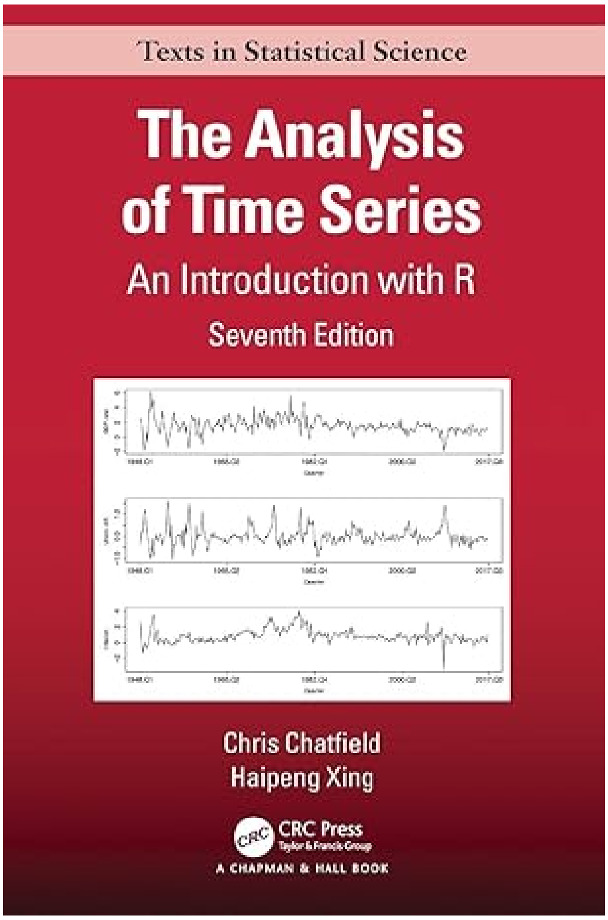
Notes for The Analysis of Time Series Seventh Edition by Chris Chatfield & Haipeng Xin

Gallant Tsao

August 6, 2025



Contents

2 Basic Descriptive Techniques

Statistical techniques for analyzing time series vary from relatively straightforward descriptive techniques to sophisticated inferential techniques. This chapters introduces the former. Descriptive techniques should be tried before attempting more complicated procedures, because they are important in 'cleaning' the data, and then getting a 'feel' for them, before trying to generate ideas as regards to a suitable model.

This chapter focuses on ways of understanding typical time-series effects, such as trend, seasonality, and correlations betyween successive observations.

2.1 Types of Variation

Traditional methods of time-series analysis are mainly concerned with decomposing the variation in a series into components representing trend, seasonal variation and other cyclic changes. Any remaining variation is attributed to 'irregular' fluctuations. This approach is not always the best but is particularly valuable when the variation is dominated by trend and seasonality.

Seasonal variation Many time series, such as sales figures and temperature readings, exhibit variation that is annual in period. For example, unemployment is typically 'high' in winter but low in summer. This yearly variation is easy to understand, and can readily be estimated if seasonality is of direct interest. Alternatively, seasonal variation can be removed from the data to give deseasonalized data.

Other cyclic variations Apart from seasonal effects, there are some other variations of time series at a fixed period due to some other physical cause. For example, daily variations in temperature depend on what time of the day it is.

Trend This may be loosely defined as 'long-term change in the mean level'. However, this largely depends how we define the term 'long-term'. It can be a year to even 50 years depending on the span of the time series.

Other irregular fluctutations After trend and cyclic variations have been removed from a set of data, we are left with residuals that may or may not be 'random'. We'll examine later whether this can be explained in terms of probability models, such as the moving average (MA) or autoregressive (AR) models.

2.2 Stationary Time Series

Broadly speaking a time series is said to be stationary if there is no systematic change in mean (no trend), if there is no systematic change in variance and if strictly periodic variations have been removed. In other words, the properties of one section of the data are much like those of any ohter section.

Much of the probability theory of time series is concerned with stationary time series, and for this reason time series analysis often requires one to transform a non-stationary time series to a stationary one so as to use the theory. For example, we can remove the trend and seasonality and model the resuals by a stationary stochastic process. However, sometimes we may be more interested in the non-stationary components too!

2.3 The Time Plot

Ths first, and most important, step in any time-series analysis is to plot the observations against time. This grap, called the time plot, will show up important features of the series such as trend, seasonality, outliers and discontinuities.

Plotting a time series is not as easy as is sounds. The choice of scales, siz eof intercept, way that points are plotted may substantially affect the way the plot looks. Not all computer software plots the time series well, so we'll have to adjust manually sometimes.

