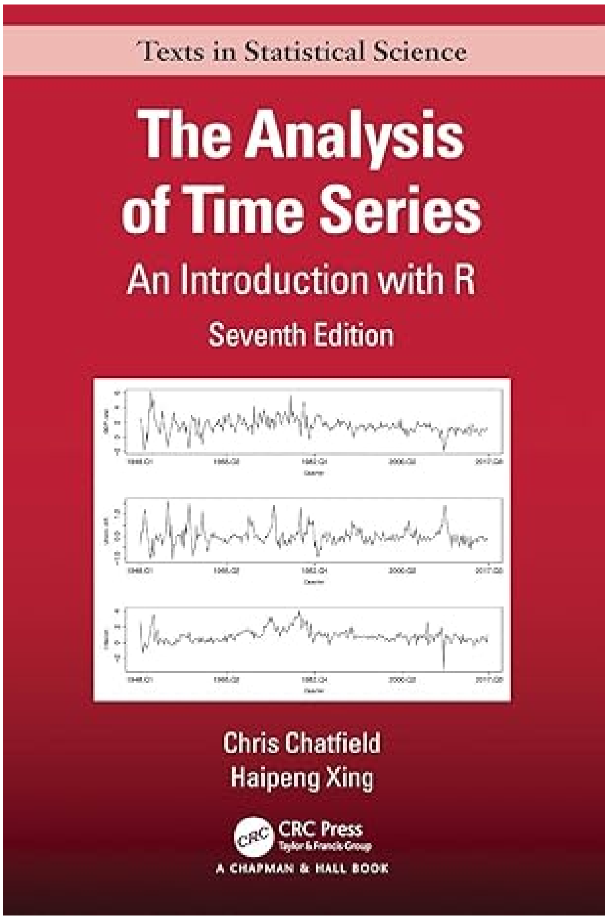
Notes for The Analysis of Time Series Seventh Edition by Chris Chatfield & Haipeng Xin

Gallant Tsao

August 6, 2025



Contents

1 Introduction

1.1 Some Representative Time Series

We begin with some examples of the sort of time series that arise in practice.

Economic and financial time series

Many time series are routinely recorded in economics and finance: Share prices on succesive days, export totals in successive months, average incomes in successive months, company profits in successive years, and so on.

The classic Beveridge wheat price index series consists of the average wheat price in nearly 50 places in various countries measured in successive years from 1500 to 1869. We can plot the series via

```
> library(tseries) # load the library
> data(bev) # load the dataset
> plot(bev, xlab="Year", ylab="Wheat price index", xaxt="n")
> x.pos = c(1500, 1560, 1620, 1680, 1740, 1800, 1860) # define x-axis labels
> axis(1, x.pos, x.pos)
```

Fig. 1.1 shows this series and some apparent cyclic behavior can be seen.

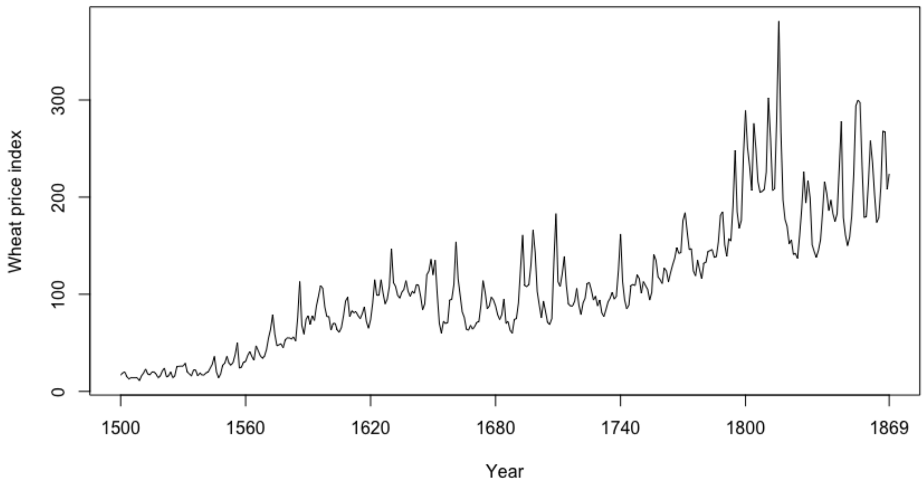


Figure 1.1: The Beverage wheat price annual index series from 1500 to 1869.

There is also an example of financial time series: Fig. 1.2 below hsows the daily returns (or percentage change) of the adjusted closing prices of the Standard & Poor's 500 (S&P) Index from January 4th, 1995 to December 30th, 2016.

