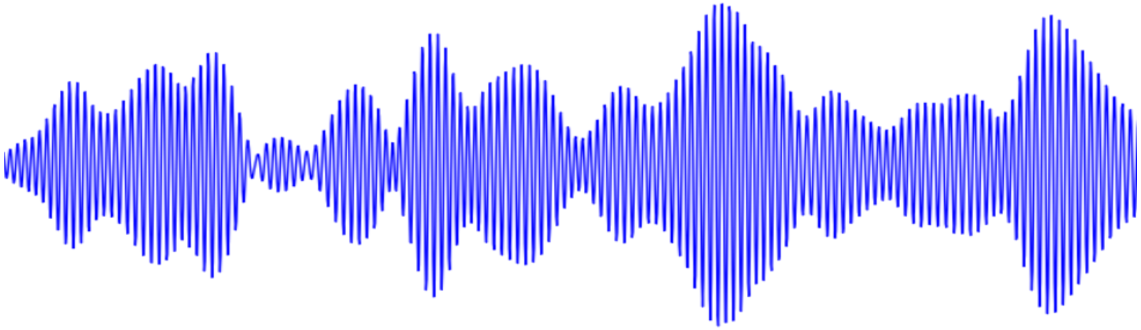
Radio Interferometers and Coherence



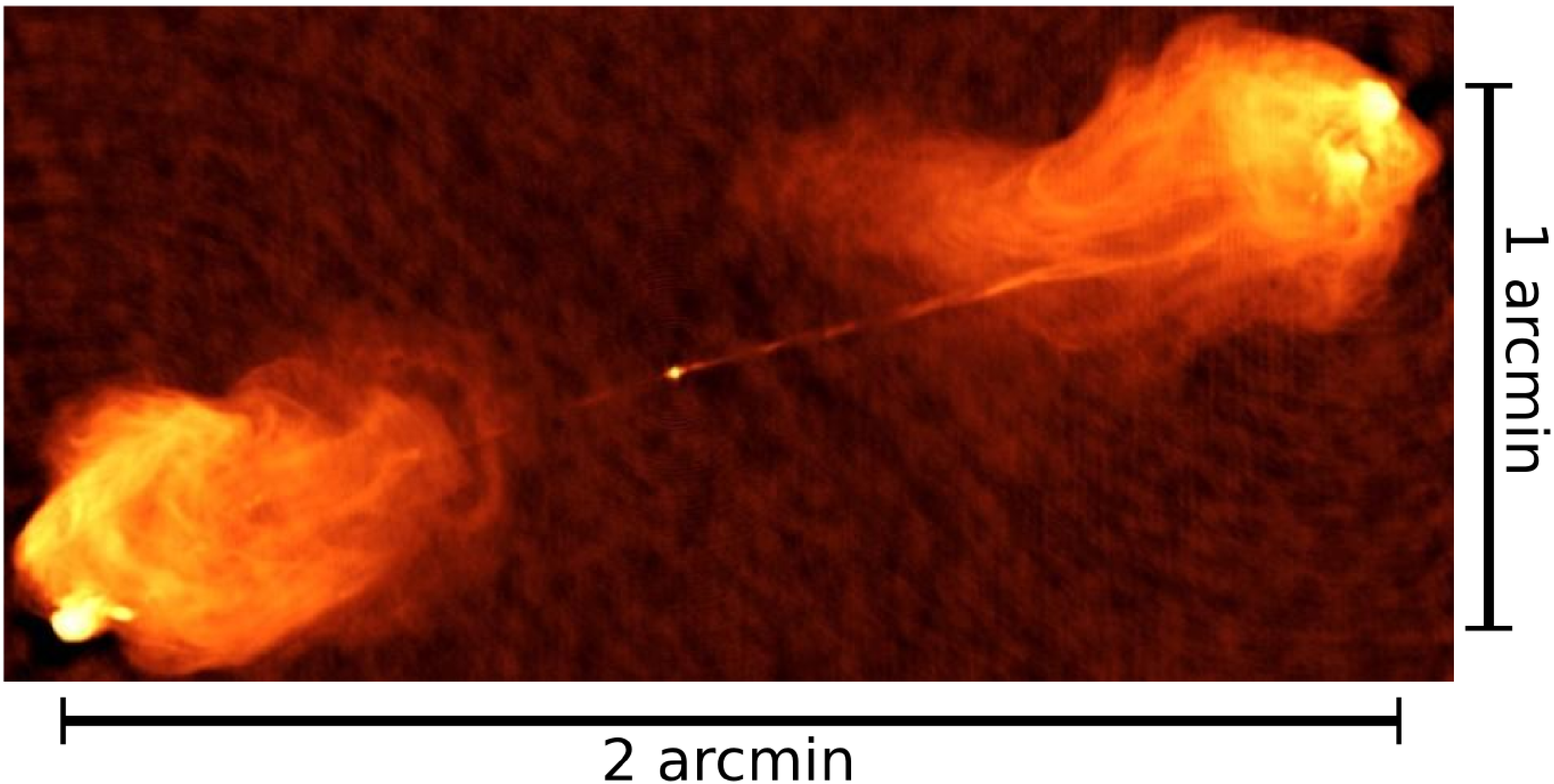


Jason Gallicchio
Harvey Mudd College
3 March 2015

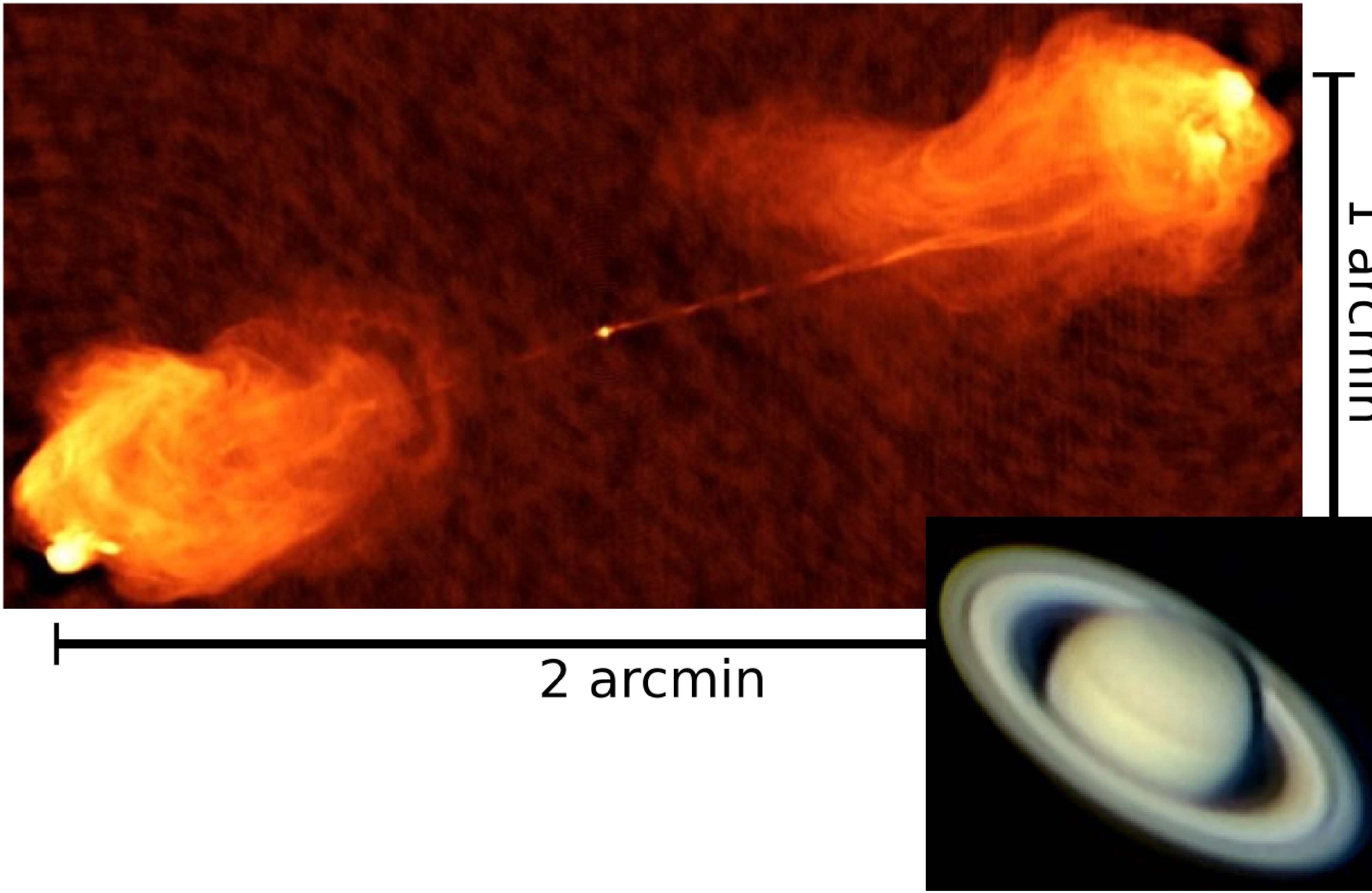
Outline

- It's completely impossible (with one dish)
  - Telescope resolution
- It's possible, but subtle with two
  - Interferometery
- It relies on coherence and incoherence

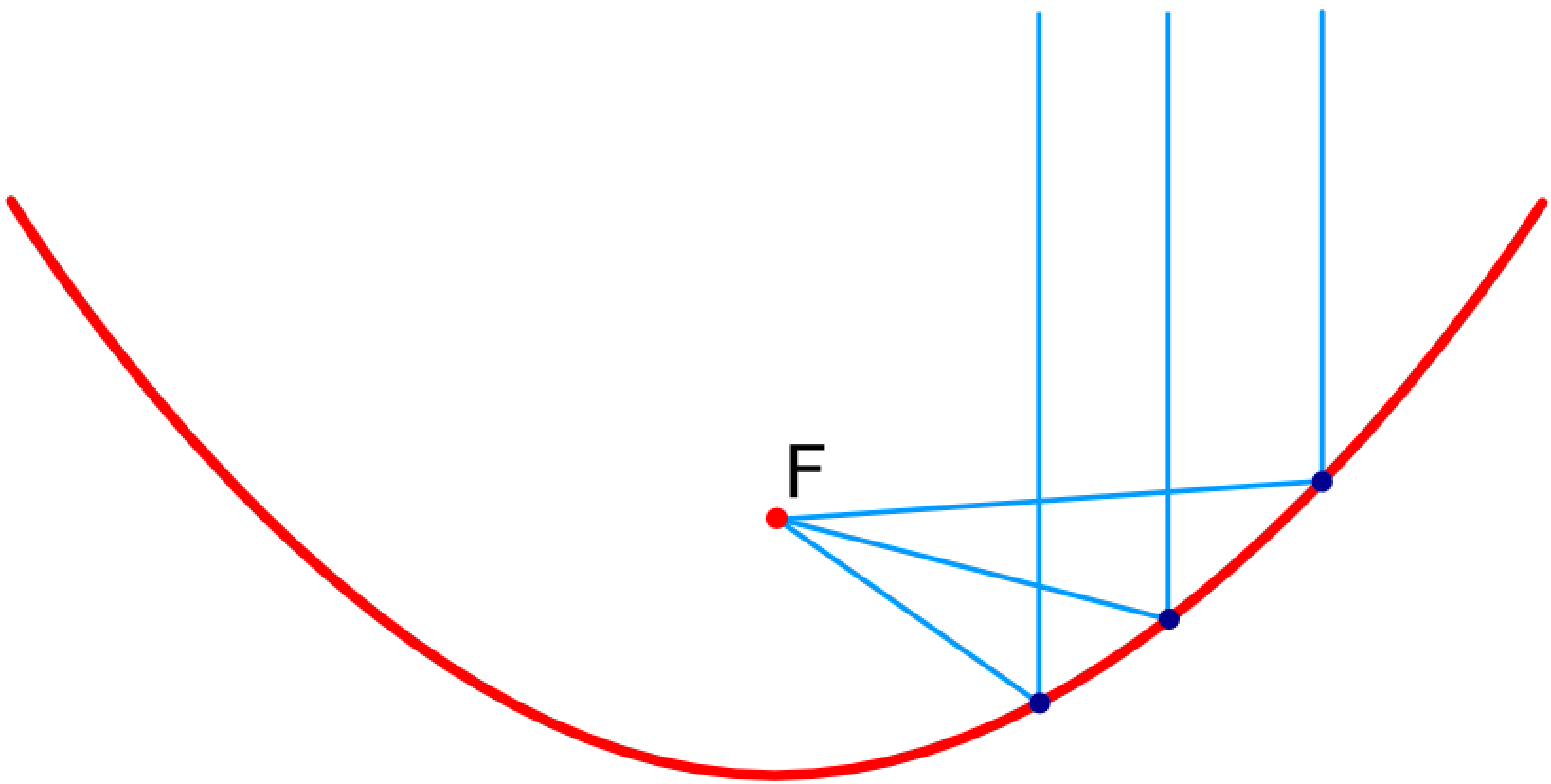
At 5 GHz (6cm) with resolution (pixel size): 0.5 arcsec



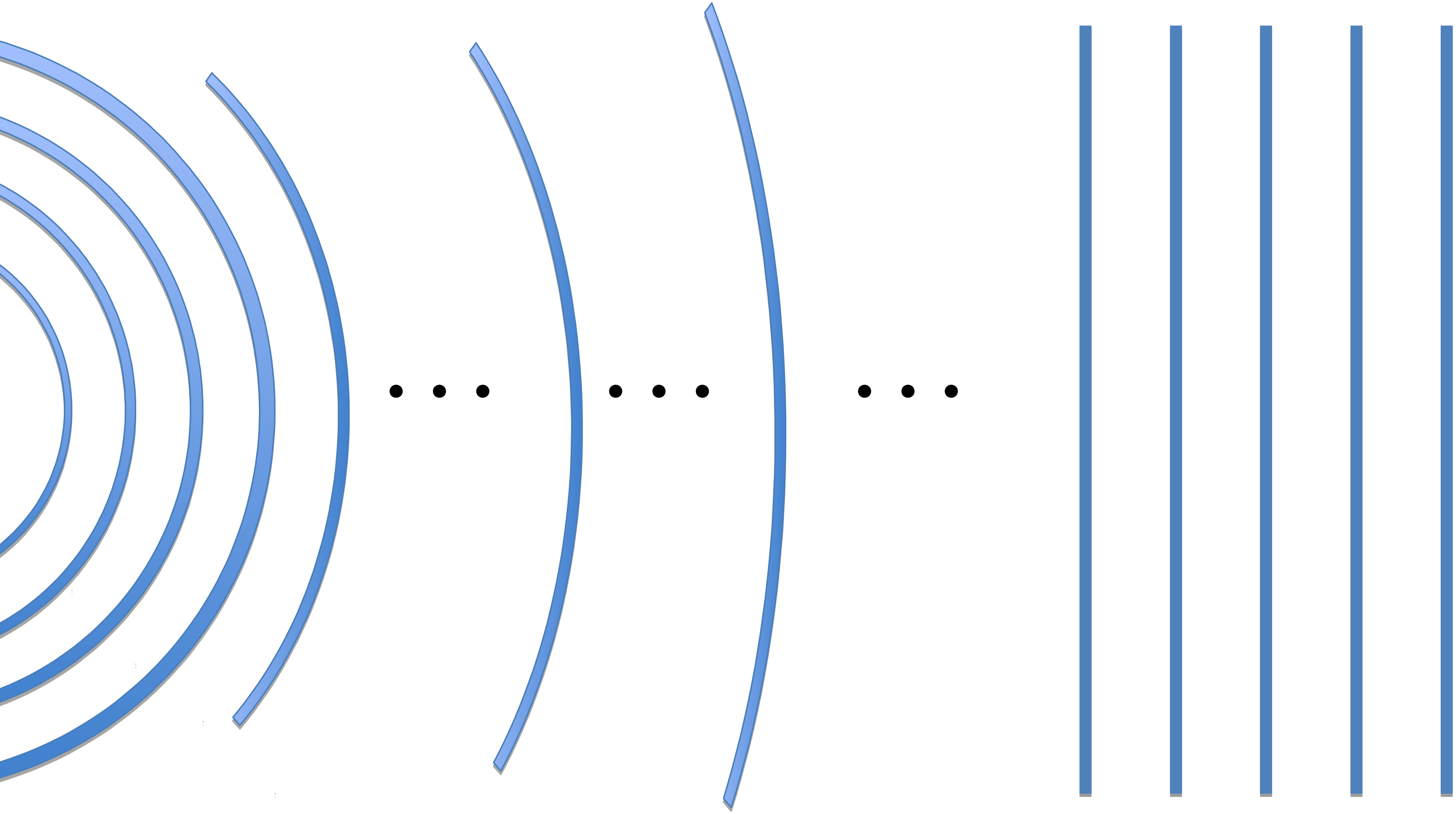
At 5 GHz (6cm) with resolution (pixel size): 0.5 arcsec



Telescope Resolution



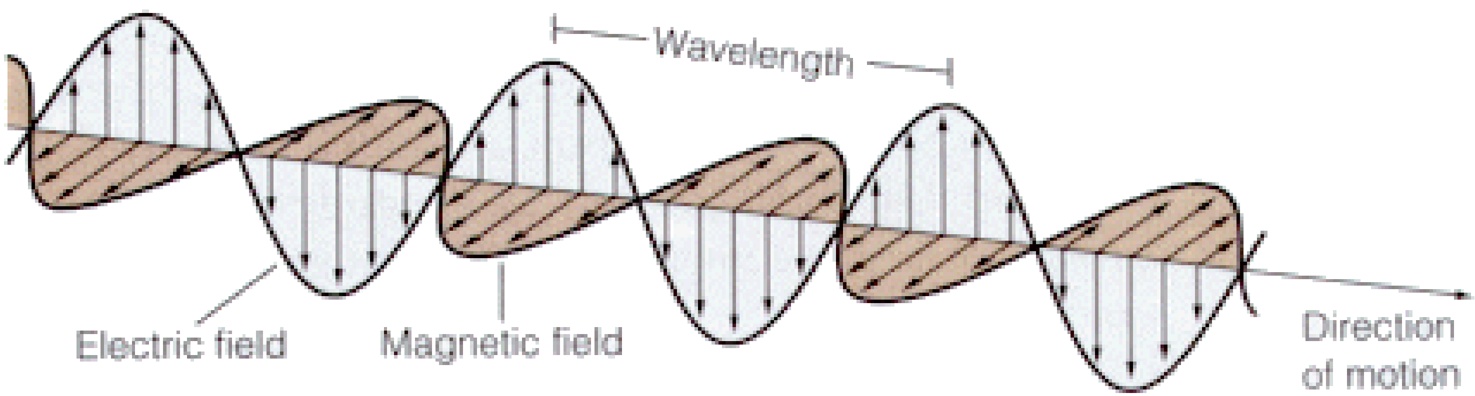
EM Waves



Energy, Intensity, Power

$$\text{Energy Density } [\text{J/m}^3] = \frac{1}{2}\epsilon_0 E^2$$

$$\text{Intensity of EM wave } [\text{W/m}^2] = \frac{1}{2}c\epsilon_0 E^2$$



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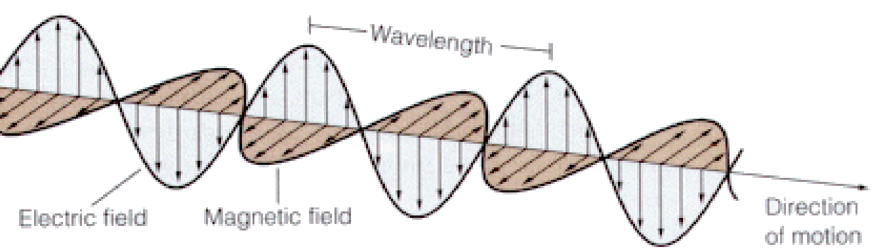
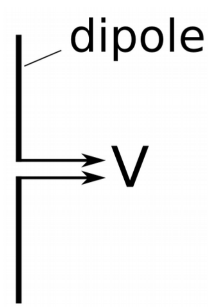