2.4 Transformations

Plotting the data may suggest that it is reasonable to transform them by, for example, taking logs or square roots. The three main reasons for making a transformation are as follows:

(i) To stabilize the variance

If there is a trend in the series and the variance appears to increase with the mean, they it may be advisable to transform the data. In particular, if the standard deviation is directly proportional to the mean, a logarithmic transformation is indicated. On the other hand, if the variance changes through time without a trend being present, then a transformation will not help.

(ii) To make the seasonal effect additive

If there is a trend in the series and the size of the seasonal effect appears to increase with the mean, then it may be advisable to transform the data so as to make the seasonal effect constant from year to year. The seasonal effect is then said to be additive. In particular, if the size of the seasonal effect is directly proportional to the mean, then the seasonal effect is said to be multiplicative and a logarithmic transformation is appropriate to make the effect additive. However, this transformation will only stabilize the variance if the error term is also thought to be multiplicative (see Section 2.6), a point that is sometimes overlooked.

(iii) To make the data normally distributed

Model building and forecasting are usually carried out on the assumption that the data are normally distributed. In practice this is not necessarily the case; there may, for example, be evidence of skewness in that there tend to be 'spikes' in the time plot that are all in the same direction (either up or down). This effect can be diffcult to eliminate with a transformation and it may be necessary to model the data using a different 'error' distribution.

The logarithmic and square root transformations above are special cases of a general class of transformations called the Box-Cox transformation. Given an observed time series $\{x_t\}$ and a transformation parameter λ, the transformed series is given by

$$y_t = \begin{cases} (x_t^\lambda - 1)/\lambda & \text{if } \lambda \neq 0, \\ \log(x_t) & \text{if } \lambda = 0. \end{cases}$$

The best value of λ is usually 'guesstimated'.

However, Nelson and Granger (1979) found that there is little improvement in forecast performance when a general Box–Cox transformation was tried on a number of series. There are cases where the transformation fails to stabilize the variance. Usually, transformations should be avoided wherever possible except where the transformed variable has a direct physical interpretation. For example, when percentage increases are of interest, then taking logarithms makes sense.

2.5 Analyzing Series that Contain a Trend and No Seasonal Variation

The simplest type of trend is the familiar 'linear trend + noise', for which the observation at time t is a random variable X_t given by

$$X_t = \alpha + \beta t + \varepsilon_t,$$

where α, β are constants and ε_t denotes a random error term with zero mean. The mean level at time t is given by $m_t = \alpha + \beta t$; which is some times called the 'trend term'. However, some authors denote β as the trend so it depends on the context.

The trend above is a deterministic function of time and is sometimes called a global linear trend. In practice, this generally provides an unrealistic model, and nowadays there is more emphasis on models that allow for local linear trends. One possibility is to fit a piecewise linear model where the trend line is locally linear but with change points where the slope and intercept change (abruptly). We can also assume that α, β evolve stochastically, giving rise to a stochastic trend.

Now we describe some methods to describing the trend.

2.5.1 Curve Fitting

A traditional method of dealing with non-seasonal data with a trend is to fit a simple function such as a polynomial, a Gompertz curve, or a logistic curve. The Gompertz curve can be written in the form

$$\log x_t = a + br^t$$

where a, b, r are parameters with $0 < r < 1$, or in the alternative form of

$$x_t = \alpha \exp\left[\beta \exp\left(-\gamma t\right)\right],$$

which is equivalent as long as $\gamma > 0$. The logistic curve is given by

$$x_t = a/(1 + be^{-ct}).$$

For curves of this type, the fitted function provides a measure of the trend, and the residuals provide an estimate of local fluctuations.

2.5.2 Filtering

A second procedure for dealing with a trend is by using a linear filter, which converts a time series $\{x_t\}$ into another time series $\{y_t\}$ by the linear operation

$$y_t = \sum_{r=-q}^{s} a_r x_{t+r},$$

where $\{a_r\}$ are a set of weights. In order to smooth out local fluctuations and estimate the local mean, we should clearly choose the weights so that $\Sigma a_r = 1$, and the operation is referred as the moving average.

Moving averages are often symmetric with $s = q$ and $a_j = a_{-j}$. The simplest example of a moving average is a symmetric moving average, defined by

$$\mathrm{Sm}(x_t) = \frac{1}{2q+1} \sum_{r=-q}^{+q} x_{t+r}.$$

The simple moving average is not generally recommended by itself for measuring trend, although it can be useful for removing seasonal variation.

Another example is to take $\{a_r\}$ to be successive terms in the expansion of $(\frac{1}{2} + \frac{1}{2})^{2q}$. As q gets large, the weights approximate a normal curve.

