

Chapter 1: A Quick Refresher on Analysis & Probability

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June 22, 2025

Convex Sets and Functions

Definition. A subset $K \subseteq \mathbb{R}^n$ is a convex set if, for any pair of points in K , the line segment connecting these two points is also contained in K , i.e.

$$\lambda x + (1 - \lambda)y \in K \quad \forall x, y \in K, \lambda \in [0, 1].$$

Let $K \subseteq \mathbb{R}^n$ be a convex subset. A function $f : K \rightarrow \mathbb{R}$ is a convex function if

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y) \quad \forall x, y \in K, \lambda \in [0, 1].$$

f is concave if the inequality above is reversed, or equivalently, if $-f$ is convex.

Norms and Inner Products