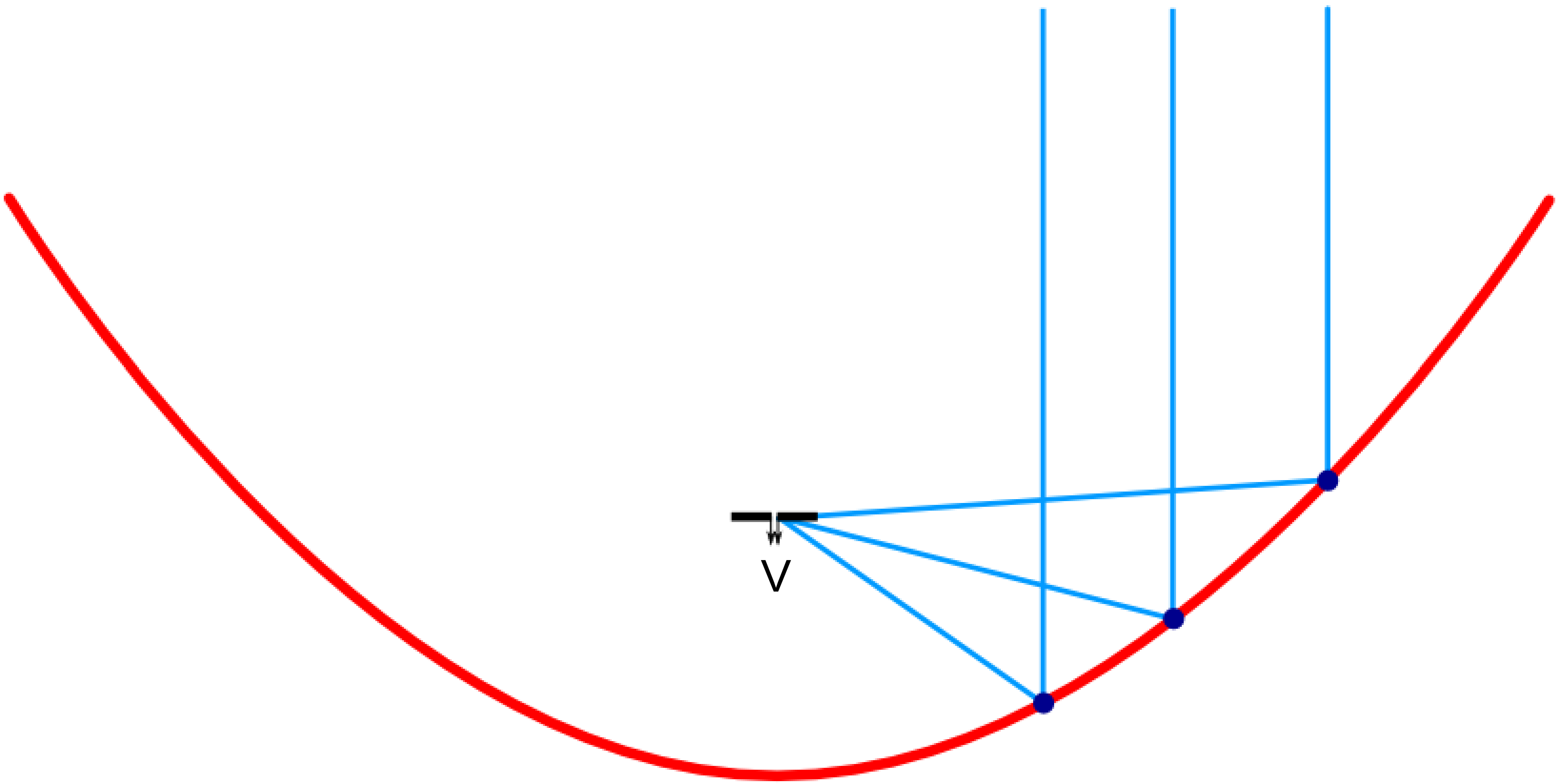


$$\text{For Antenna: } V \propto E$$

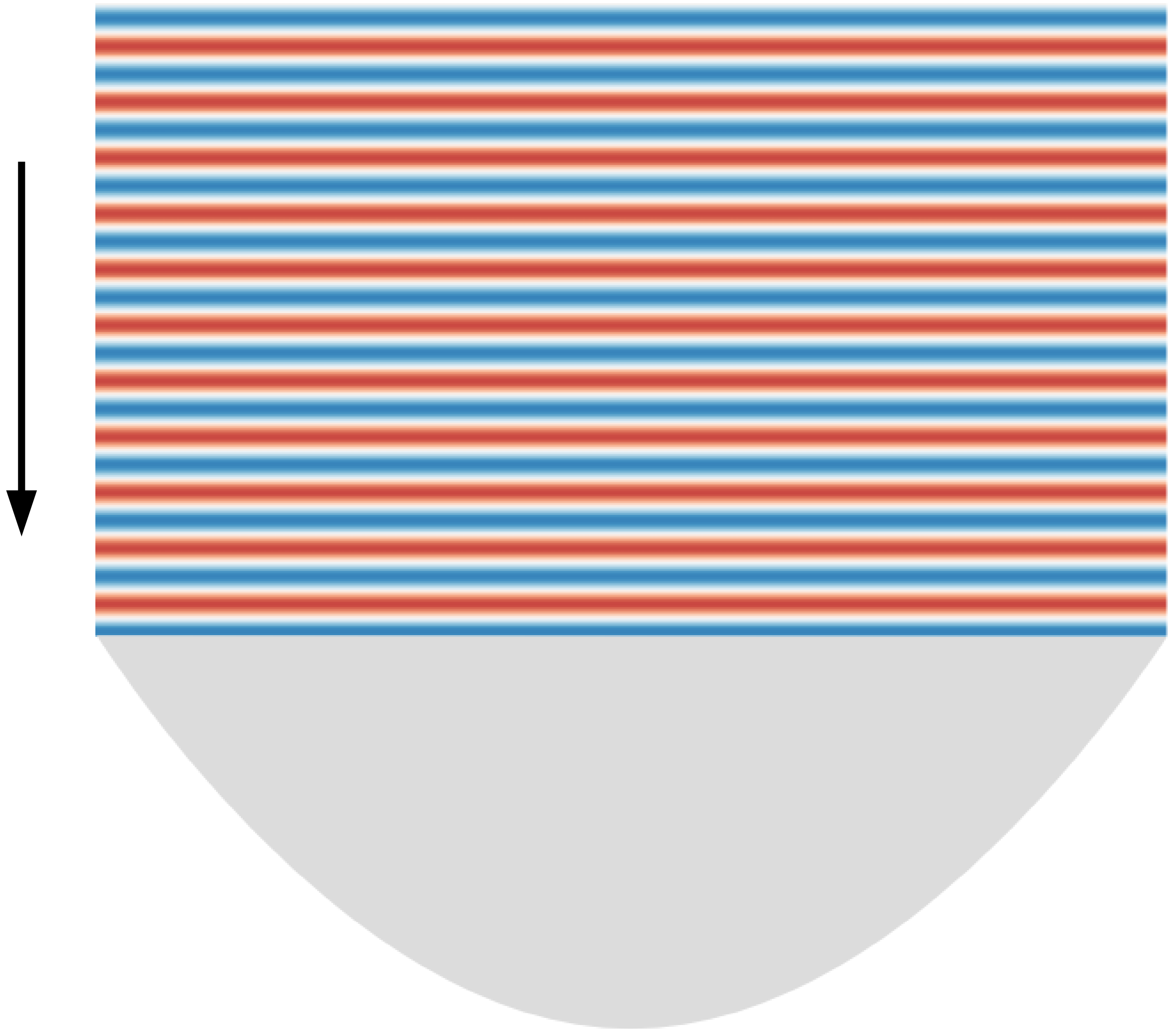
$$\text{Power} = \frac{V^2}{R} \propto E^2$$

Simplify: Treat Electric field as a scalar (i.e. consider just one polarization)