A third example is Spencer's 15-point moving average, which is used for smoothing mortality statistics to get life tables. This covers 15 consecutive points with $q = 7$, and the symmetric weights are

$$\frac{1}{320}[-3, -6, -5, 3, 21, 46, 67, 74, 67, 46, 21, \dots]$$

A fourth example is the Henderson moving average. This moving average aims to follow a cubic polynomial trend without distortion, and the choice of q depends on the degree of irregularity. The symmetric nine-term moving average, for example, is given by

$$[-0.041, -0.010, 0.119, 0.267, 0.330, \dots]$$

To demonstrate this effect of moving averages, we use the Beveridge wheat price annual index series from 1500 to 1869 as an example (Fig. 1.1).

The top panel of Fig. 2.1 shows the original time series, while the middle and bottom ones show the smoothed series via the simple moving average and Spencer's 15-point moving average.

To reproduce Fig. 2.1, we can use the following piece of code:

```
> smooth.sym<-function(my.ts, window.q){
    window.size<-2*window.q+1
    my.ts.sm<-rep(0, length(my.ts)-window.size)
    for (i in 1:length(my.ts.sm)) {
      my.ts.sm[i]<-mean(my.ts[i:(i+window.size-1)])
    }
    my.ts.sm
  } # moving average with equal and symmetric weights
```

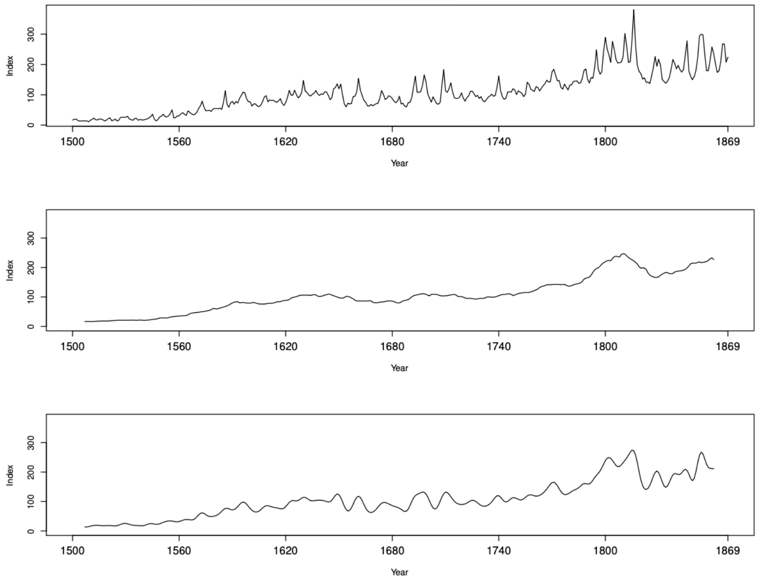


Figure 2.1: The original and smoother Beveridge wheat price annual index series from 1500 to 1869 (Top: The raw data; Middle: Smoothed series with simple moving average and $q = 7$; Bottom: Smoothed series with Spencer's 15-point moving average).

```
> smooth.spencer<-function(my.ts){
    weight<-c(-3,-6,-5,3,21,46,67,74,67,46,21,3,-5,-6,-3)/320
    my.ts.sm<-rep(0, length(my.ts)-15)
    for (i in 1:length(my.ts.sm)) {
          my.ts.sm[i]<-sum(my.ts[i:(i+14)]*weight)
    }
    my.ts.sm
  } # Spencer's 15-point moving average

> library(tseries)
> data(bev)
> bev.sm<-smooth.sym(bev, 7)
> bev.spencer<-smooth.spencer(bev)
> x.pos<-c(1500, 1560, 1620, 1680, 1740, 1800, 1869)
> par(mfrow=c(3,1), mar=c(4,4,4,4))
> plot(bev, type="l", xlab="Year", ylab="Index", xaxt="n")
> axis(1, x.pos, x.pos)
> plot(c(1, length(bev)), c(0, max(bev)), type="n", xlab="Year",
ylab="Index", xaxt="n")
> lines(seq(8, length(bev)-8), bev.sm)
> axis(1, x.pos-1500+1, x.pos)
> plot(c(1, length(bev)), c(0, max(bev)), type="n", xlab="Year",
ylab="Index", xaxt="n")
> lines(seq(8, length(bev)-8), bev.spencer)
> axis(1, x.pos-1500+1, x.pos)
```

Whenever a symmetric filter is chose, there is like to be an end-effects problem, since the symmetric filter does not take care of the end. Ideally we would want smoothened values all the way to the end of the time series, especially when doing forecasting. The analyst can project the smoothed values by eye or, alternatively, use an asymmetric filter that only involves present and past values, like exponential smoothing (Section 5.2.2):

$$\mathrm{Sm}(x_t) = \sum_{j=0}^{\infty} \alpha(1-\alpha)^j x_{t-j},$$

where α is a constant such that $0 < \alpha < 1$.

