# Statistical Inference Chapter 3

## Gallant Tsao

## July 8, 2024

1. We first note that the pmf of $X$ is

$$p_X(x) = \frac{1}{N_1 - N_0 + 1}, \ x \in \{N_0, N_0 + 1, ..., N_1\}.$$

   Then we get the expectation to be

$$
\begin{aligned}
\mathbb{E}[X] &= \sum_{x=N_0}^{N_1} x \frac{1}{N_1 - N_0 + 1} \\
&= \frac{1}{N_1 - N_0 + 1} \cdot \frac{N_1 - N_0 + 1}{2}(2N_0 + (N_1 - N_0 + 1 - 1)) \\
&= \frac{N_1 + N_0}{2}.
\end{aligned}
$$

   As for the variance, we get

$$
\begin{aligned}
\mathbb{E}[X^2] &= \sum_{x=N_0}^{N_1} x^2 \frac{1}{N_1 - N_0 + 1} \\
&= \frac{1}{N_1 - N_0 + 1}\Big(\sum_{x=1}^{N_1} x^2 - \sum_{x=1}^{N_0-1} x^2\Big) \\
&= \frac{1}{N_1 - N_0 + 1}\Big(\frac{N_1(N_1+1)(N_1+2) - (N_0-1)(N_0)(2N_0-1)}{6}\Big)
\end{aligned}
$$

   So that

$$
\begin{aligned}
Var(X) &= \mathbb{E}[X^2] - (\mathbb{E}[X])^2 \\
&= 1
\end{aligned}
$$

2. Let $X$ = number of defective parts in the sample. Then
   $X \approx \text{Hypergeometric}(100, n, K)$.

   (a) Firstly, we need $n = 6$ because for the same $K$, increasing $n$ decreases the value
       of the Hypergeometric pmf. Then with $n = 6$,