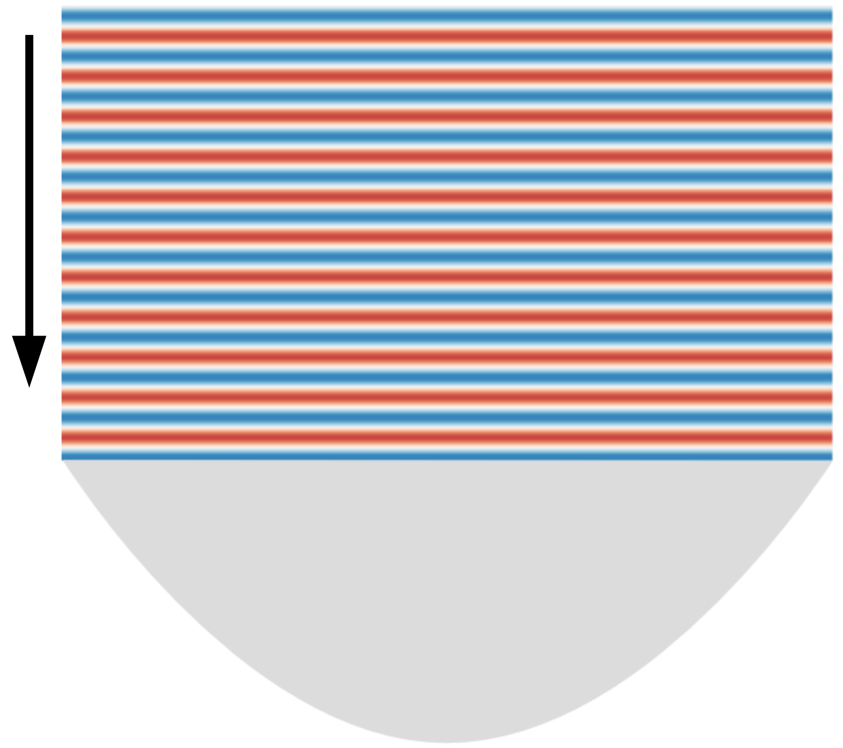
Parabolic Mirror



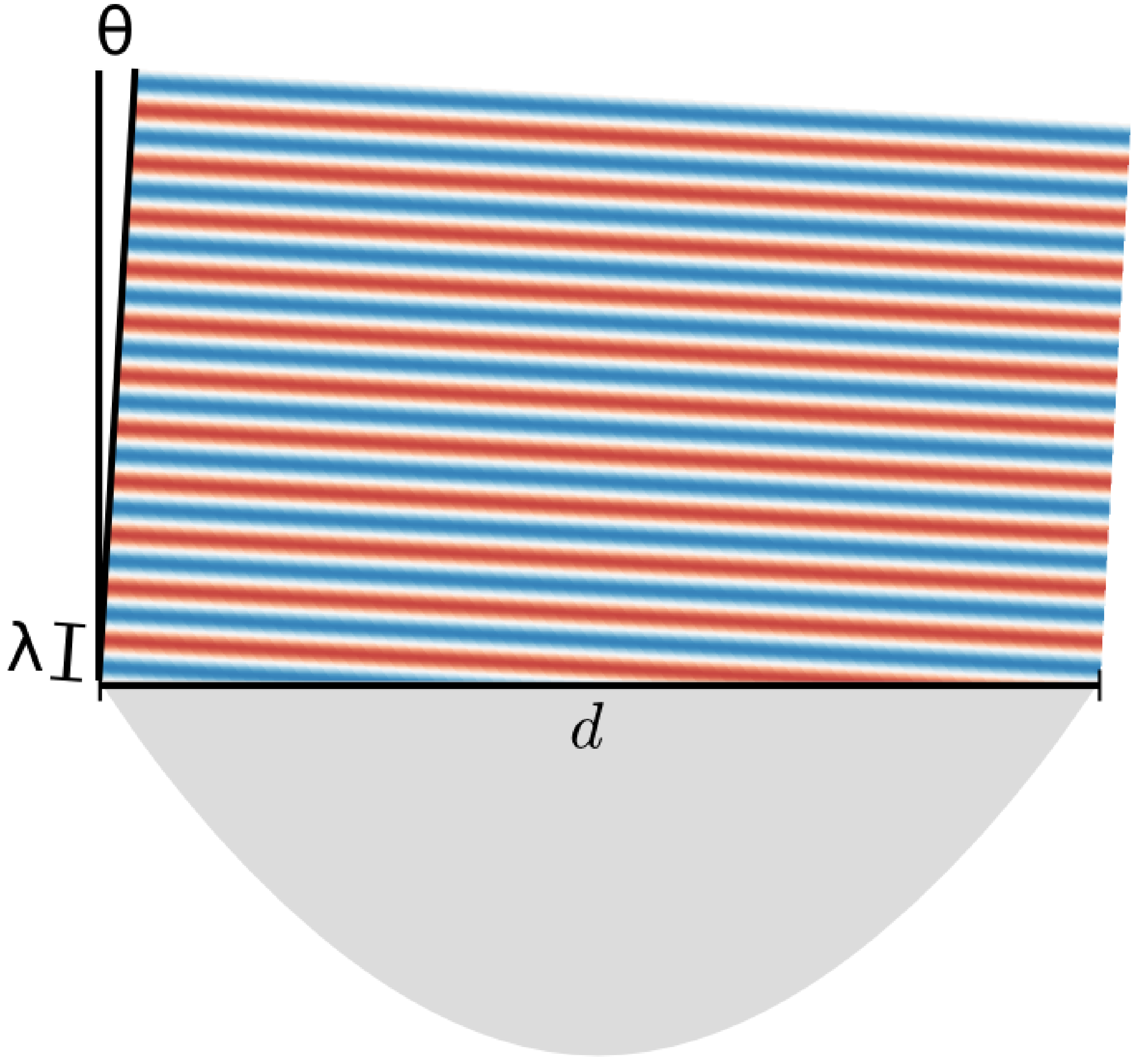
Incoming Plane Wave



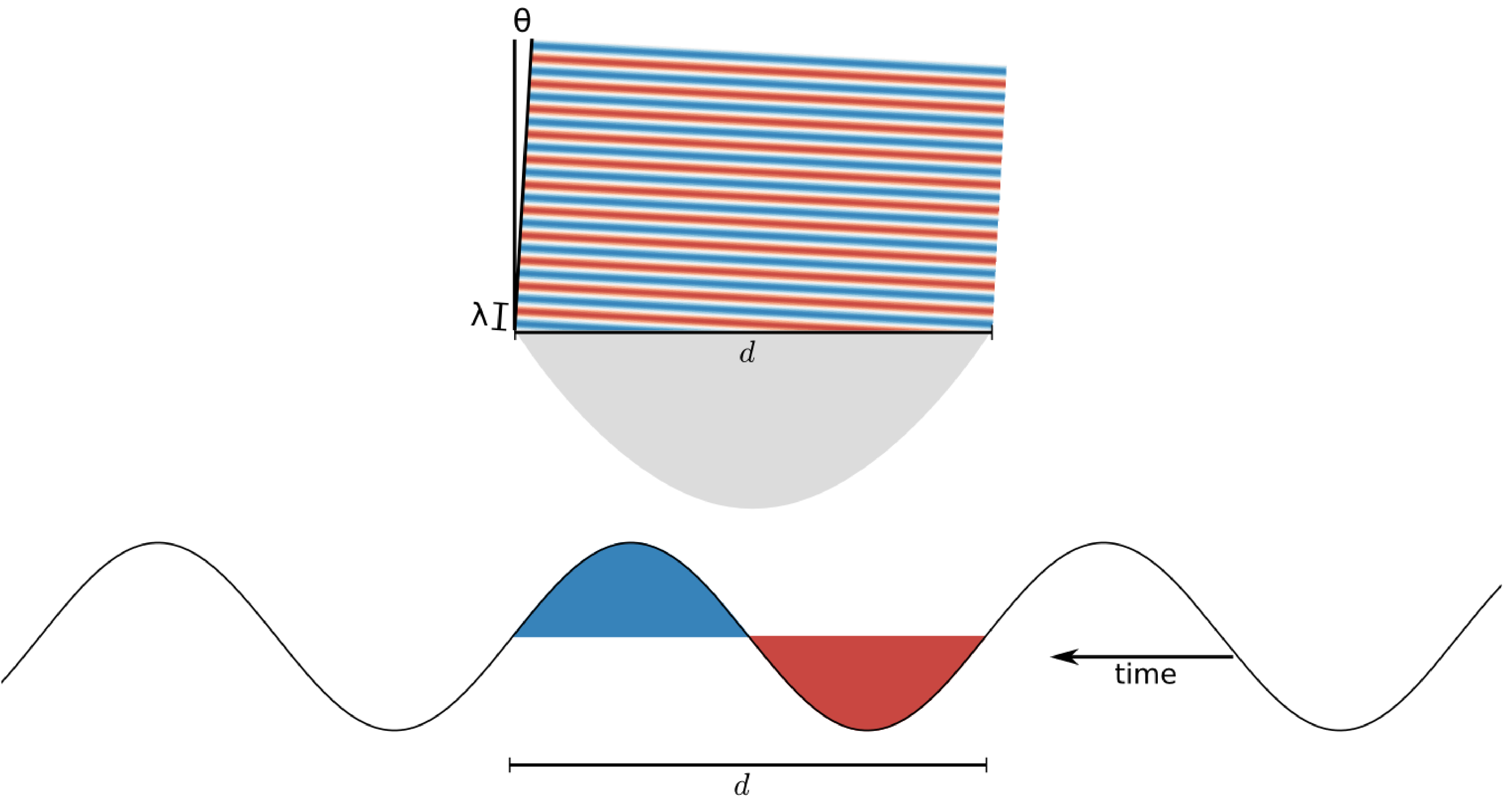
At the Entrance Aperture



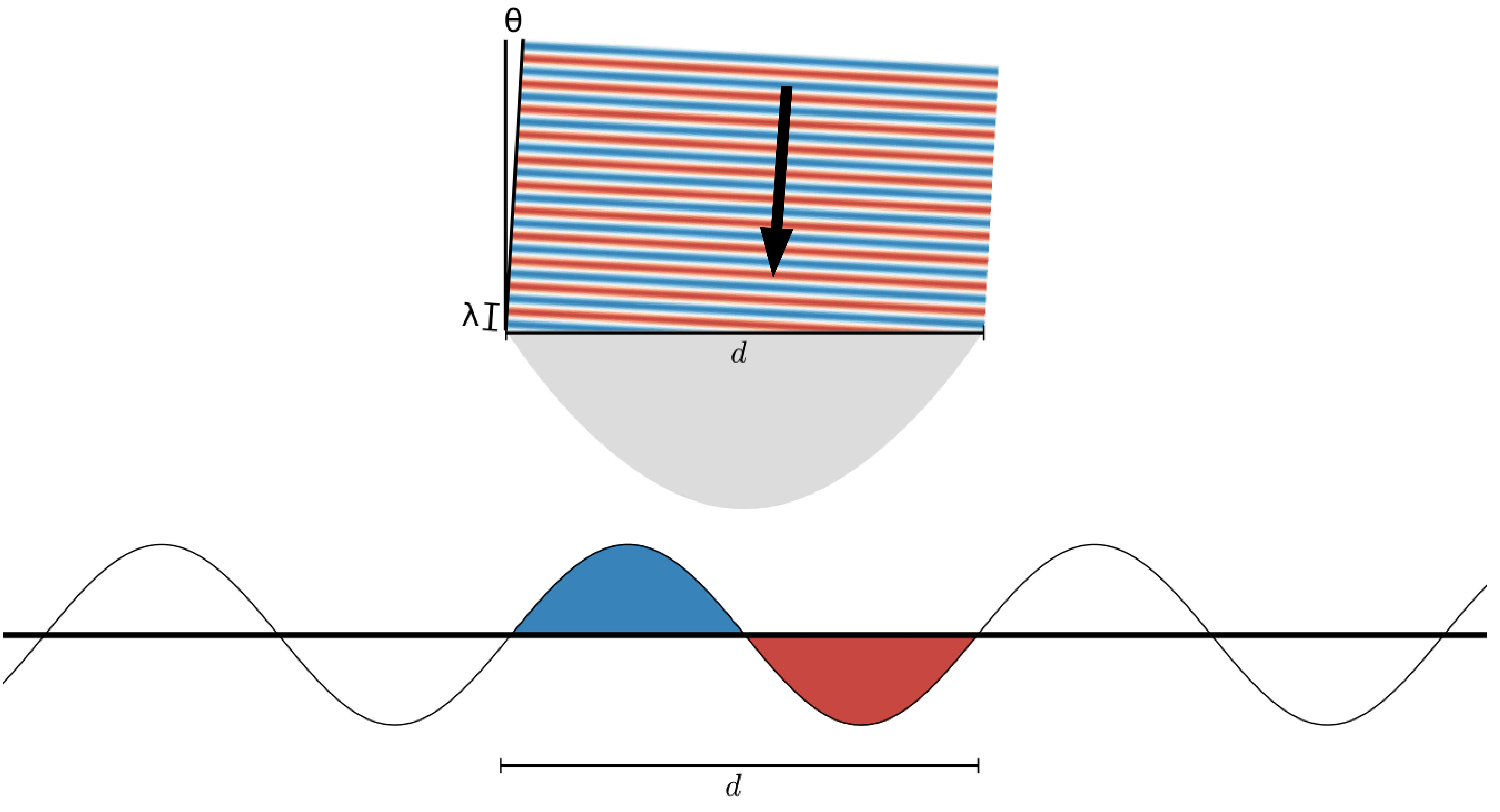
Telescope Resolution

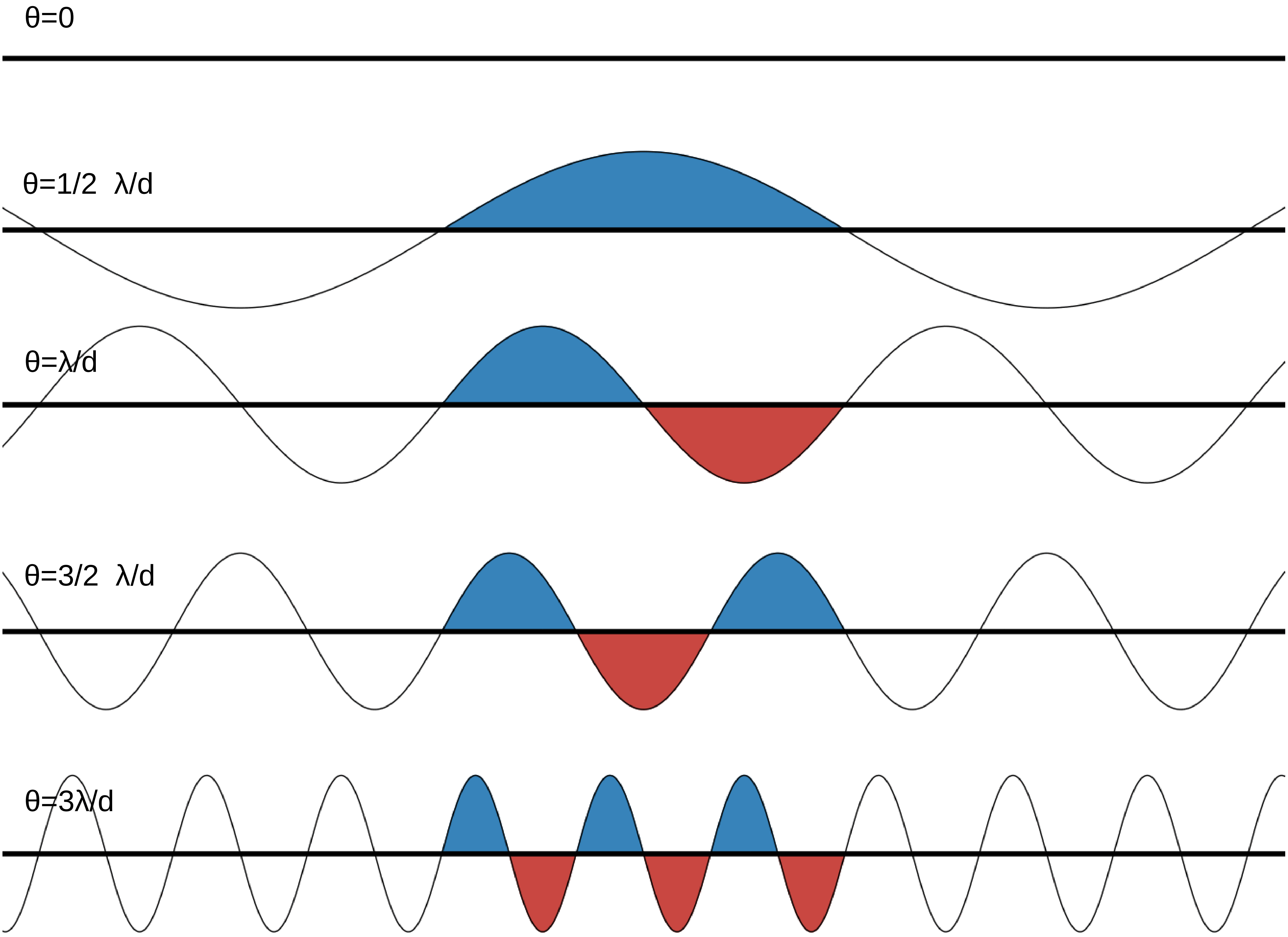


Telescope Resolution

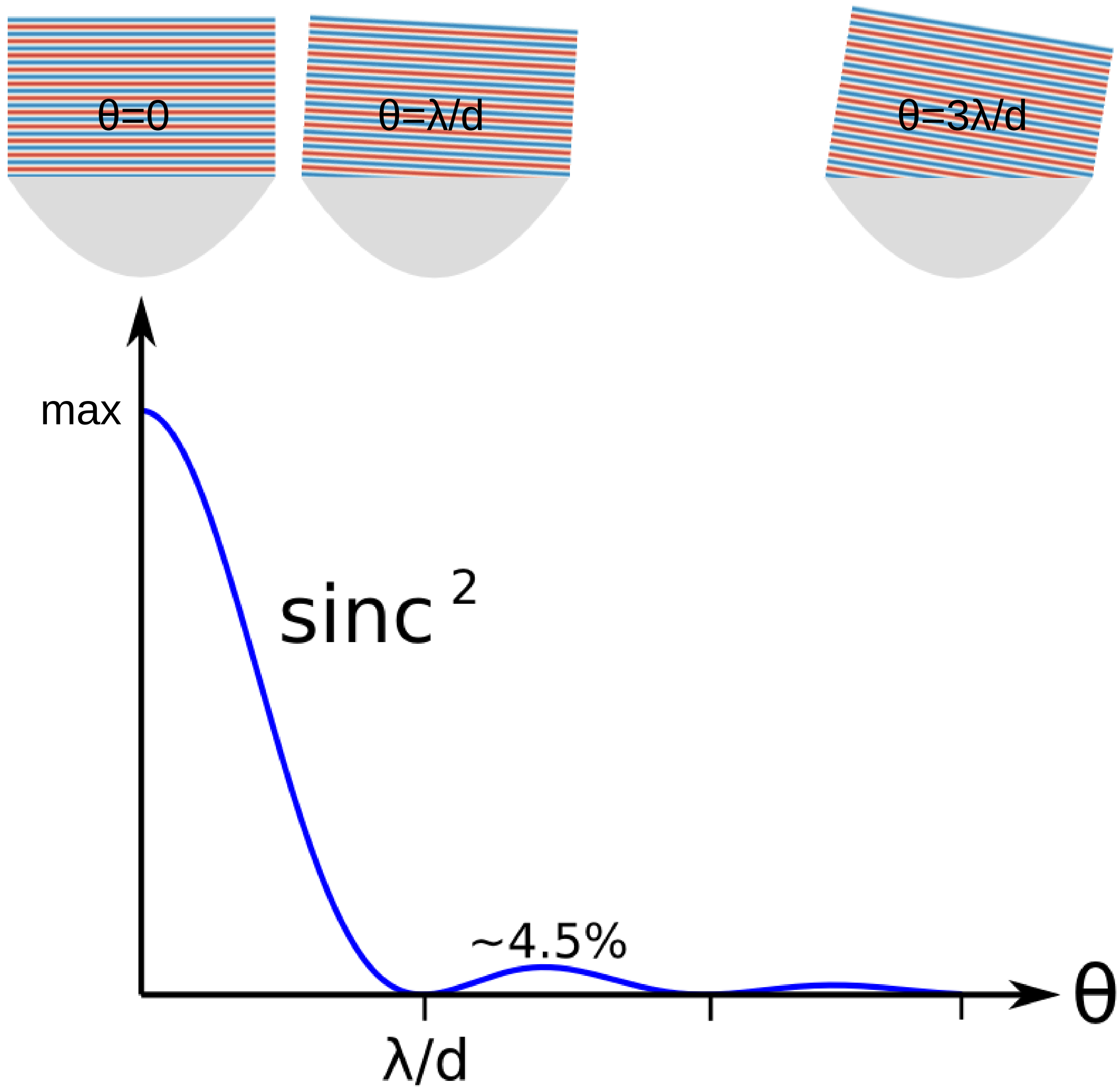


Telescope Resolution

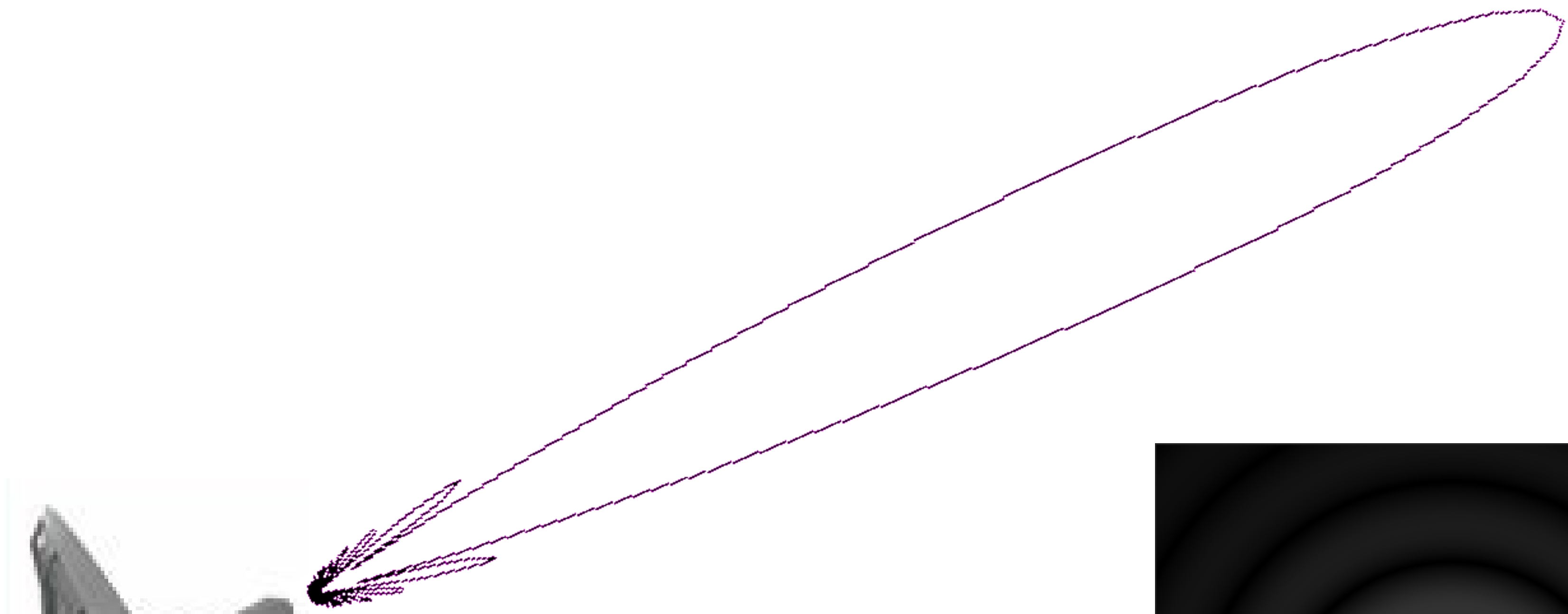


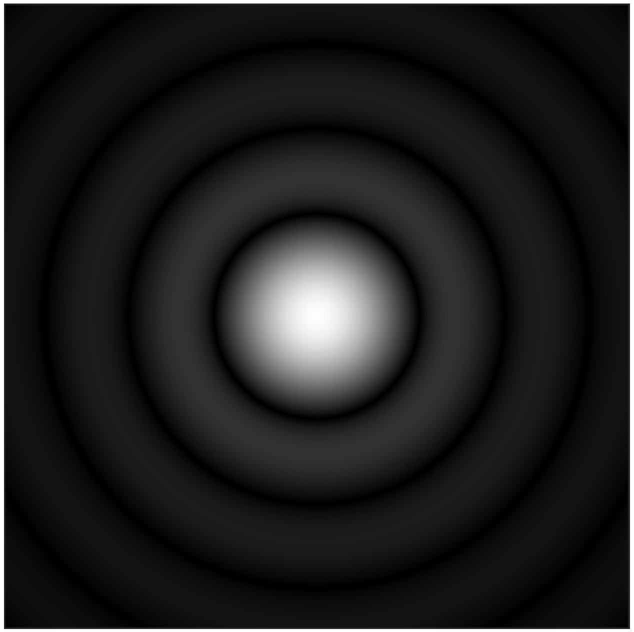

θ=0
θ=1/2 λ/d
θ=λ/d
θ=3/2 λ/d
θ=3λ/d

Telescope Response

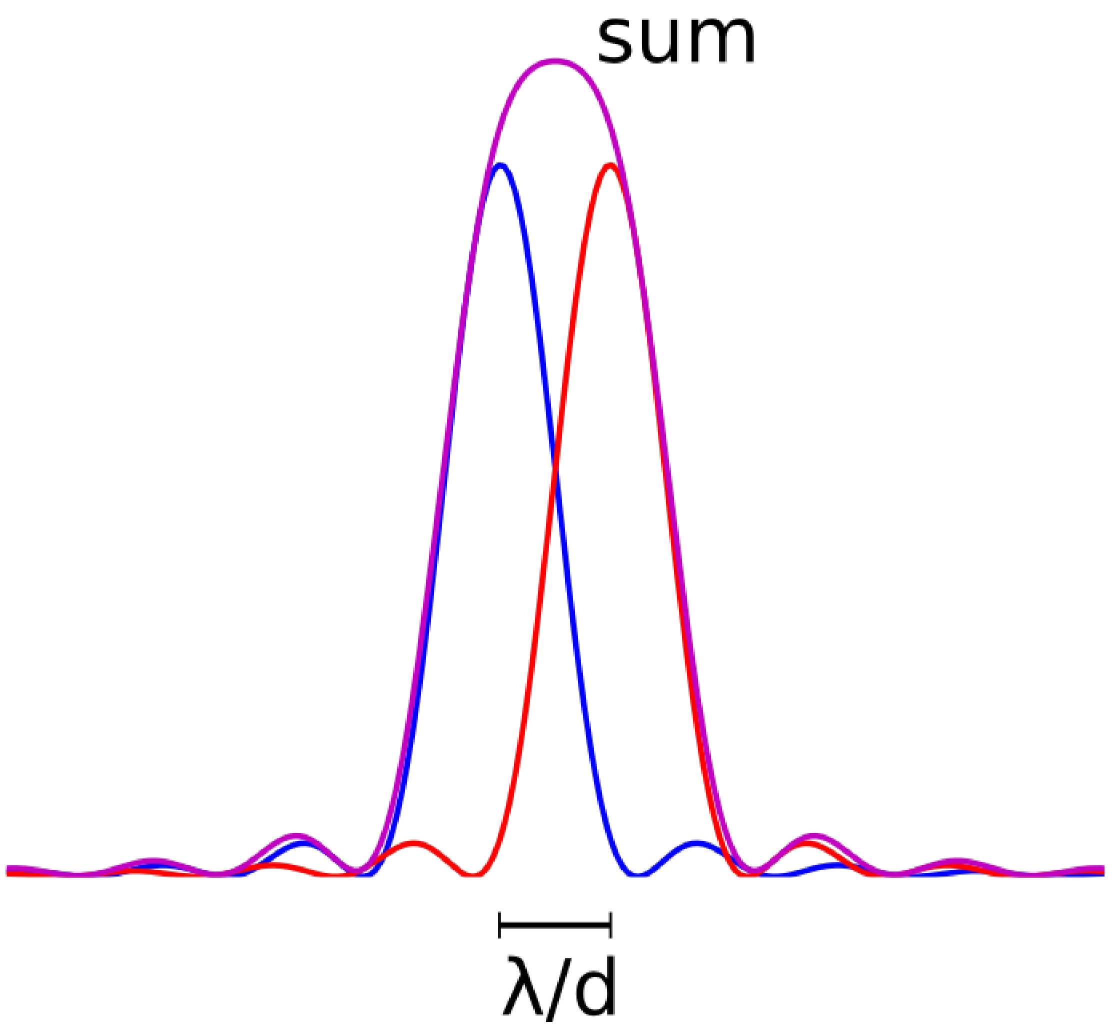


Telescope Resolution