Having estimated the trend, we can look at the local fluctuations by examining

$$\begin{aligned}\mathrm{Res}(x_t) &= \text{residual from smoothed value} \\ &= x_t - \mathrm{Sm}(X_t) \\ &= \sum_{r=-q}^{+s} b_r x_{t+r}.\end{aligned}$$

This is also a linear filter with $b_0 = 1 - a_0$, and $b_r = -a_r$ for $r \neq 0$. If $\Sigma a_r = 1$, then $\Sigma b_r = 0$ and the filter is a trend remover.

How to choose the appropriate filter? The answer to this question requires considerable experience plus knowledge of the frequency aspects of time-series analysis. Some times we may want to get rid of local fluctuations to get smoothed values (low-pass filter), while other times, we want to investigate the residuals and remove the long-term fluctuations (high-pass filter).

Filters in series A smoothing prodedure may be carried out in two or more stages:

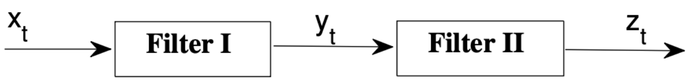


Figure 2.2: Two filters in series.

It is easy to show that a series of linear operations is still a linear filter overall. Suppose $\{a_r\}$ are the coefficients of the first filter which gets $\{y_t\}$ from $\{x_t\}$, and $\{b_j\}$ are the coefficients of the second filter which gets $\{z_t\}$ from $\{y_t\}$. Then

$$\begin{aligned}z_t &= \sum_j b_j y_{t+j} \\ &= \sum_j b_j \sum_r a_r x_{t+j+r} \\ &= \sum_k c_k x_{t+k}\end{aligned}$$

where

$$c_k = \sum_r a_r b_{k-r}$$

are the weights for the overall filter. In particular, the weights $\{c_k\}$ are obtained by a procedure called convolution, and write

$$\{c_k\} = \{a_r\} * \{b_j\}.$$

For example,

$$\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right) = \left(\frac{1}{2}, \frac{1}{2}\right) * \left(\frac{1}{2}, \frac{1}{2}\right)$$

The Spencer 15-point moving average is a convolution of four filters:

$$\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right) * \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right) * \left(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right) * \left(-\frac{3}{4}, \frac{3}{4}, 1, \frac{3}{4}, -\frac{3}{4}\right)$$

2.5.3 Differencing

A special type of filtering is to take the difference of a given time series until it becomes stationary. For instance, first differencing is defined as

$$\nabla x_t = x_t - x_{t-1} \text{ for } t = 2, 3, \ldots, N$$

while second differencing is defined as

$$\nabla^2 x_t = \nabla x_t - \nabla x_{t-1} = x_t - 2x_{t-1} + x_{t-2}.$$

Seasoning differences will be introduced in the next section.

2.5.4 Other approaches

More complicated approaches to handling trend will be mentiond later. In particular, state-space models involving trend terms will be mentioned in Chapter 10.

2.6 Analyzing Series that Contain a Trend and Seasonal Variation

Three seasonal models in common use are

$$\begin{aligned} &A \quad X_t = m_t + S_t + \varepsilon_t \\ &B \quad X_t = m_t S_t + \varepsilon_t \\ &C \quad X_t = m_t S_t \varepsilon_t \end{aligned}$$

where m_t is the deseasonalized mean level at time t, S_t is the seasonal effect at time t, and ε_t is the random error.

Model A describes the additive case, while models B and C describe the multiplicative case. The analysis of time series, which exhibit seasonal variation, depends on whether one wants to (1) measure the seasonal effect and/or (2) eliminate seasonality.

For series that do contain a substantial trend, a more sophisticated approach is required. With monthly data, the most common way of eliminating the seasonal effect is to calculate

$$\mathrm{Sm}(x_t) = \frac{\frac{1}{2}x_{t-6} + x_{t-5} + \cdots + x_{t+5} + \frac{1}{2}x_{t+6}}{12}.$$

For quarterly data, the seasonal effect can be eliminated by calculating

$$\mathrm{Sm}(x_t) = \frac{\frac{1}{2}x_{t-2} + x_{t-1} + x_t + x_{t+1} + \frac{1}{2}x_{t+2}}{4}.$$

These smoothing procedures all effectively estimate the local (deseasonalized) level of the series. The seasonal effect itself can then be estimated by calculating $x_t - \mathrm{Sm}(x_t)$ or $x_t/\mathrm{Sm}(x_t)$ depending on whether the seasonal effect is additive or multiplicative.

