Statistical Inference Chapter 3

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1. We first note that the pmf of X is

$$p_X(x) = \frac{1}{N_1 - N_0 + 1},\ x \in \{N_0, N_0 + 1, ..., N_1\}.$$

Then we get the expectation to be

$$\begin{aligned}
\mathbb{E}[X] &= \sum_{x=N_0}^{N_1} x \frac{1}{N_1 - N_0 + 1} \\
&= \frac{1}{N_1 - N_0 + 1} \cdot \frac{N_1 - N_0 + 1}{2}(2N_0 + (N_1 - N_0 + 1 - 1)) \\
&= \frac{N_1 + N_0}{2}.
\end{aligned}$$

As for the variance, we get

$$\begin{aligned}
\mathbb{E}[X^2] &= \sum_{x=N_0}^{N_1} x^2 \frac{1}{N_1 - N_0 + 1} \\
&= \frac{1}{N_1 - N_0 + 1}\Big(\sum_{x=1}^{N_1} x^2 - \sum_{x=1}^{N_0-1} x^2\Big) \\
&= \frac{1}{N_1 - N_0 + 1}\Big(\frac{N_1(N_1 + 1)(N_1 + 2) - (N_0 - 1)(N_0)(2N_0 - 1)}{6}\Big)
\end{aligned}$$

So that

$$\begin{aligned}
Var(X) &= \mathbb{E}[X^2] - (\mathbb{E}[X])^2 \\
&= 1
\end{aligned}$$

2. Let X = number of defective parts in the sample. Then $X \approx \text{Hypergeometric}(100, M, K)$.

   (a)