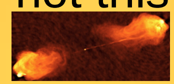


Rayleigh Criterion

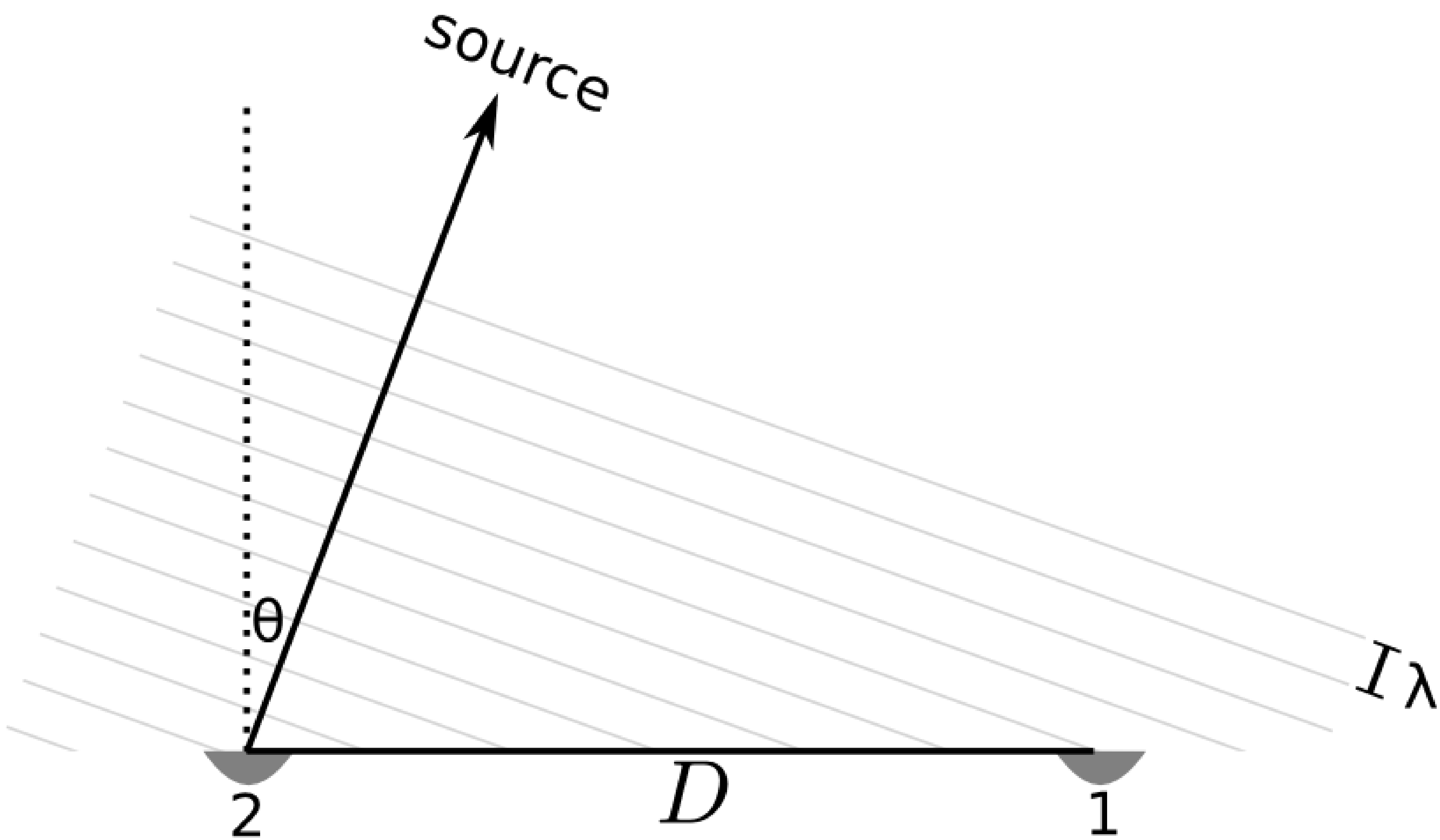


One Dish Sees

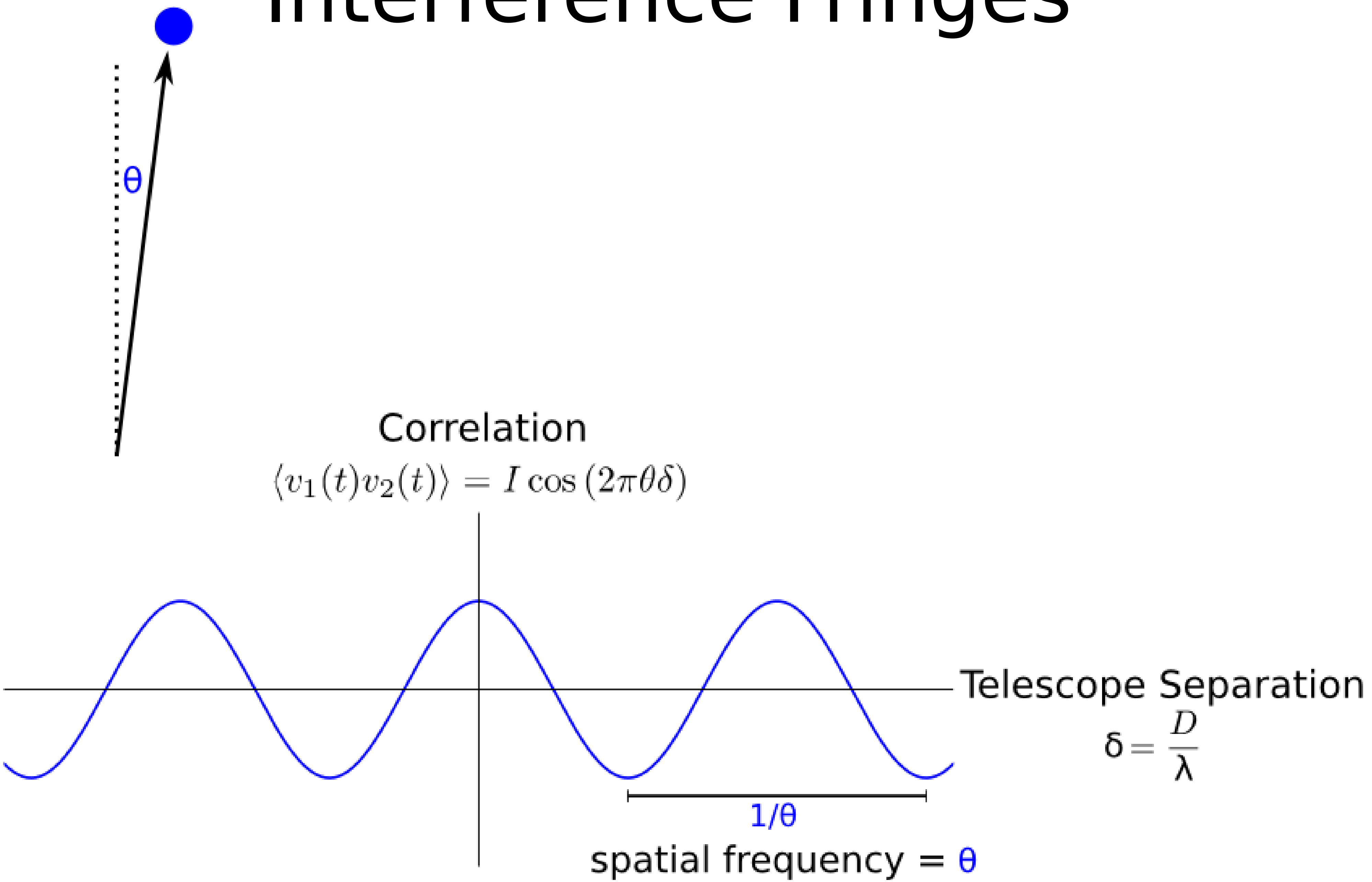
not this

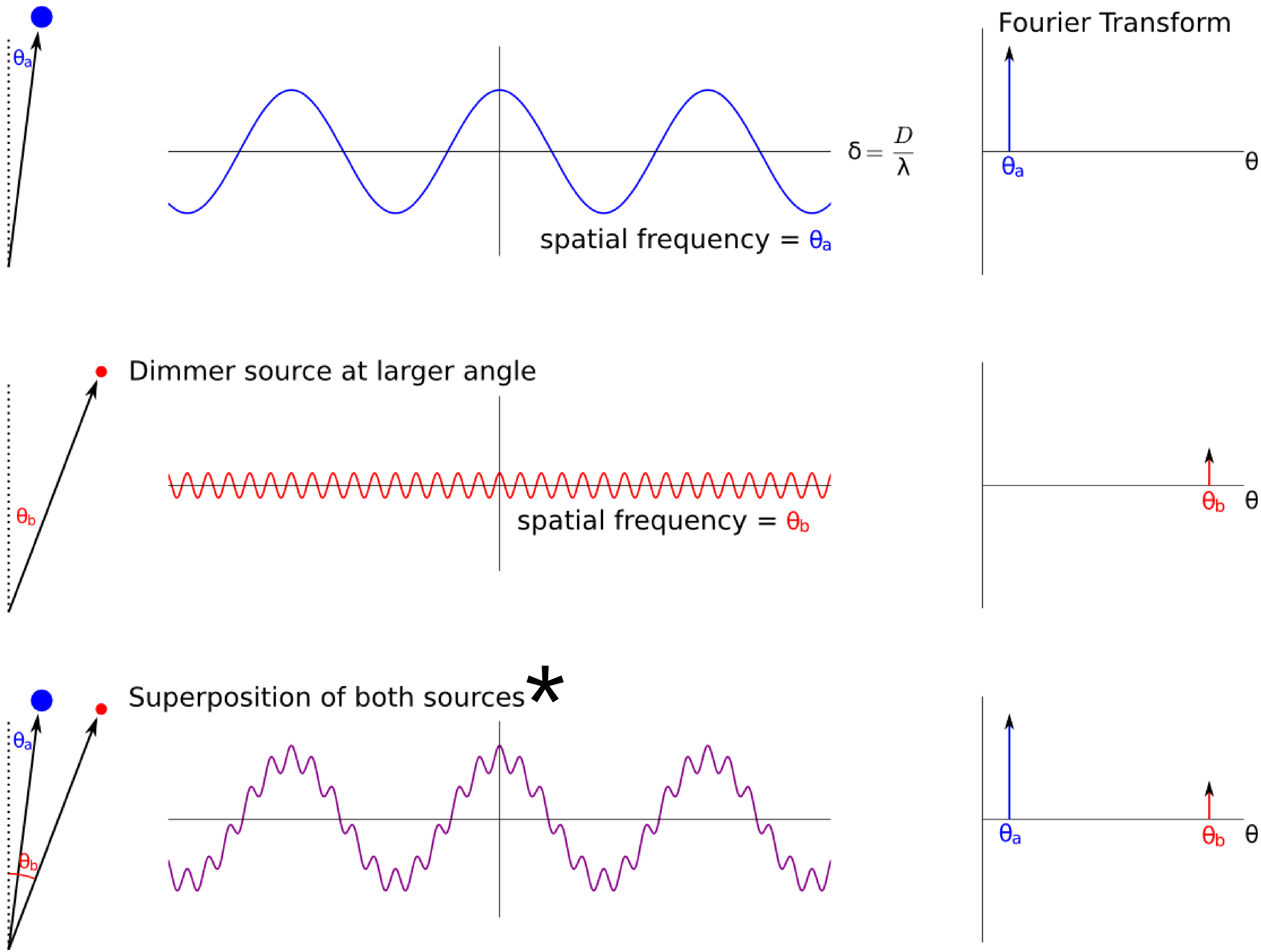


Two Telescopes



Interference Fringes



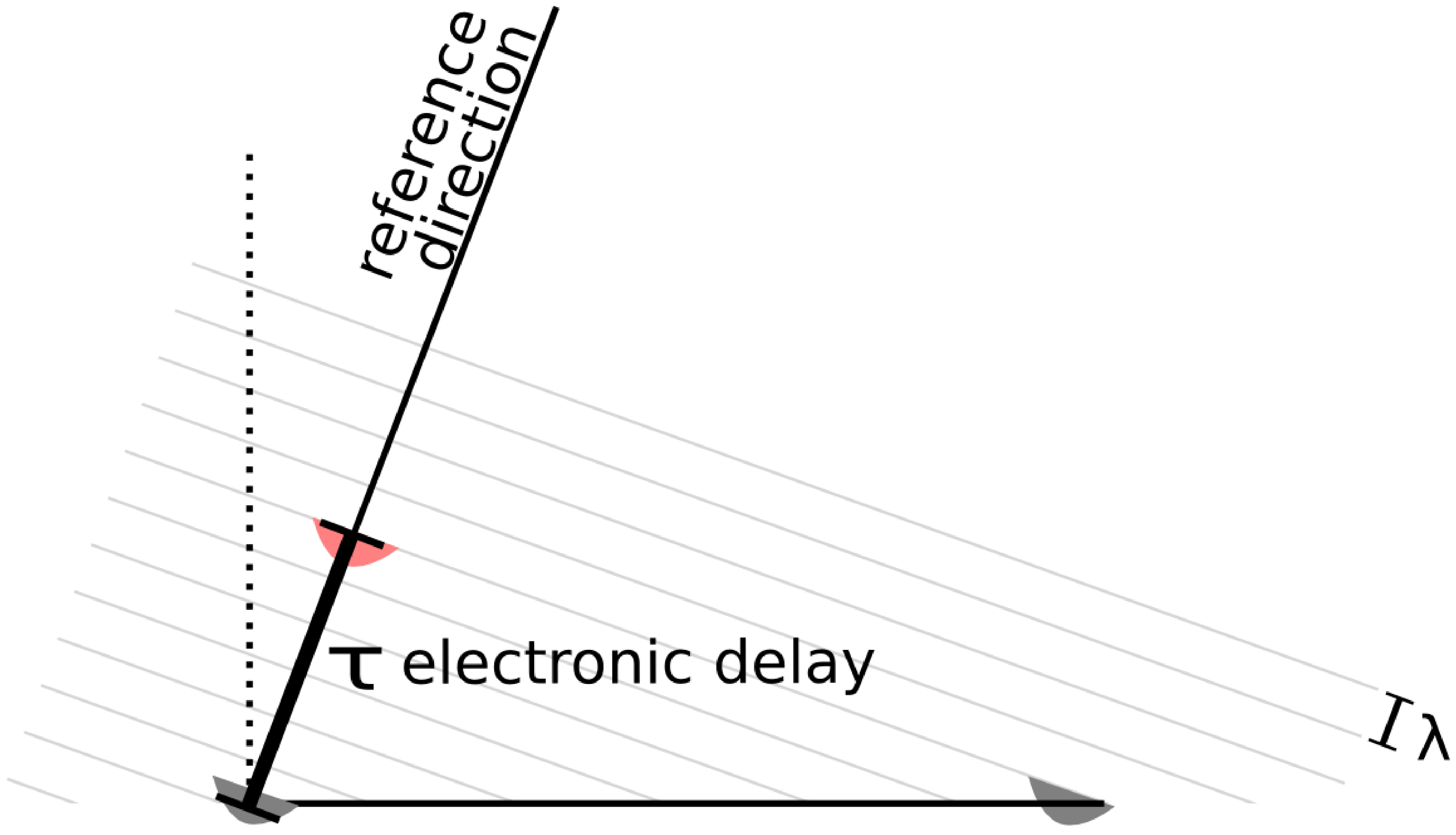


Fourier Transform
θ_a
$\delta = \frac{D}{\lambda}$
spatial frequency = θ_a
θ
Dimmer source at larger angle
θ_b
spatial frequency = θ_b
Superposition of both sources*

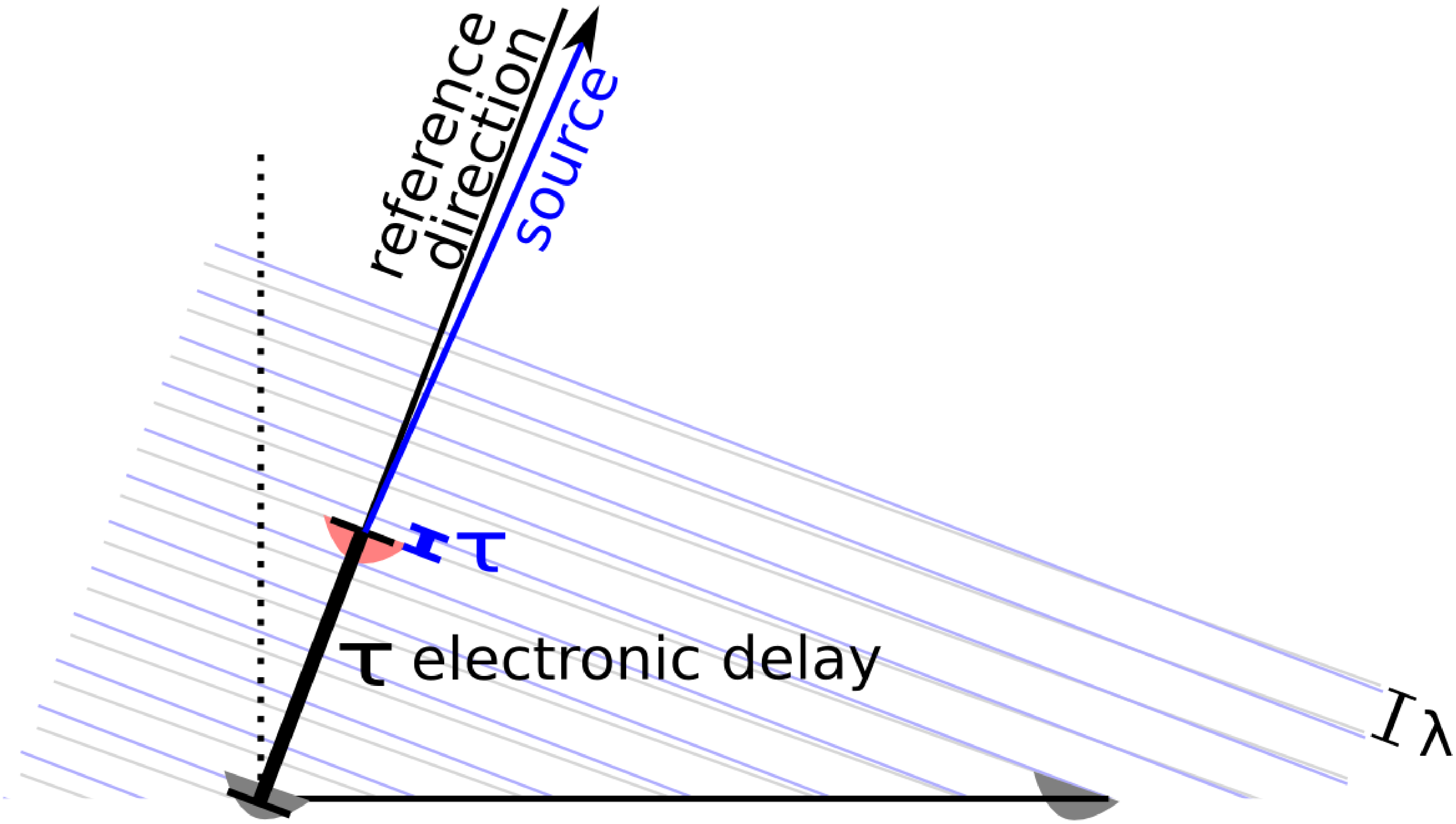
Two Related Questions

- Sources that aren't almost straight up?
- How do you tell left vs right?