As an example, Fig. 2.3 illustrates the decomposition of the domestic monthly sales series of Australian fortified wine by winemakers into trend, additive, seasonal, and remainder series. We can do this via the "decompose" command in R:

```
> wine<-read.csv("../data/aus\_wine\_sales.csv", header=F)
> wine.ts<-ts(wine[,2], frequency=4, start=c(1985,1))
# create a time series object
> wine.de<-decompose(wine.ts, type="additive")
> plot(wine.de)
```

Another technique of eliminating seasonal effects is seasonal filtering:

$$\nabla_{12} x_t = x_t - x_{t-12}.$$

Two general reviews of methods for seasonal adjustment are Butter and Fase (1991) and Hylleberg (1992). Currently, the most common package for removing trend and seasonal effects is called the X-12 method.

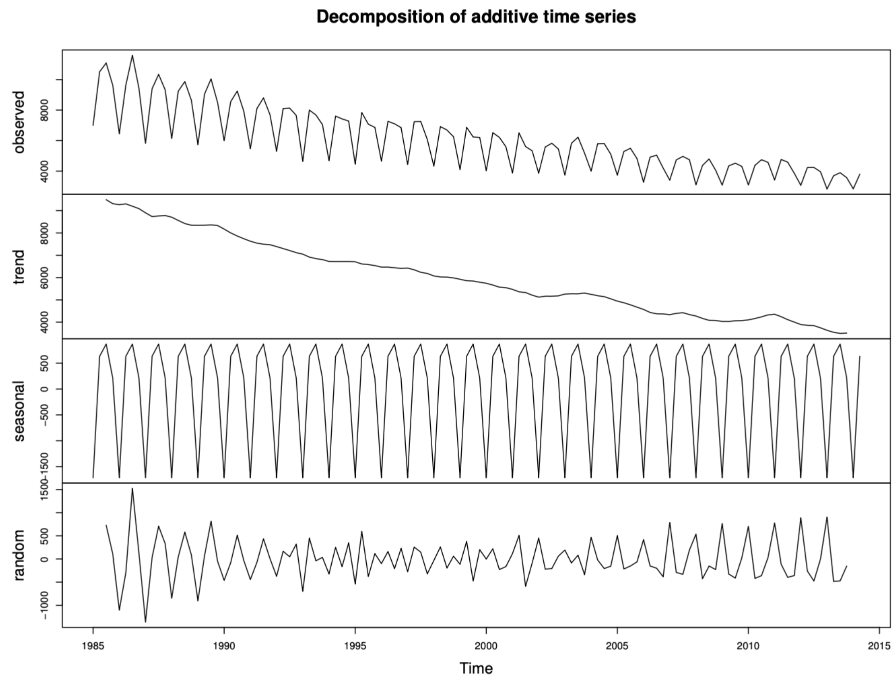


Figure 2.3: The decomposition of the domestic sales of Australian fortified wine.

It is a fairly complicated procedure that implements a series of linear filters and adopts a recursive approach. The new software for X-12 also gives more flexibility in handling outliers, and also allows the users to deal with the possible issue of calendar effects. The X-12 method is often combined with ARIMA modeling, which gives rise to the package X-12-ARIMA. Some other countries use SEATS (Signal Extraction in ARIMA Time Series) or TRAMO (Time-series Regression with ARIMA Noise).

2.7 Autocorrelation and the Correlogram

Let's first review on the regular correlation coefficient. Given N pairs of variables x and y, say $\{(x_1, y_1), \ldots, (x_N, y_N)\}$, the sample correlation coefficient is given by

$$r = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{N}(x_i - \bar{x})^2 \sum_{i=1}^{N}(y_i - \bar{y})^2}}.$$

This value lies in $[-1, 1]$ and measures the strength of linear association between the two variables.

Now, given N observations $x_1, \ldots, x_N$ of a time series, we can form $N-1$ pairs of observation by getting $\{(x_1, x_2), \ldots, (x_{N-1}, x_N)\}$, where each pair of observations is seperated by 1 time interval. Then we can get a sample correlation coefficient

$$r_1 = \frac{\sum_{t=1}^{N-1}(x_t - \bar{x}_{(1)})(x_{t+1} - \bar{x}_{(2)})}{\sqrt{\sum_{t=1}^{N-1}(x_t - \bar{x}_{(1)})^2 \sum_{i=1}^{N-1}(x_{t+1} - \bar{x}_{(2)})^2}},$$

where

$$\bar{x}_{(1)} = \frac{1}{N-1}\sum_{t=1}^{N-1} x_t$$

is the mean of the first $N-1$ observations, while

$$\bar{x}_{(2)} = \frac{2}{N}\sum_{t=1}^{N-1} x_t$$

is the mean of the last $N-1$ observations. The quantity above is called the sample autocorrelation coefficient or a serial correlation coefficient at lag one.

