Statistical Inference Chapter 5

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1. Let X be the number of color blinded people in that population of size n. Then $X \sim \text{Binomial}(n, 0.01)$. We want

$$\begin{aligned} P(X \geq 1) &= 1 - P(X = 0) \\ &= 1 - \binom{n}{0}(0.01)^0(0.99)^n \\ &= 1 - 0.99^n \\ &> 0.95. \end{aligned}$$

   For the inequality above to be true, we need $n > \log_{0.99}(0.05) \implies n \geq 299$.

2. (a)

3. From the definition, we can see that $Y_i \sim \text{Bernoulli}(1 - F(\mu))$. Since $\sum_{i=1}^n Y_i$ is a sum of independent Bernoulli random variables, we can get that

$$\sum_{i=1}^n Y_i \sim \text{Binomial}(n, 1 - F(\mu)).$$

4. (a)

5. Let $Y = \sum_{i=1}^n X_i$. We can see that $\bar{X} = (1/n)Y$ is a scale transformation. Then the pdf of $\bar{X}$ is

$$f_{\bar{X}}(x) = \frac{1}{1/n} f_Y(\frac{x}{1/n}) = n f_X(nx).$$

6. (a)

7. (a)