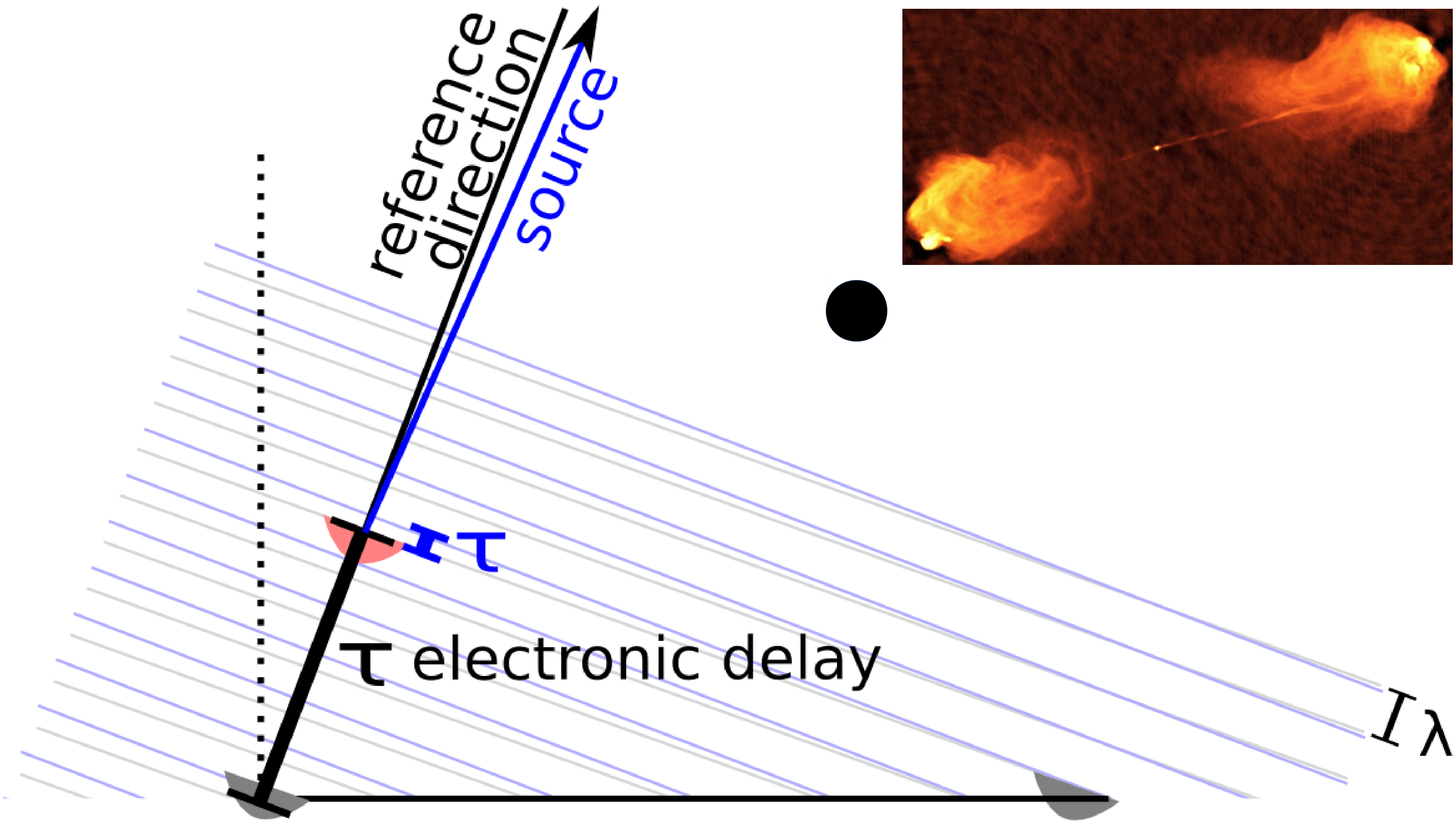
Sources at Realistic Angles



Sources at Realistic Angles

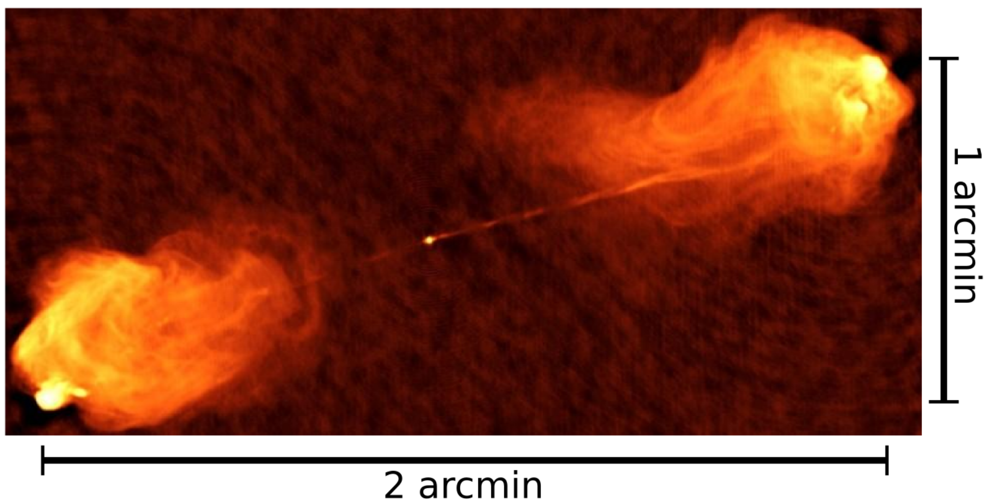


Sources at Realistic Angles



1D version of our example...

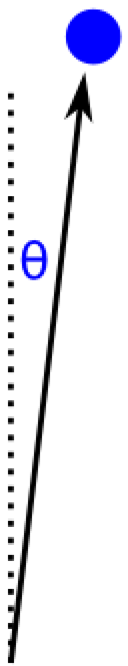
At 5 GHz (6cm) with resolution (pixel size): 0.5 arcsec

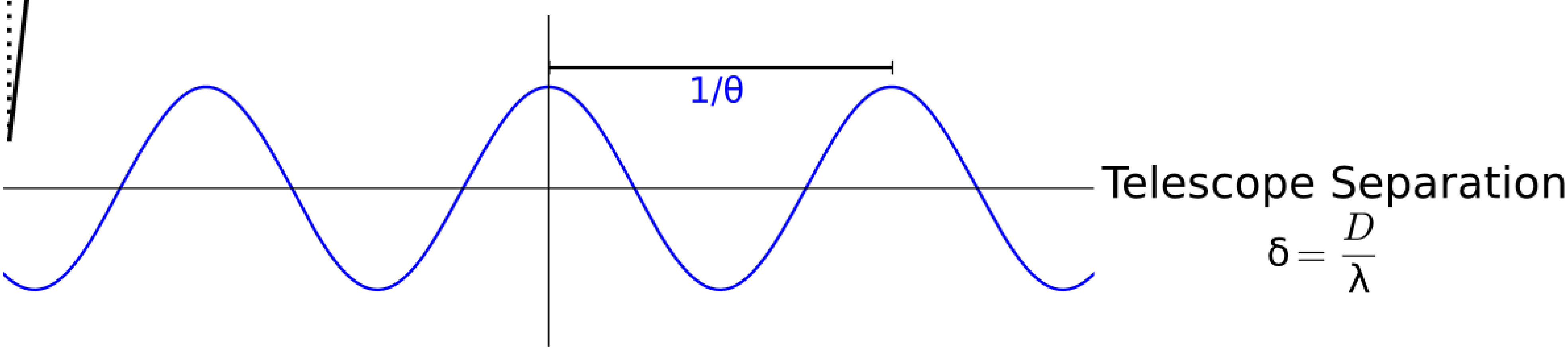


Questions:

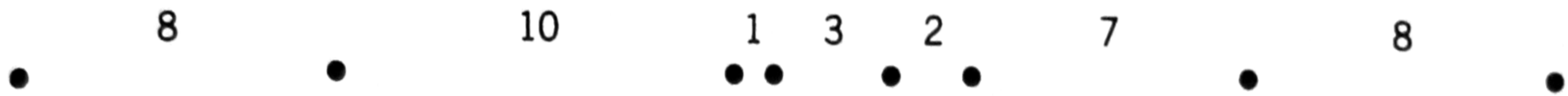
For resolution of 0.5 arcsec, what is smallest D required to see **one** fringe?

For separation of 2 arcmin, how many fringes at this D?

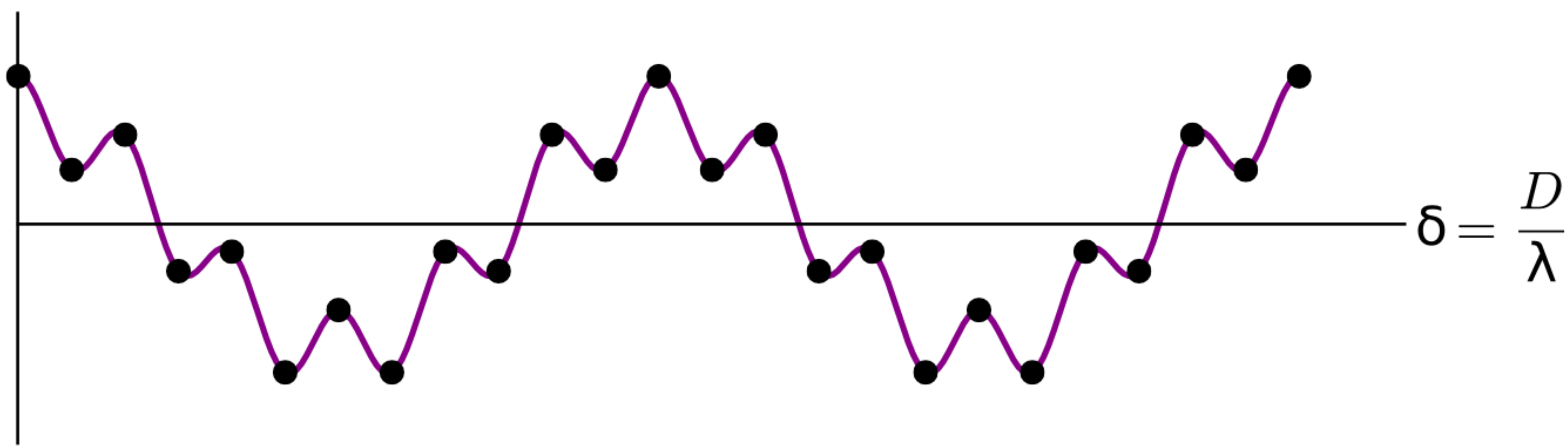




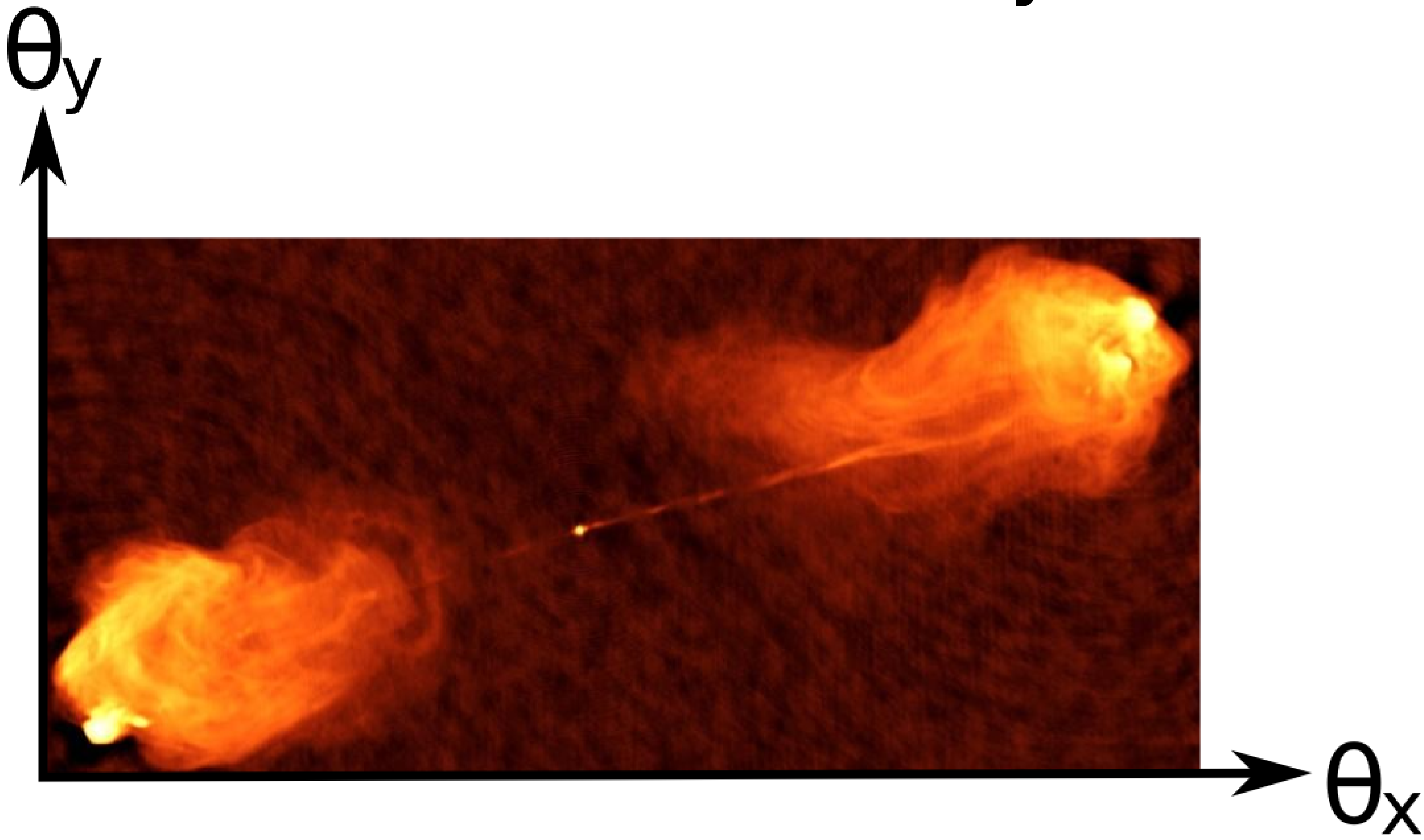
Few Antennas, Many Baselines

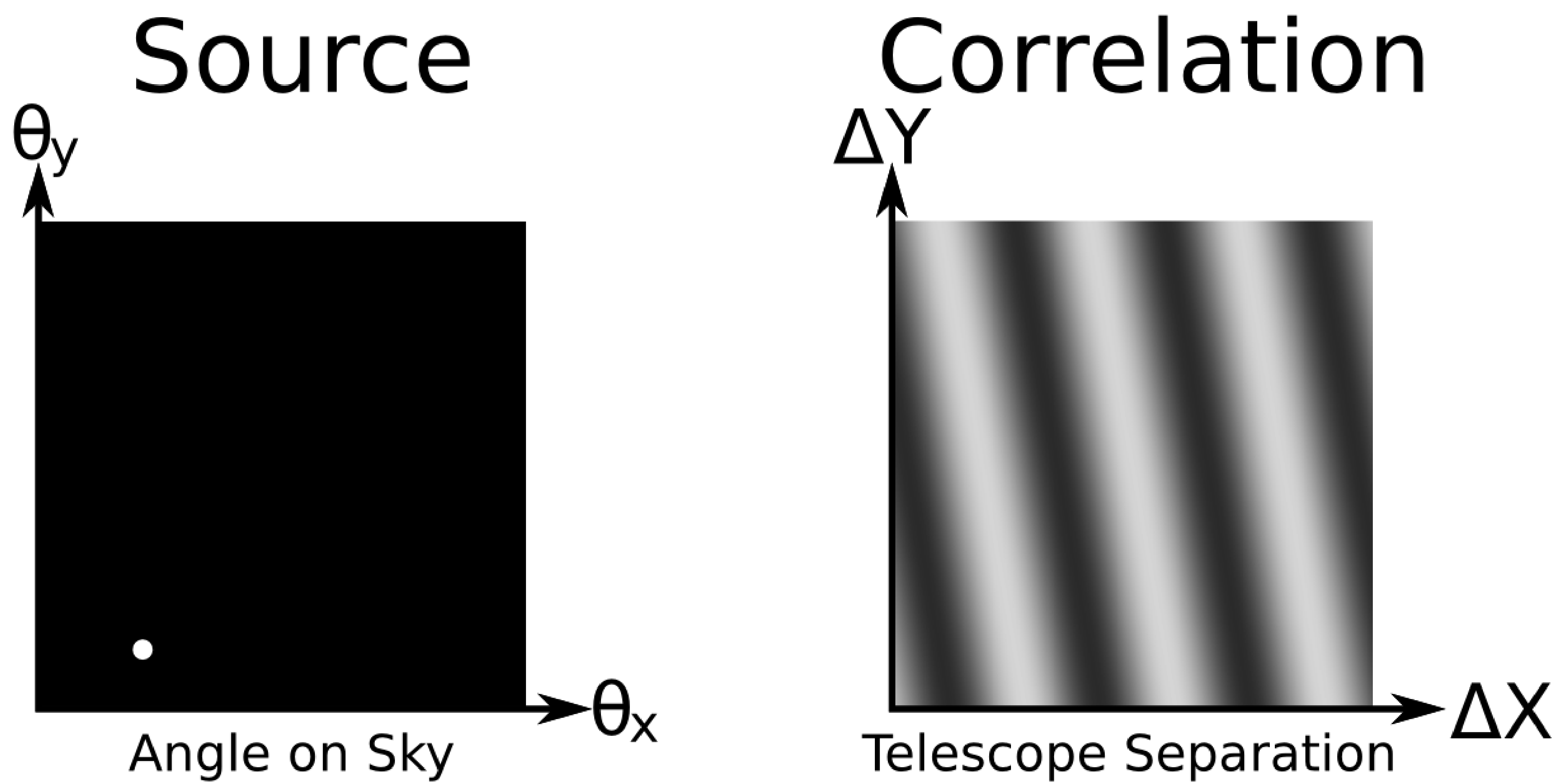


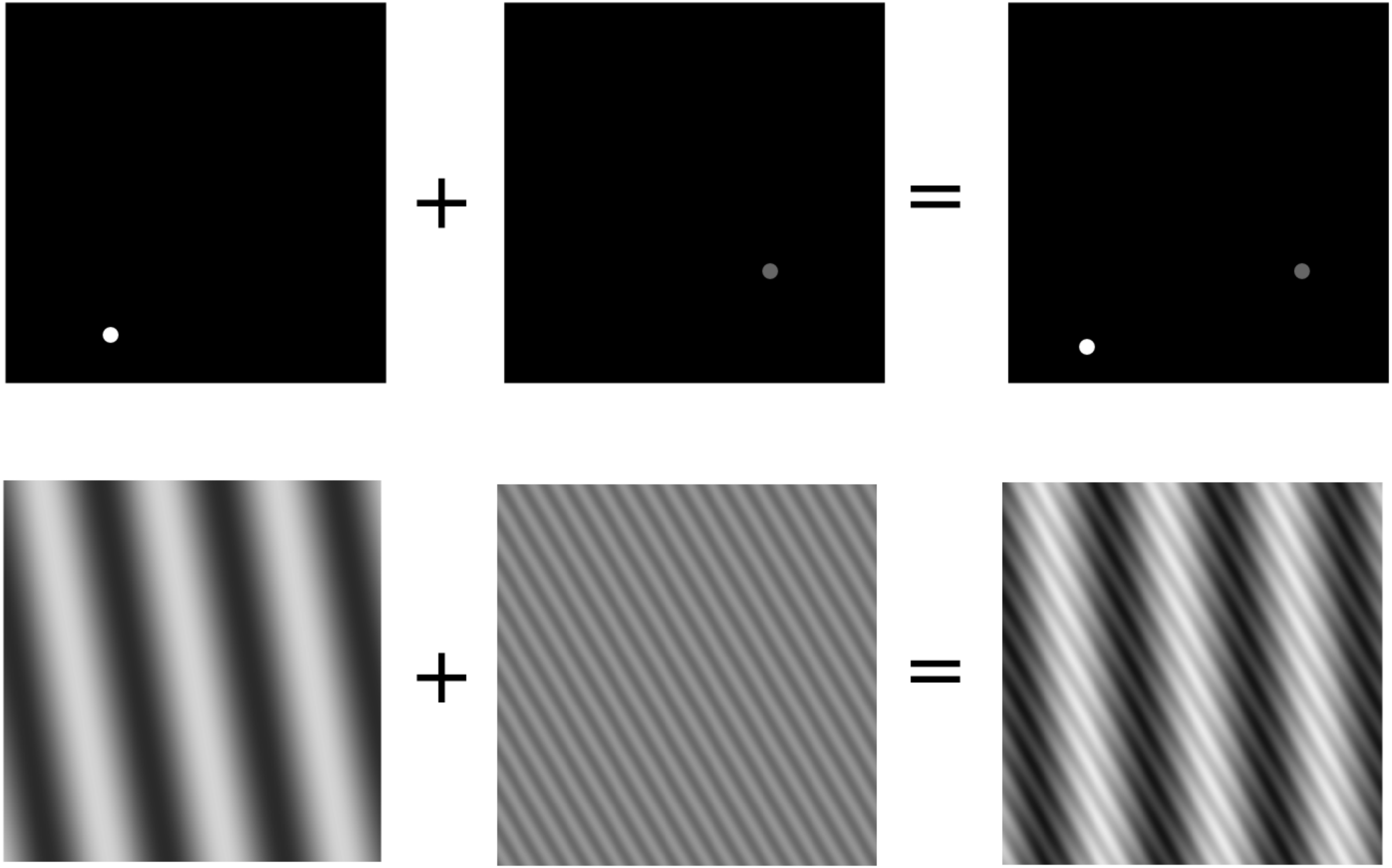
All spacings between 1 and 24 need only 8 antennas!