However, the formula is rather complicated, so instead we can estimate using

$$r_1 = \frac{\sum_{t=1}^{N-1}(x_t - \bar{x})(x_{t+1} - \bar{x})}{(N-1)\sum_{t=1}^{N-1}(x_t - \bar{x})^2/N}$$

where $\bar{x}$ is just the overall mean. We can also simplify by dropping the factor $N/(N-1)$, which is close to one if N is large. This gives the even simpler formula

$$r_1 = \frac{\sum_{t=1}^{N-1}(x_t - \bar{x})(x_{t+1} - \bar{x})}{\sum_{t=1}^{N}(x_t - \bar{x})^2} \quad (*)$$

which is the form that will be used in this book.

In a similar way, we can find the correlation between observations of lag k, meaning

$$r_k = \frac{\sum_{t=1}^{N-k}(x_t - \bar{x})(x_{t+k} - \bar{x})}{\sum_{t=1}^{N}(x_t - \bar{x})^2}.$$

In practice, the autocorrelation coefficients are calculated from the autocovariance coefficients, $\{c_k\}$, which we define by analogy using the usual covariance formula:

$$c_k = \frac{1}{N}\sum_{t=1}^{N-k}(x_t - \bar{x})(x_{t+k} - \bar{x}).$$

We then compute

$$r_k = c_k/c_0, \ \ k = 0, 1, \ldots, M, \ \text{ where } M < N.$$

Some authors use this equation instead (NOT used in this book!):

$$c_k = \frac{1}{N-k}\sum_{t=1}^{N-k}(x_t - \bar{x})(x_{t+k} - \bar{x}).$$

2.7.1 The correlogram

A useful aid in interpreting a set of autocorrelation coefficients it the correlogram. It contains the sample correlation coefficients r_k for $k = 0, 1, \ldots, M$ for some M which is usually much less than N. For example, if $N = 200$ then we can use $M = 30$. Examples are shown below, from Fig. 2.4 toFig. 2.8.

2.7.2 Interpreting the correlogram

Interpreting the coefficients from the correlogram is not easy. Here are some situations:

Random series

A time series is said to be completely random (i.i.d.) if it consists of a series of independent observations having the same distribution. For large N, we would expect that $r_k \approx 0$ for all $k \neq 0$. In fact, later we will see that, for a random time series, $r_k, k \geq 1$ is approximately $N(0, 1/N)$. Thus, if a time series, is random, we can expect 19 out of 20 of the values of r_k to lie between $\pm 1.96/\sqrt{N}$. As a result, it is common to regard any values of r_k outside of this range to be statistically significant. This also means that even if the time series is completely random, we can find statistically significant values of r_k, which is a type I error.

Fig. 2.4 can be reproduced using the following R code:

```
> set.seed(1)
> x<-rnorm(400)
> par(mfrow=c(2,1), mar=c(3,4,3,4))
> plot(x, type="l", xlab="", ylab="")
> title(xlab="Time", ylab="Series", line=2, cex.lab=1.2)
> acf(x, ylab="", main="")
> title(xlab="Lag", ylab="ACF", line=2)
```

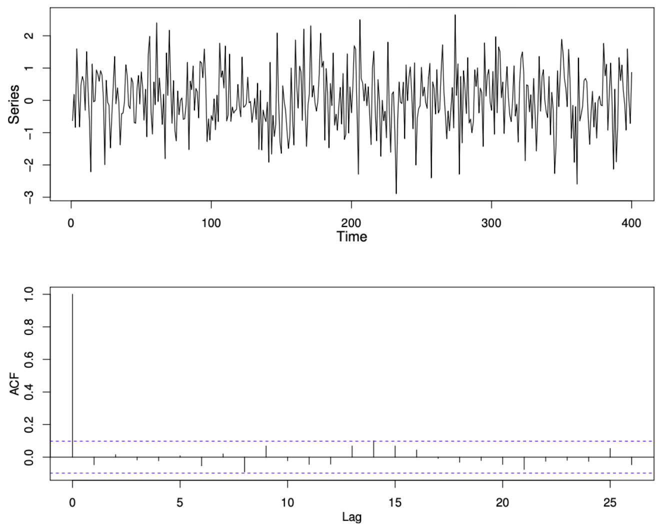


Figure 2.4: A completely random series together with its correlogram. The dotted lines in the correlogram are at $\pm 1.96/\sqrt{N}$. Values outside these lines are said to be significantly different from zero.

