

## Statistical Inference Chapter 3

Gallant Tsao

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1. We first note that the pmf of  $X$  is

$$p_X(x) = \frac{1}{N_1 - N_0 + 1}, \quad x \in \{N_0, N_0 + 1, \dots, N_1\}.$$

Then we get the expectation to be

$$\begin{aligned} \mathbb{E}[X] &= \sum_{x=N_0}^{N_1} x \frac{1}{N_1 - N_0 + 1} \\ &= \frac{1}{N_1 - N_0 + 1} \cdot \frac{N_1 - N_0 + 1}{2} (2N_0 + (N_1 - N_0 + 1 - 1)) \\ &= \frac{N_1 + N_0}{2}. \end{aligned}$$

As for the variance, we get

$$\begin{aligned} \mathbb{E}[X^2] &= \sum_{x=N_0}^{N_1} x^2 \frac{1}{N_1 - N_0 + 1} \\ &= \frac{1}{N_1 - N_0 + 1} \left( \sum_{x=1}^{N_1} x^2 - \sum_{x=1}^{N_0-1} x^2 \right) \\ &= \frac{1}{N_1 - N_0 + 1} \left( \frac{N_1(N_1 + 1)(N_1 + 2)}{6} - \frac{(N_0 - 1)(N_0)(2N_0 - 1)}{6} \right) \end{aligned}$$

So that

$$\begin{aligned} Var(X) &= \mathbb{E}[X^2] - (\mathbb{E}[X])^2 \\ &= 1 \end{aligned}$$

2. Let  $X$  = number of defective parts in the sample. Then  $X \approx \text{Hypergeometric}(100, M, K)$ .

(a)