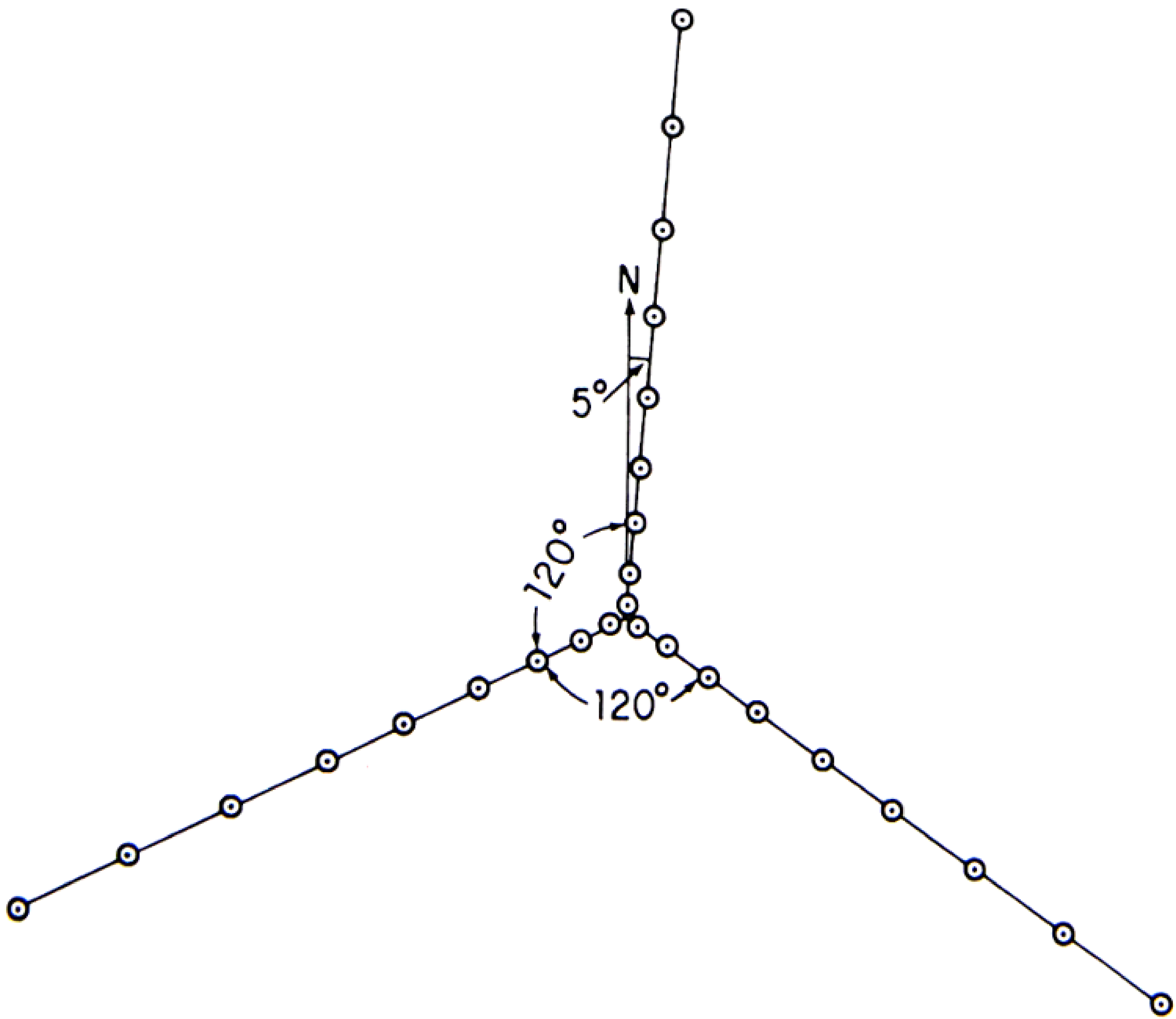
2D Source on Sky



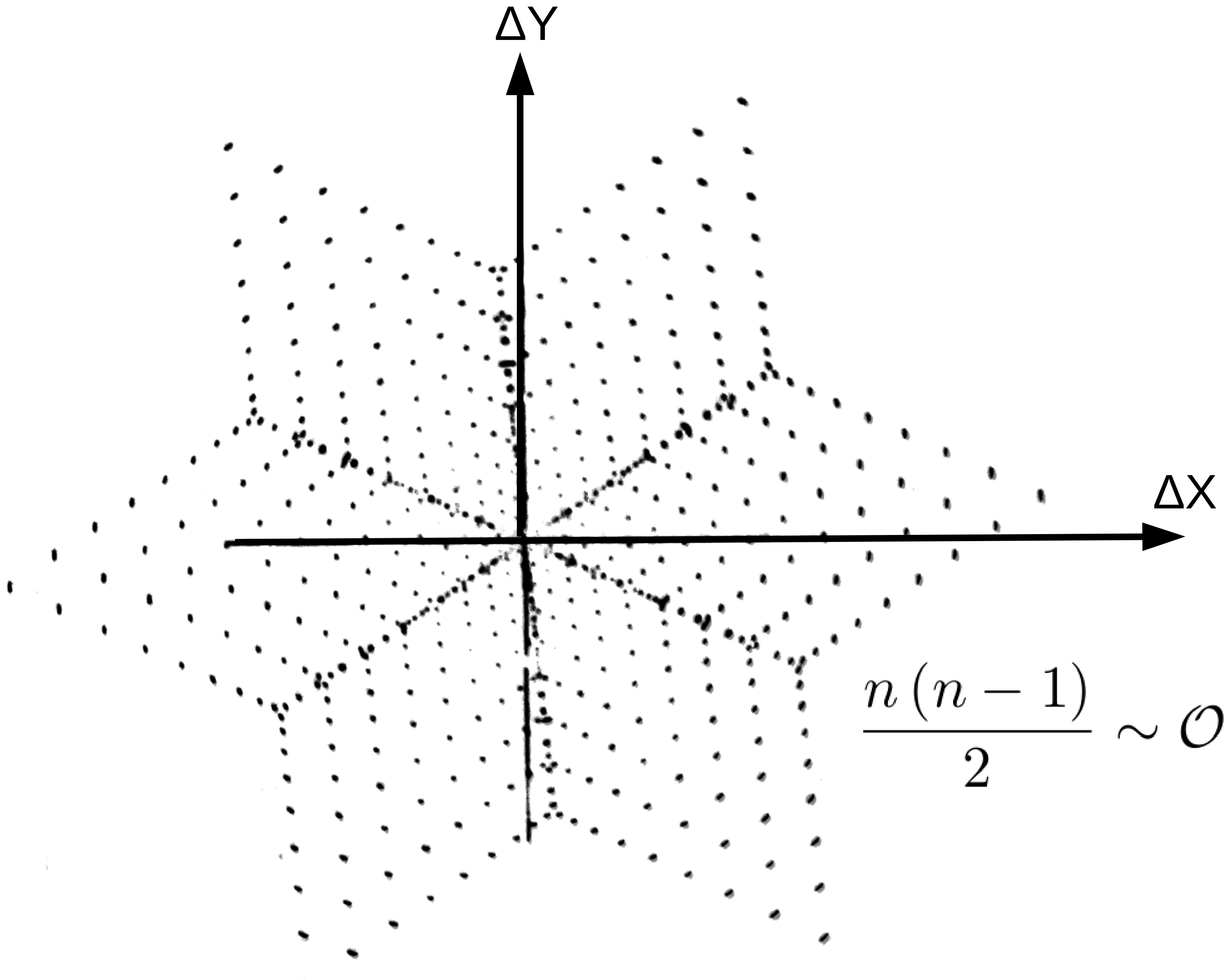

Source
θy
θx
Angle on Sky
Correlation
ΔY
ΔX
Telescope Separation




y
x

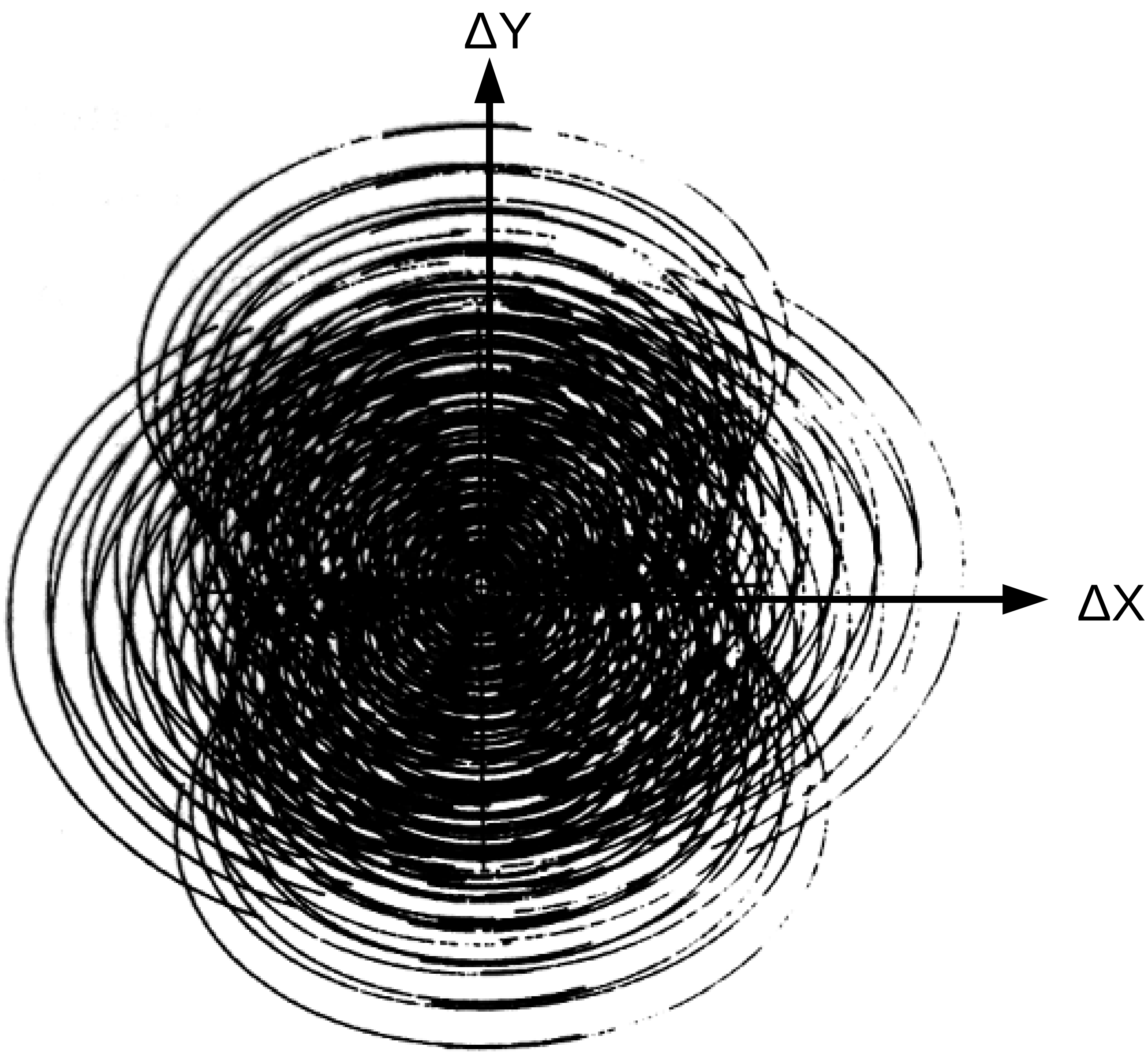

N
5°
120°
120°
120°

Few Antennas, Many Baselines



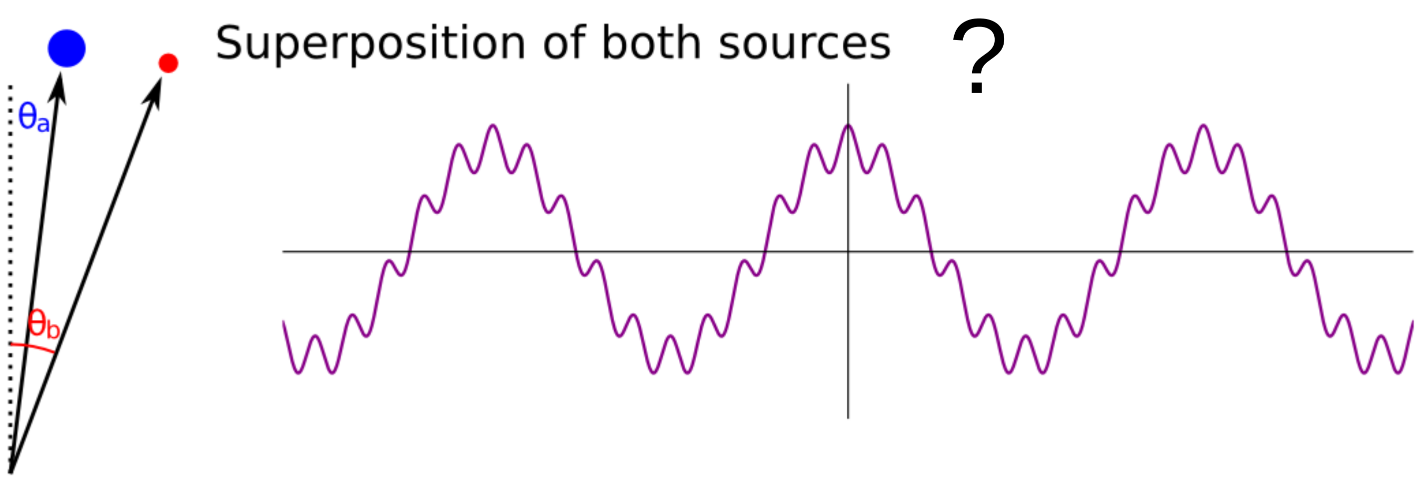
$$\frac{n\,(n-1)}{2} \sim \mathcal{O}(n^2)$$

Earth Rotation



*Superposition

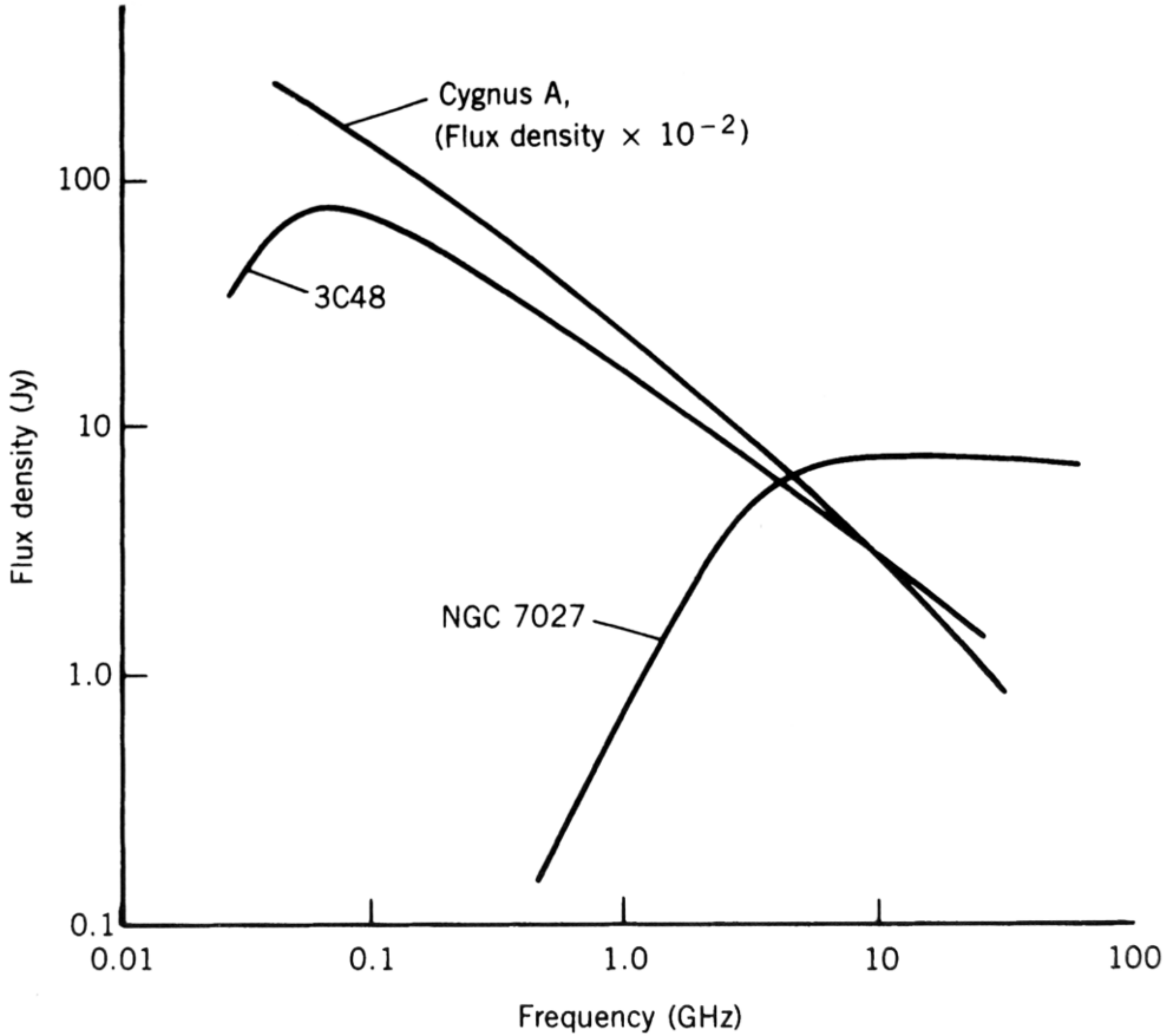
- E-fields and voltages add.
  - But not the “square-and-average” correlation
  - Right? (To the board!)
- So far we considered pure plane waves:
  - sines at a definite frequency (zero bandwidth)



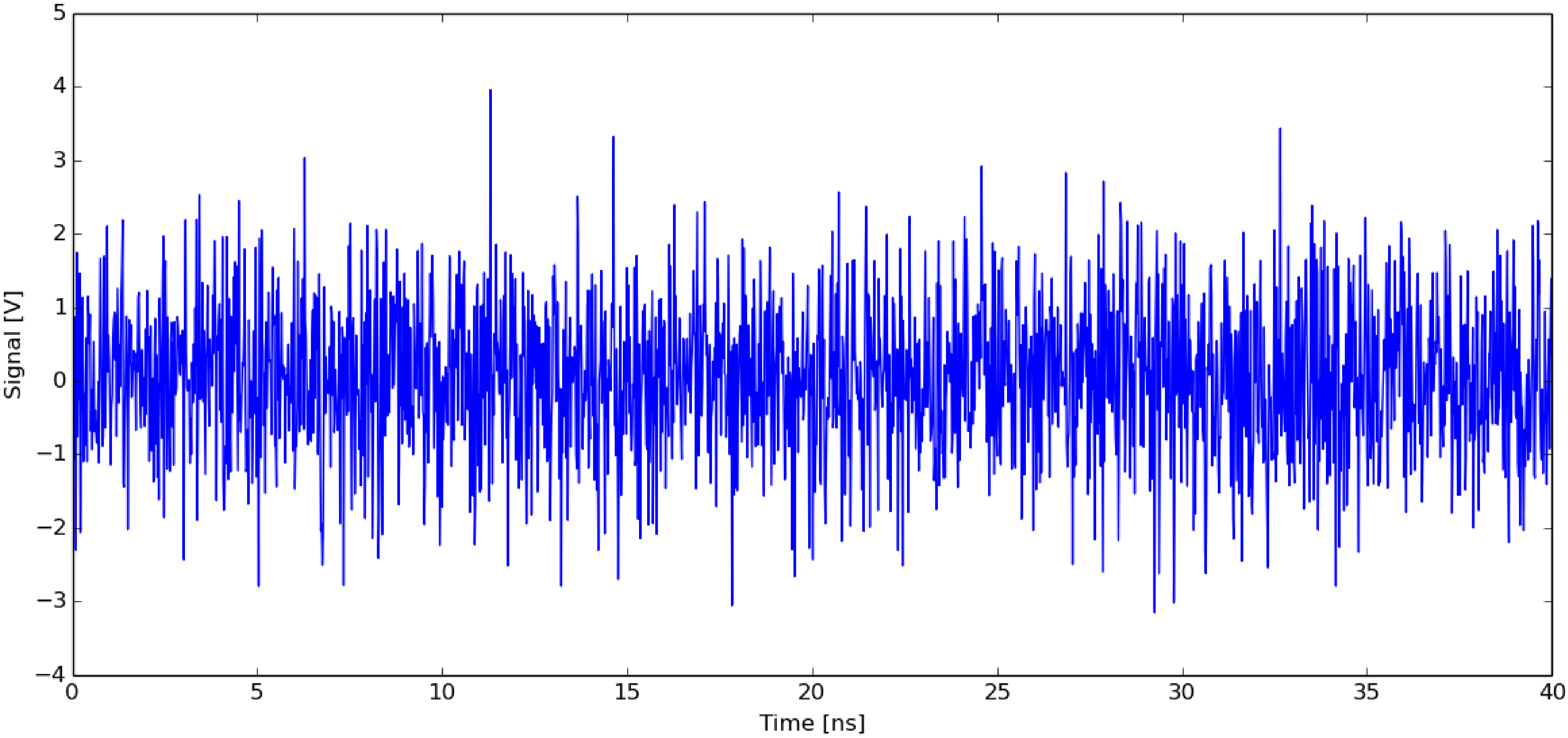
?