Short term correlation

Stationary series often exhibit short-term correlation characterized by a fairly large value of r_1 followed by one or two further coefficients that tend to decrease. Values of r_k for longer lags tend to be approximately zero. An example is shown in Fig. 2.5.

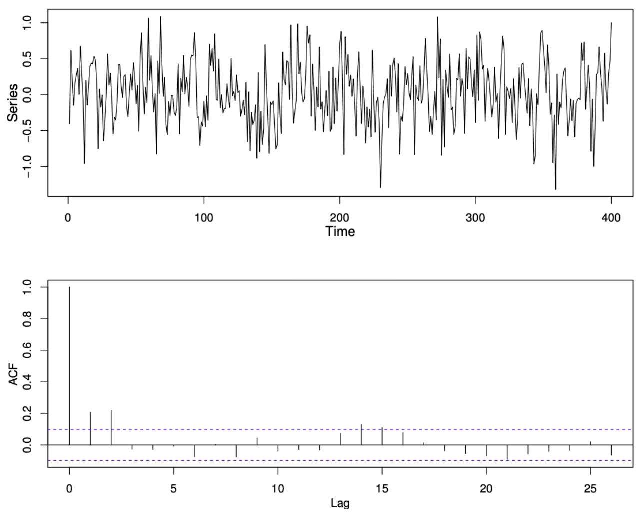


Figure 2.5: A time series showing short-term correlation together with its correlogram.

A time series that gives rise to such a correlogram is one for which an observation above the mean tends to be followed by one or more further observations above the mean, and similarly for observations below the mean.

Alternating series

If the time series tends to alternate, so will its corrologram. An example is shown in Fig. 2.6. The time series plots in Fig. 2.5 and Fig. 2.6 suggest that it is hard to distinguish between a time series with short-term correlation from that with alternating correlation.

Non-stationary series If a time series contains a trend, then the value of r_k will not come down to zero except for very large values of the lag. This is because an observation on one side of the overall mean tends to be followed by a large number of further observations on the same side of the mean because

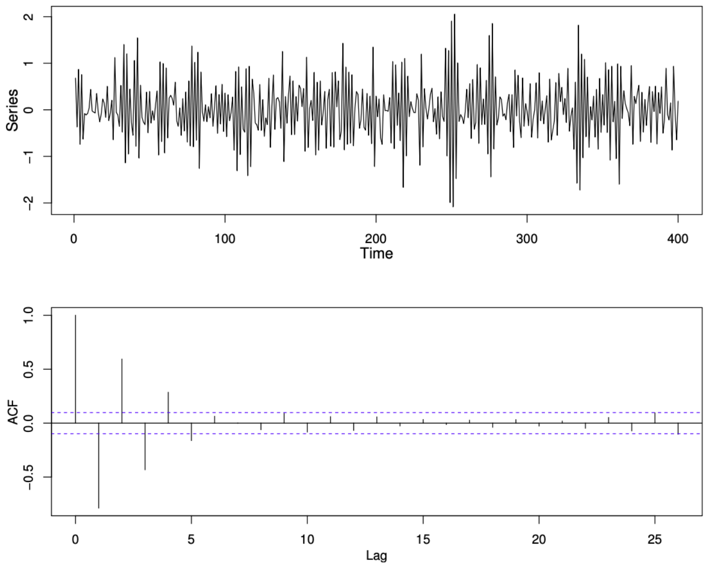


Figure 2.6: An alternating time series together with its correlogram.

of the trend. See Fig. 2.7 for an example. Little can be inferred from the corrologram, and any trend should be removed before calculating the set of autocorrelation coefficients $\{r_k\}$.

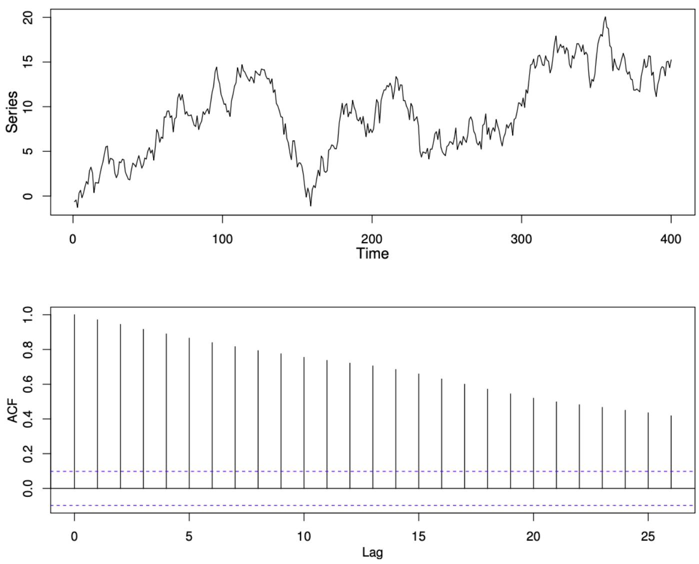


Figure 2.7: A non-stationary time series together with its correlogram.

Fig. 2.7 can be reproduced via the following R code:

```
> set.seed(1)
> ts.sim3<-cumsum(rnorm(400))
> par(mfrow=c(2,1), mar=c(3,4,3,4))
> plot(ts.sim3, type="l", xlab="", ylab="")
> title(xlab="Time", ylab="Series", line=2, cex.lab=1.2)
> acf(ts.sim3, ylab="",main="")
> title(xlab="Lag", ylab="ACF", line=2)
```

Seasonal series If a time series contains seeasonal variation, the corrologram will also exhibit oscillation at the same frequency. For example, with monthly observations, we expect r_6 to be 'large' and negative, while $r_1 2$ will be 'large' and positive. In particular, if x_t follows a sinusoidal pattern, so does r_k. For example, if

$$x_t = a \cos t\omega$$

where a is a constant and the frequency ω is such that $0 < \omega < \pi$, then it can be shown (Exercise 2.3) that

$$r_k \approxeq \cos k\omega \text{ for large } N.$$

See Fig. 2.8 for the corrologram of the monthly air temperature data shown in Fig. 1.3.

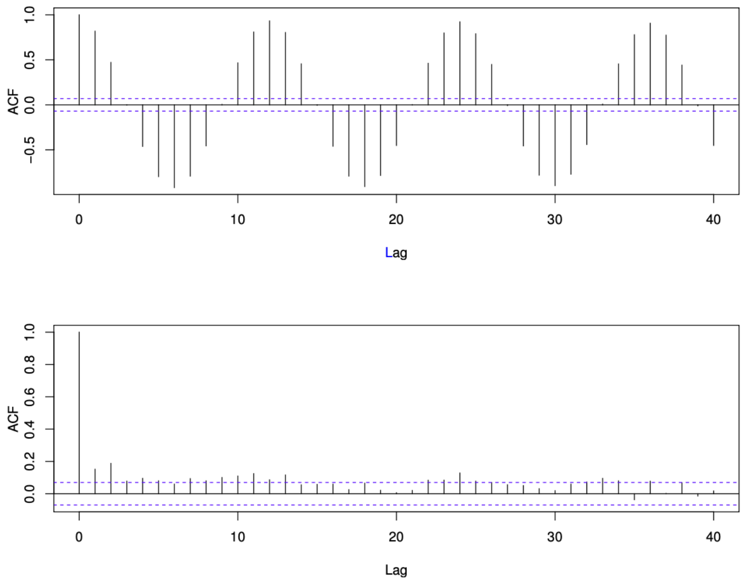


Figure 2.8: The correlograms of monthly observations on air temperature in Anchorage, Alaska for the raw data (top) and for the seasonally adjusted data (bottom).

The sinusoidal pattern of the correlogram is clearly evident, but for seasonal data of this type the correlogram provides little extra information, because the seasonal pattern is already apparent in the data.

If the seasonal variation is removed from seasonal data, then the correlogram may provide more useful information. For example, after the seasonal effect is removed from Fig. 1.3, the correlogram of the resulting series (bottom panel of Fig. 2.8) shows that the first three coefficients are significantly differnt from zero, meaning there is short-term correlation.

Outliers

If a a time series contains one or more outliers, the correlogram may be seriously affected and it is advisable to adjust these outliers before doing the formal analysis. For example, if there is one outlier in the time series at, say, time t_0, and if it is not adjusted, then the plot of x_t against x_{t+k} will contain two 'extreme' points, namely, (x_{t_0-k}, x_{t_o}) and (x_{t_0}, x_{t_0+k}).

General remarks Considerable experience is required to interpret sample autocorrelation coefficients. In addition, we need to study the probability theory of stationary series and learn about the classes of models that may be appropriate.

2.8 Other Tests of Randomness

In most cases, visual examination is enough to determine that the series is NOT random (there is trend/seasonality/short-term correlation). However, we can occasionally test whether the apparent stationary time series it 'random'. One type of approach is to carry out the test of randomness in which we test whether the observations $x_1, \ldots, x_N$ could have arisen in that order by chance. We'll mention a few such tests here.

2.9 Handling Real Data