$$\delta = \frac{D}{\lambda}$$

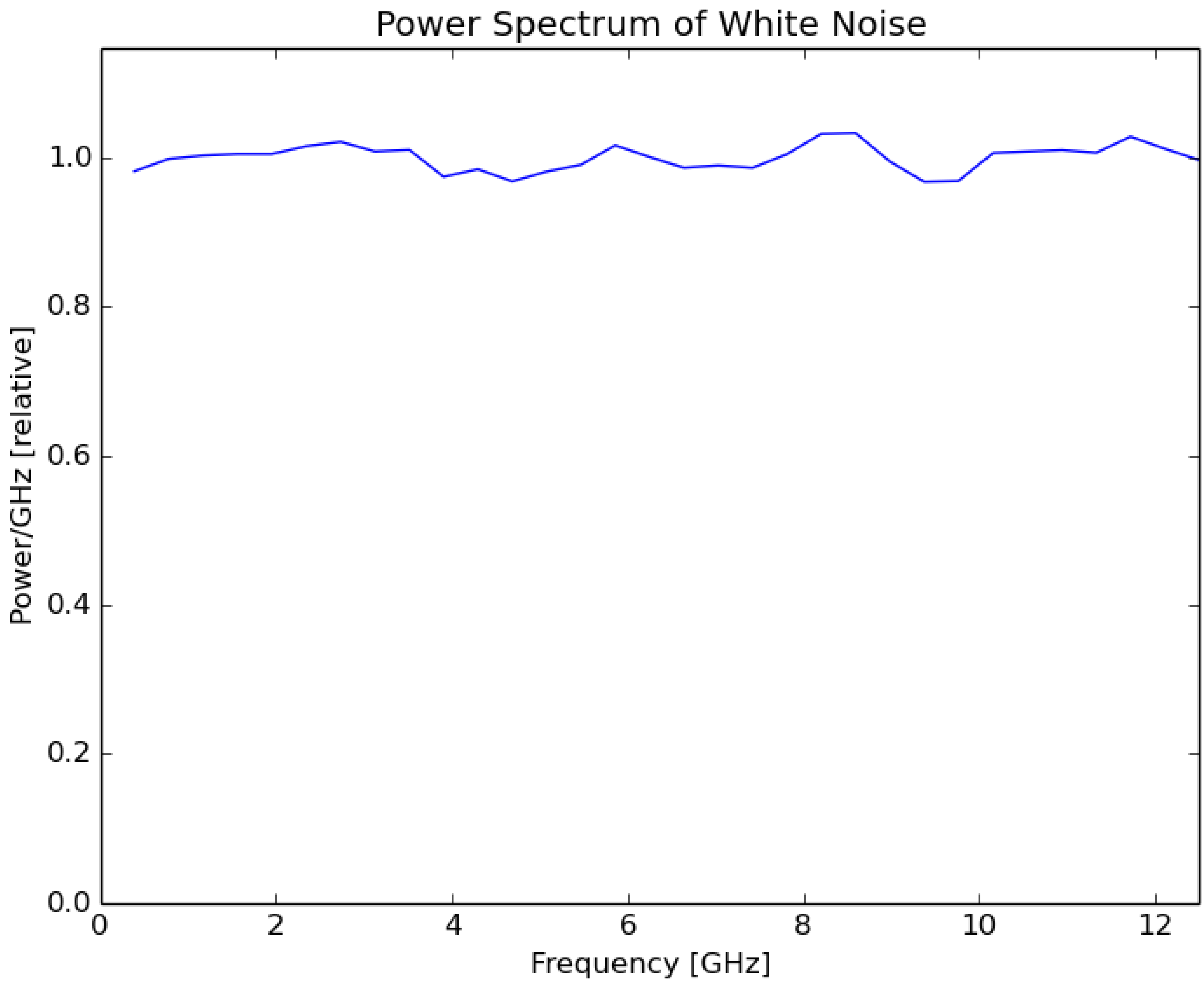
Spectra of Sources



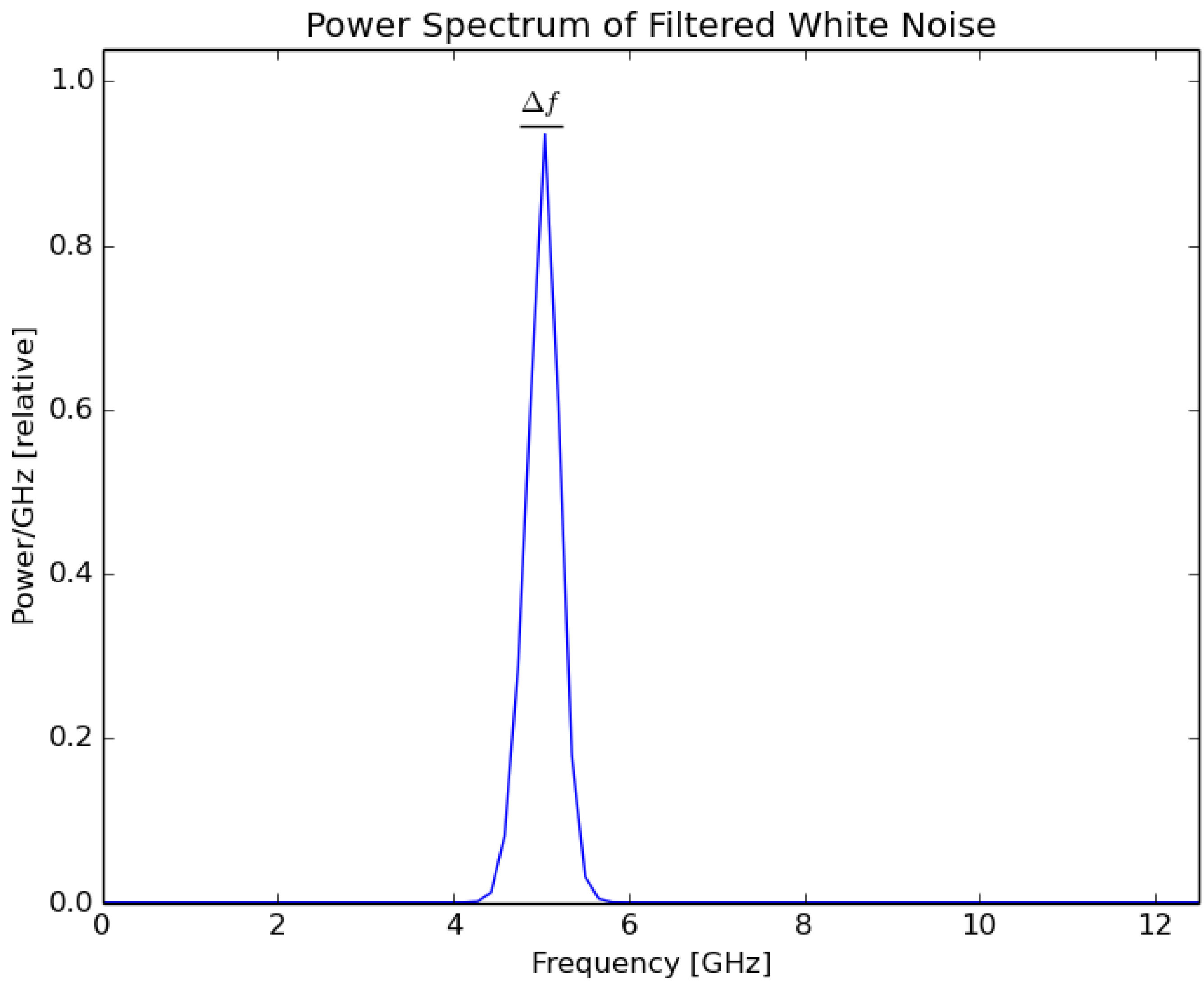
White Noise



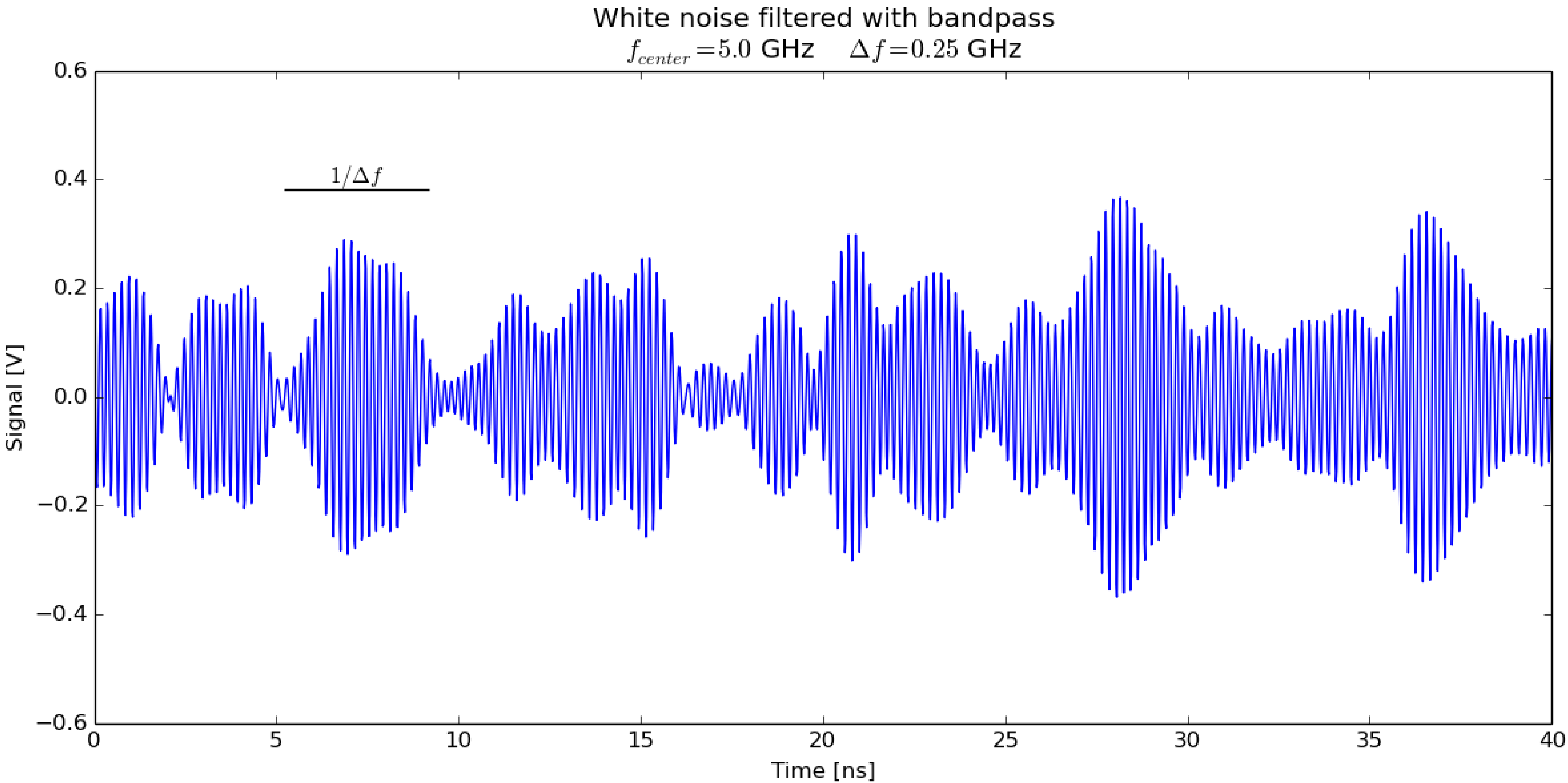
Flat Spectrum



Bandpass Spectrum (Quasi-monochromatic)



Narrow-Band Noise (Quasi-monochromatic Light)

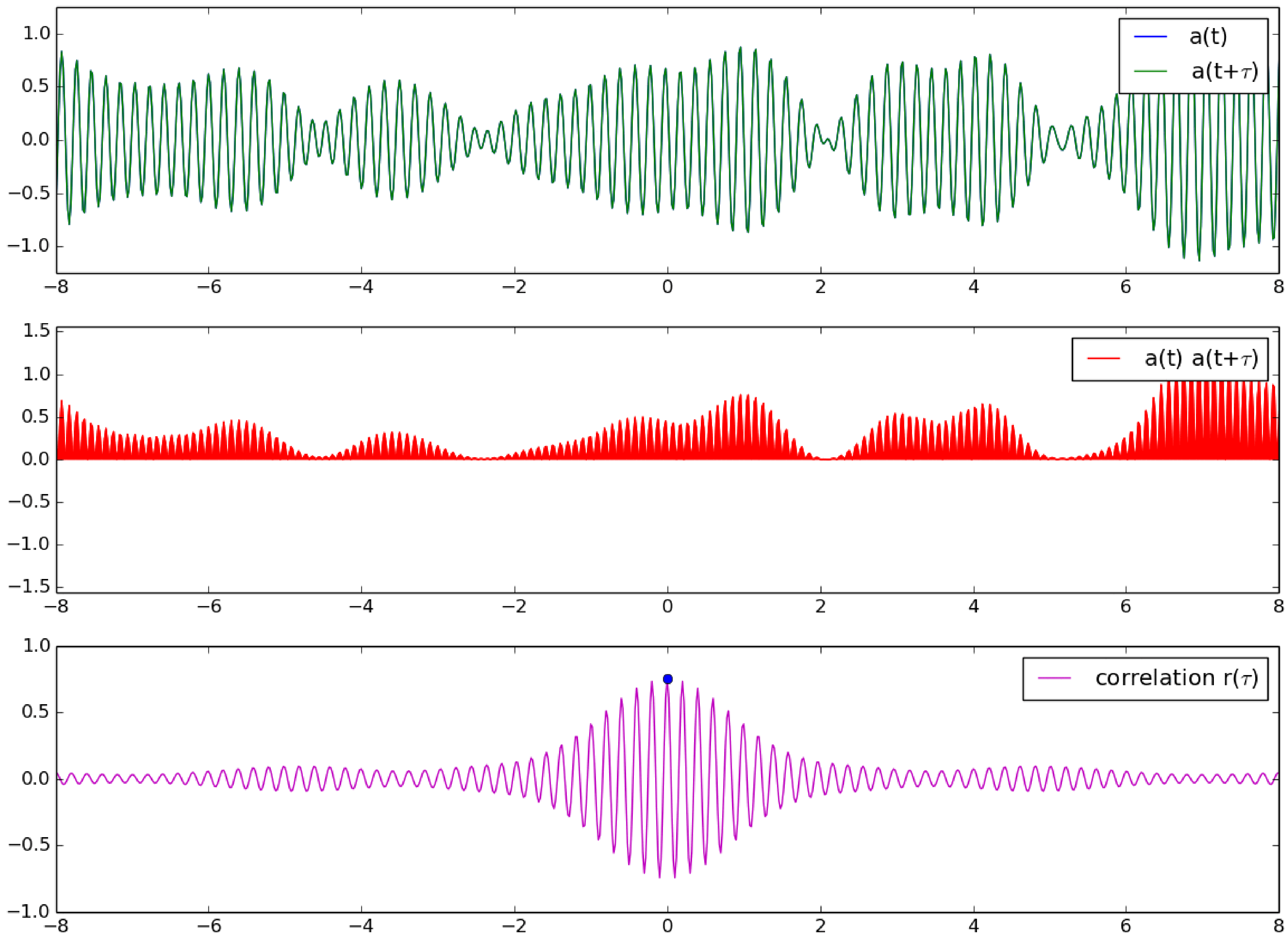


Narrow-Band Noise

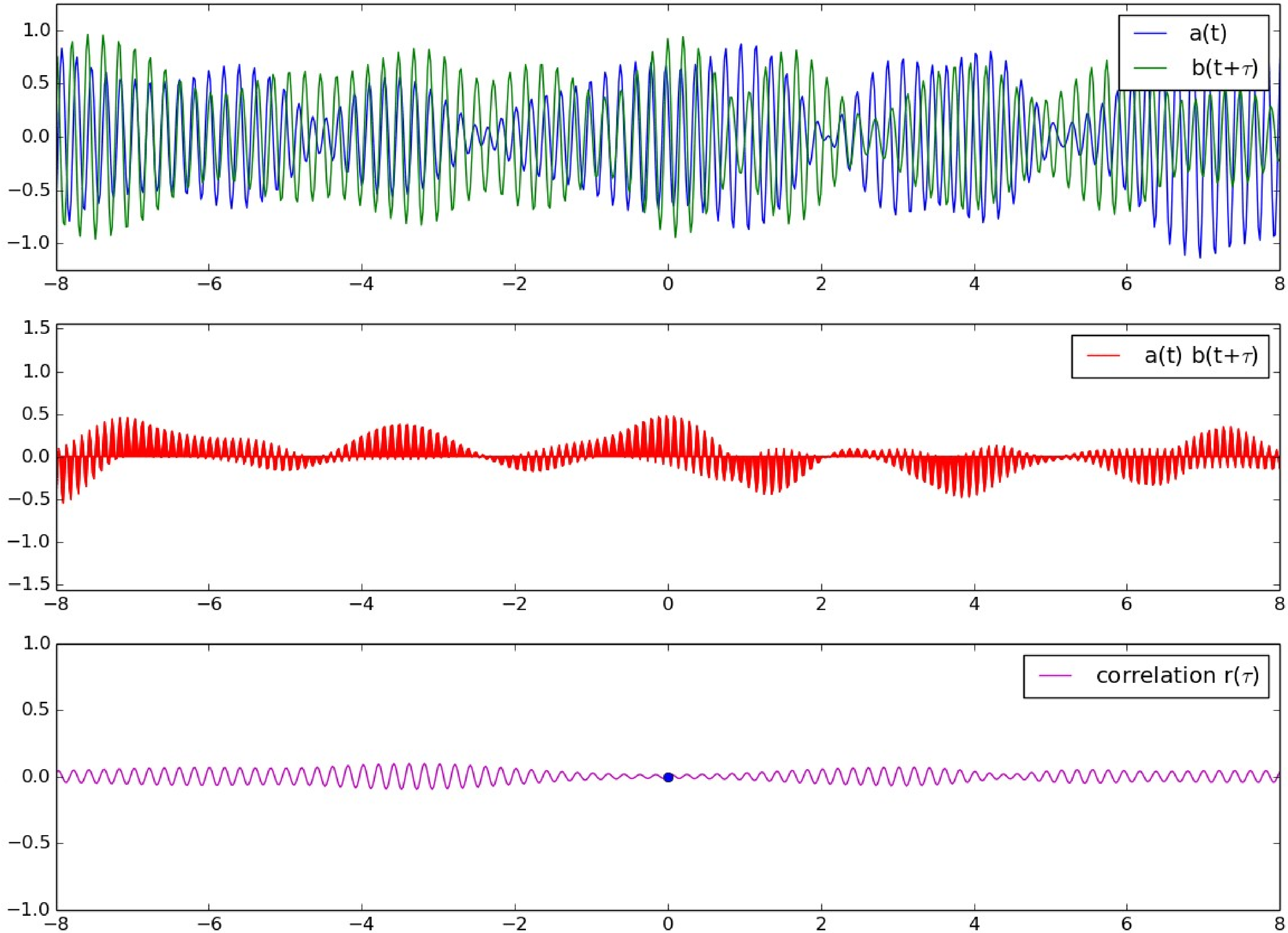
- Average period is ~1/f
- Amplitude constant for t~1/bandwidth
- Similar for phase.

Narrow-Band Correlations

- Auto-correlation
- Cross-correlation



a(t)
a(t+τ)
a(t) a(t+τ)
correlation r(τ)

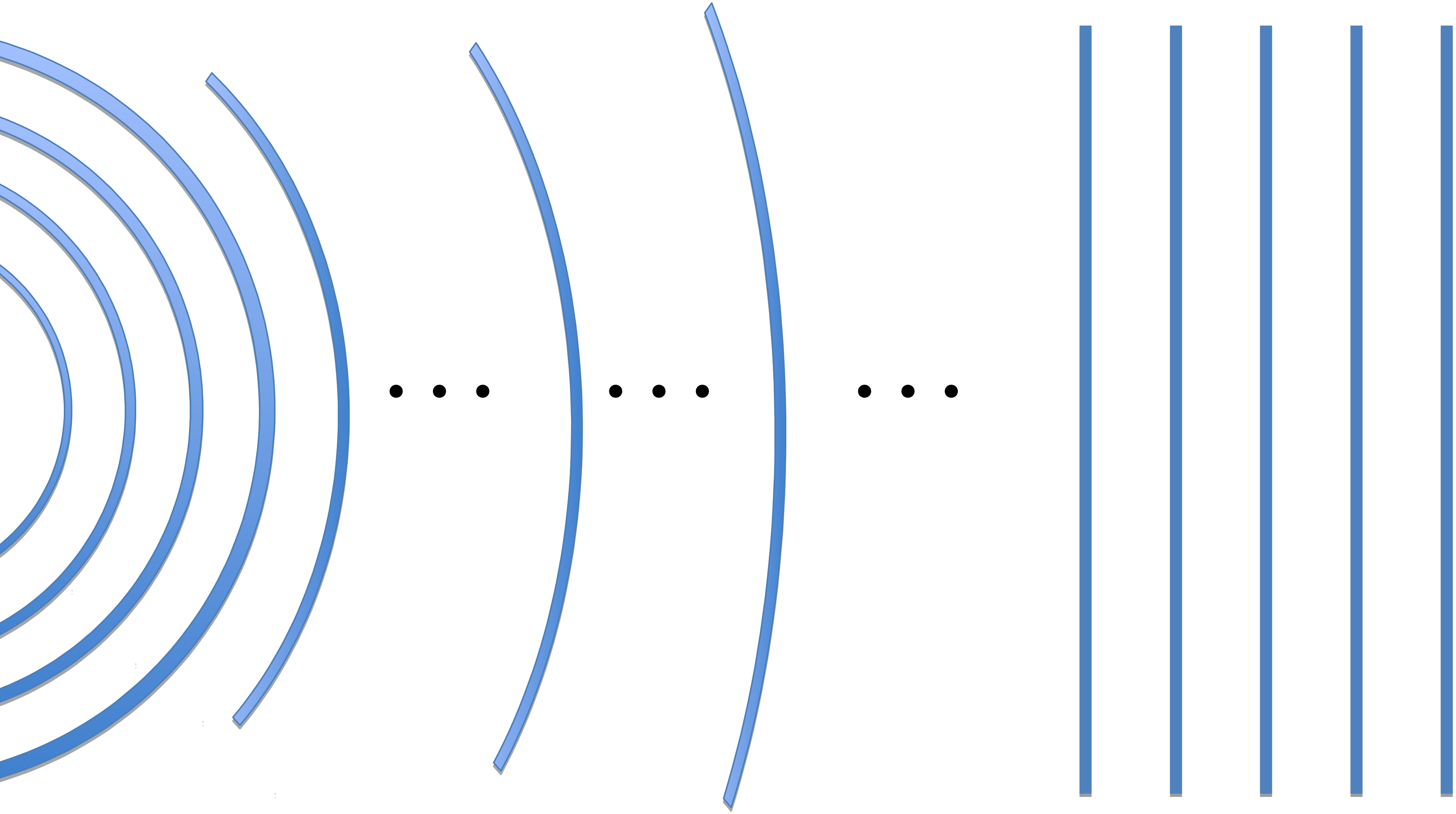


a(t)
b(t+τ)
a(t) b(t+τ)
correlation r(τ)

Review

- Coherence
  - *Spatially incoherent* sources make coherent fields far away.

EM Waves



Review

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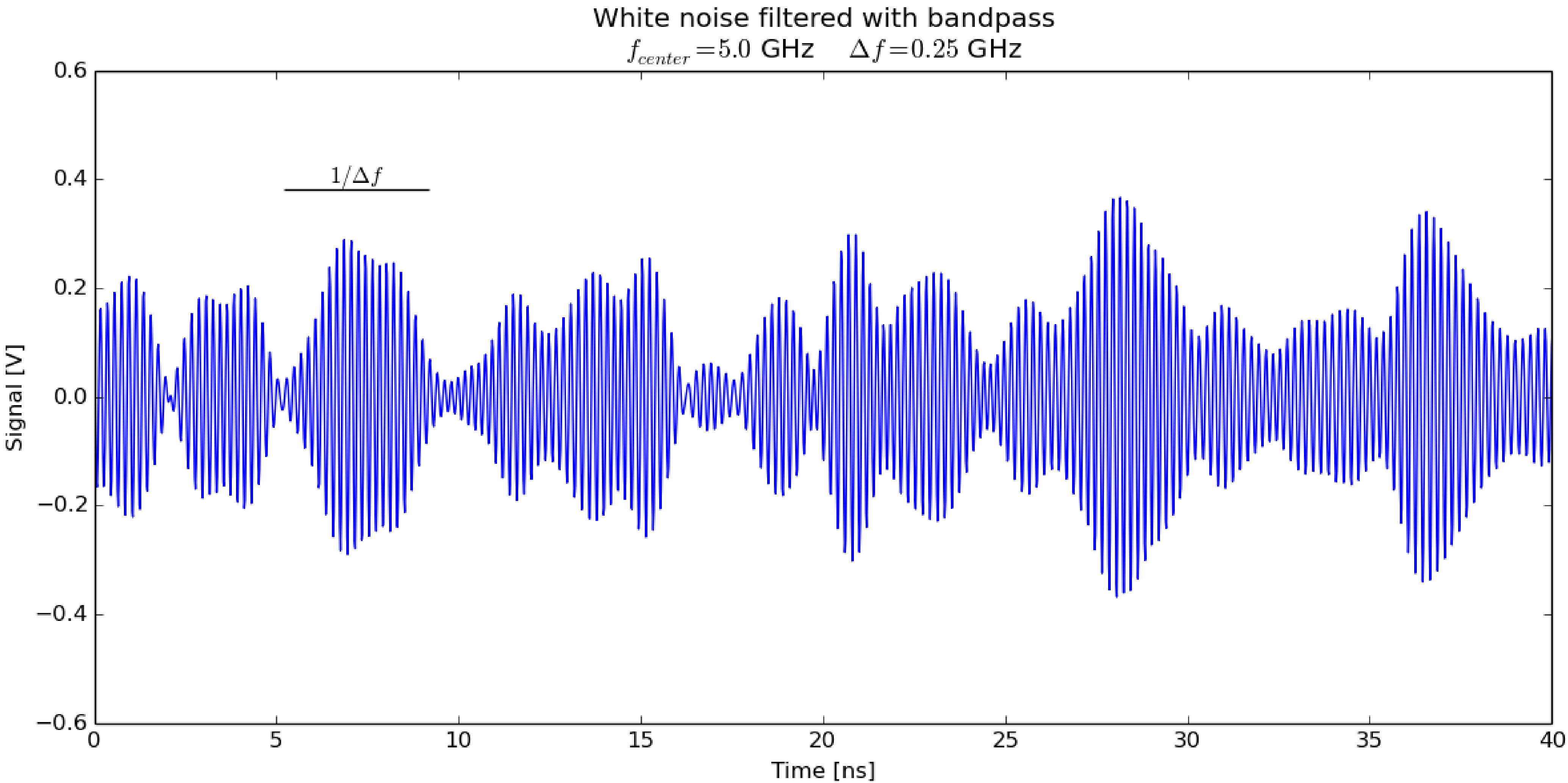
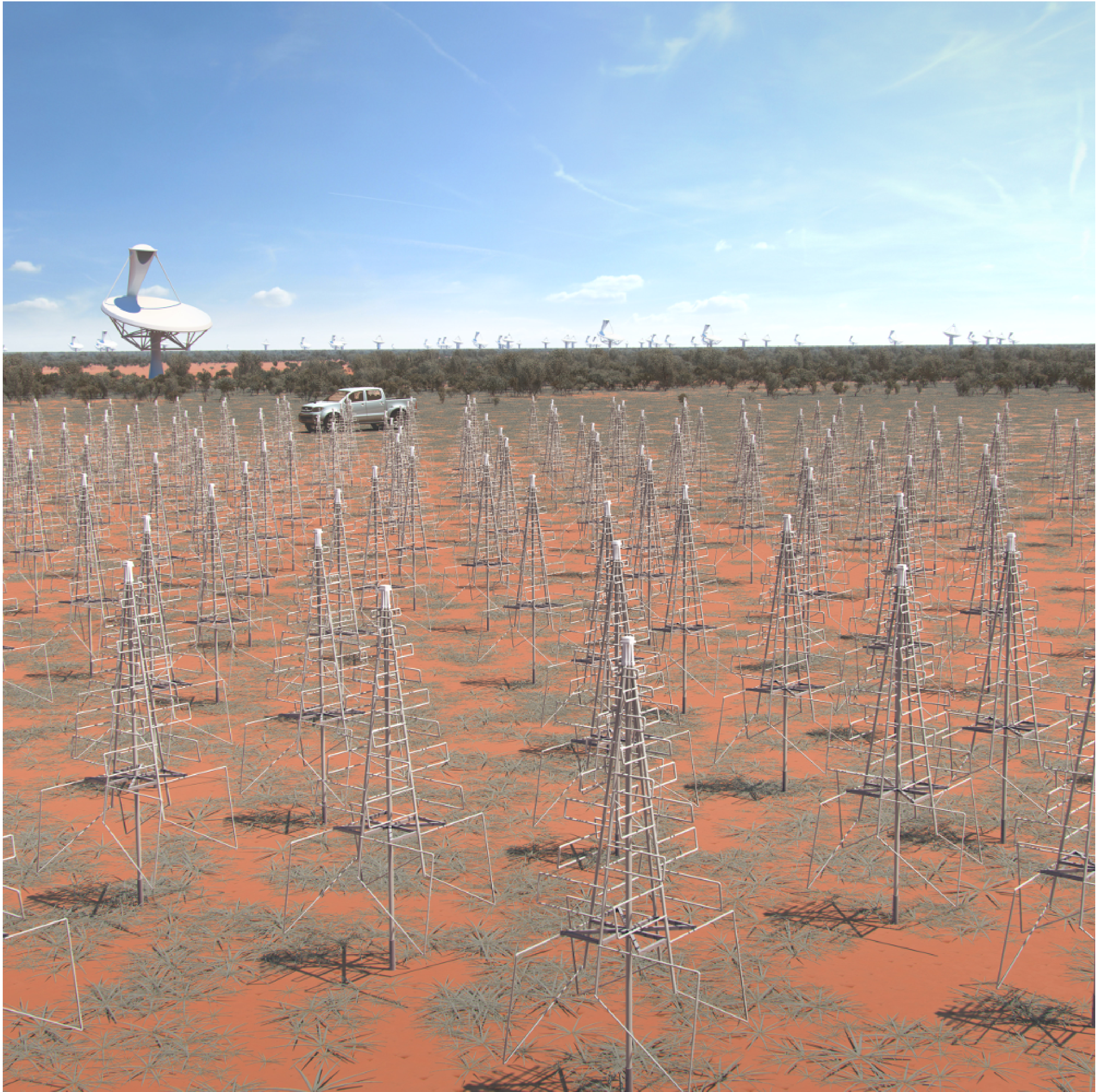
  - *Temporally incoherent* sources become coherent through filtering (even photons!)

Temporal Coherence

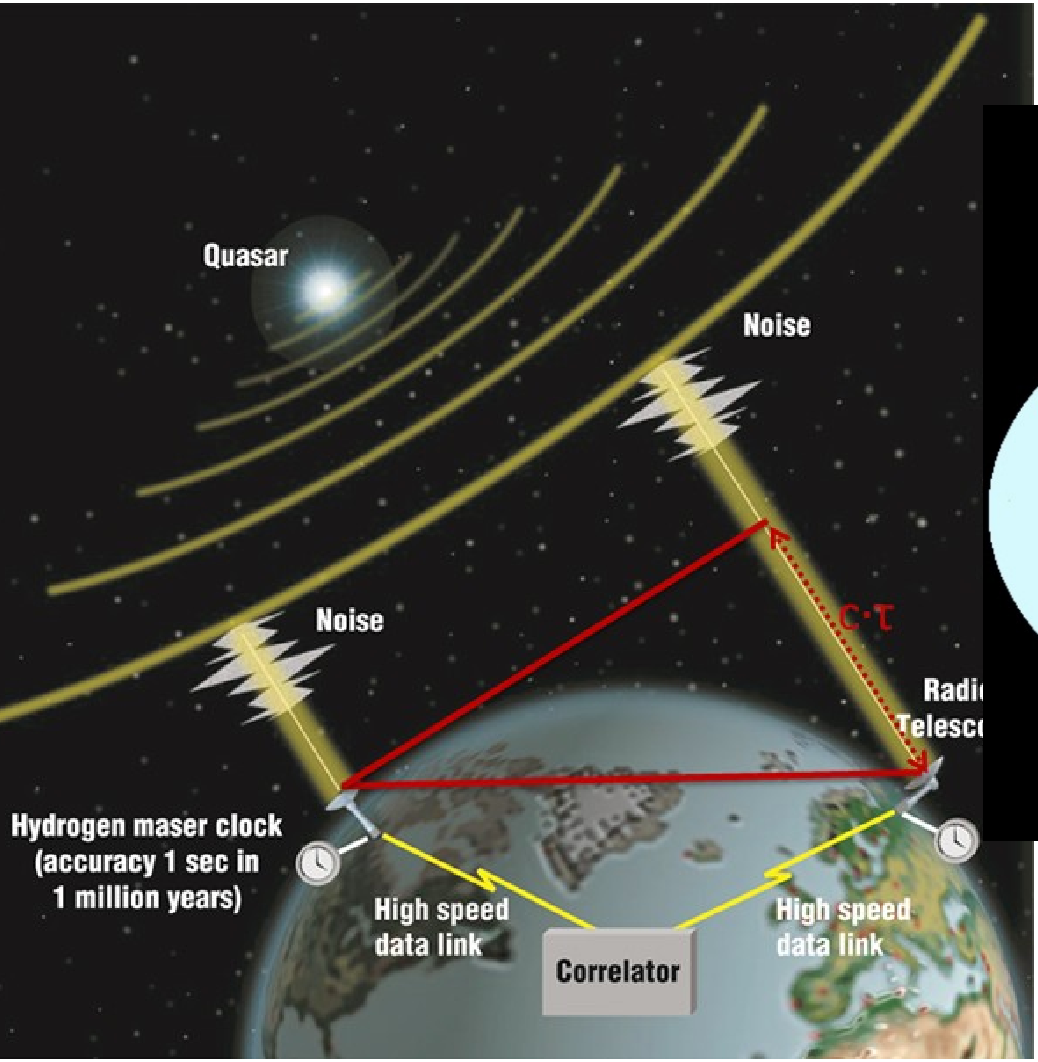


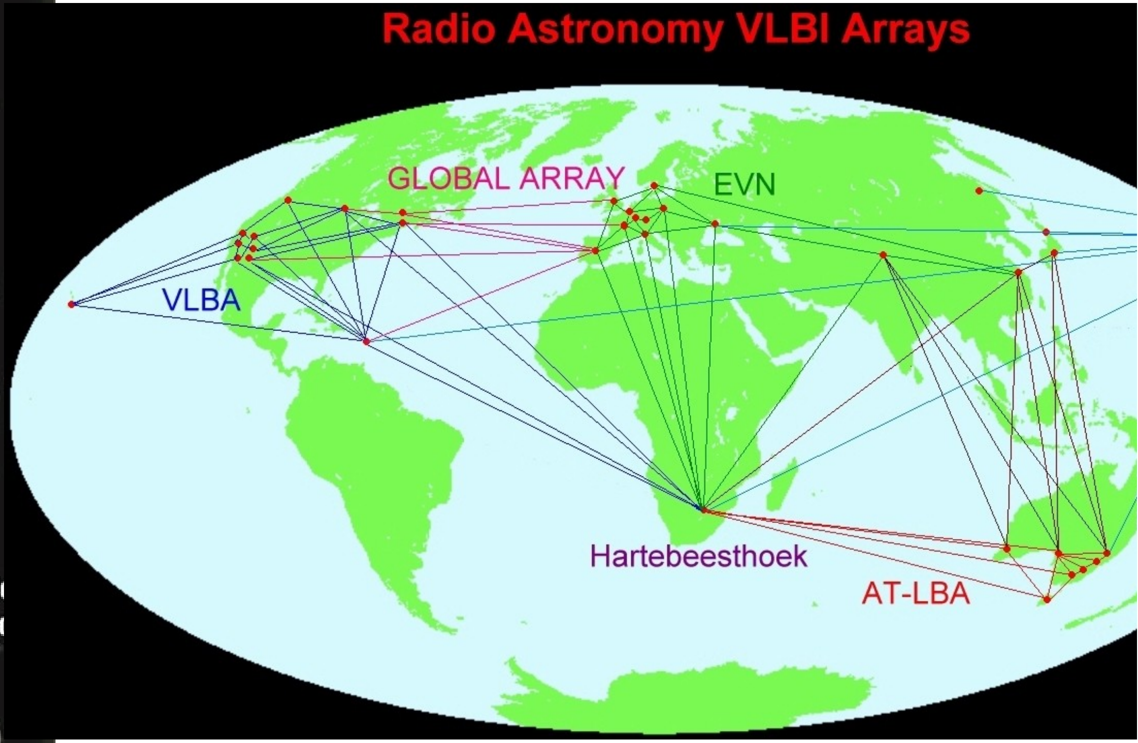
Review

- Coherence
  - *Spatially incoherent* sources make coherent fields far away
  - *Temporally incoherent* sources become coherent through filtering (even photons)
- Interferometry is a novel and efficient way to make images:
  - No lens or mirror necessary!
  - Measure enough pairwise correlations, Fourier Transform
  - “Van Cittert–Zernike theorem”



Very Long Baseline Interferometry





D = 10,000 km
λ = 1 mm
θ_r = 20 μarcsec

Review

- Coherence
  - *Spatially incoherent* sources make coherent fields far away
  - *Temporally incoherent* sources become coherent through filtering (even photons)
- Interferometry is a novel and efficient way to make images:
  - No lens or mirror necessary!
  - Measure enough pairwise correlations, Fourier Transform
  - “Van Cittert–Zernike theorem”
- A *limitation* became key to new technique.

Chem Nobel: Super-Resolved Fluorescence Microscopy