To reproduce Fig. 1.2 in R, suppose we have the data as sp500_ret_1995–2016.csv in the directory mydata. Then we can load the data via the following piece of code:

```
> sp500<-read.csv("mydata/sp500\_ret\_1995-2016.csv")
> n<-nrow(sp500)
> x.pos<-c(seq(1,n,800),n)
> plot(sp500\$Return, type="l", xlab="Day", ylab="Daily return", xaxt="n")
> axis(1, x.pos, sp500\$Date[x.pos])
```

Physical time series Many types of times series occur in the physical sciences, particularly in meteorology, marine sciences, and geophysics. Examples are rainfall on succesive days, and air temperature measured in successive hours, days, or months. Fig. 1.3 shows the average air temperature in Anchorage, Alaska in the US in successive months over a 16 year time period. The series can be downloaded from the

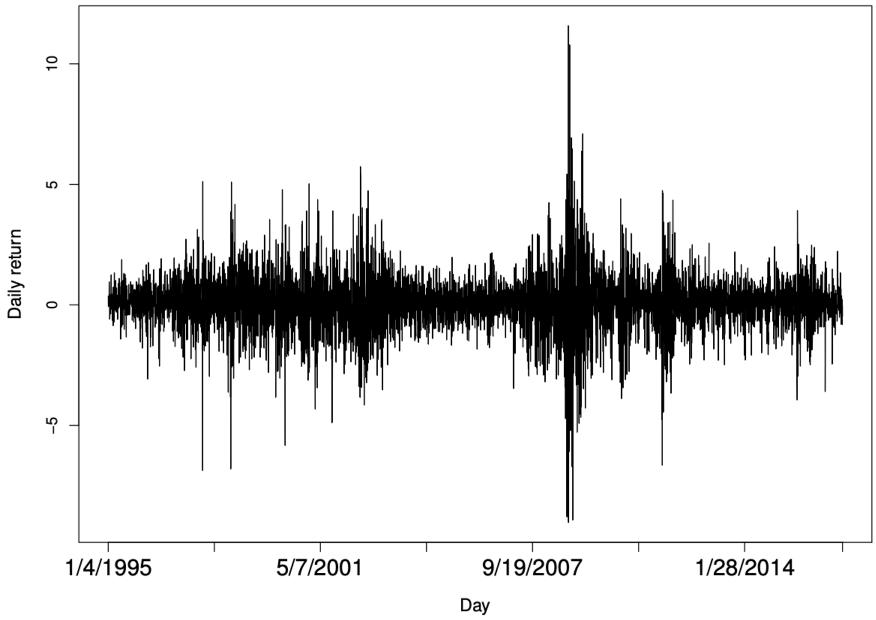


Figure 1.2: Daily returns of the adjusted closing proces of the S&P index from January 4th, 1995 to December 40th, 2016.

U.S. National Centers for Environmental Information. Seasonal fluctuations can be clearly seen in the series.

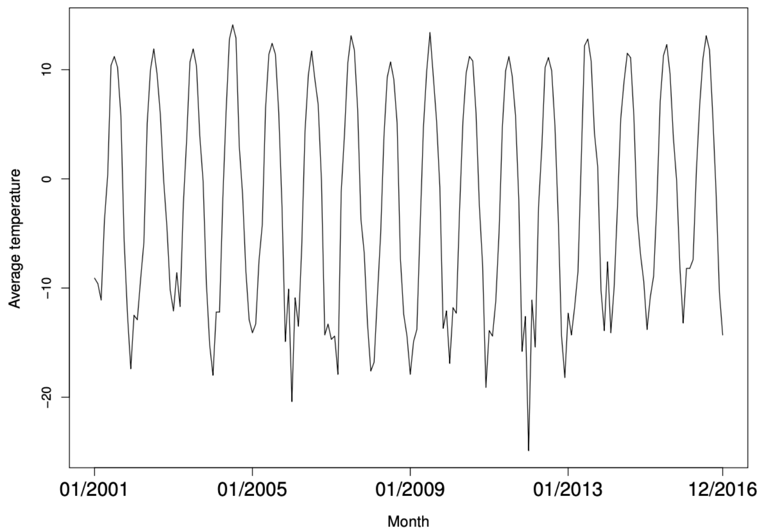


Figure 1.3: Monthly average air temperature (degrees Celcius) in Anchorage, Alaska, in successive months from 2001 to 2016.

Some mechanical recorders take measurements continuously and produce a continuous trace rather than observations at discrete intervals of time. However, for more detailed analysis, it is customary to convert the continuous trace to a series in discrete time by sampling the trace at appropriate intervals of time. The resulting analysis is more straightforward and can be handled by standard time series software.

Marketing time series

The analysis of time series arising in marketing is an important problem in commerce. Observed variables could include sales figures in successive weeks or months, monetary receipts, advertising costs and so on. As an example, Fig. 1.4 shows the domestic sales of Australian fortified wine by winemakers in successive quarters over a 30-year period, which are available at the Australian Bureau of Statistics.

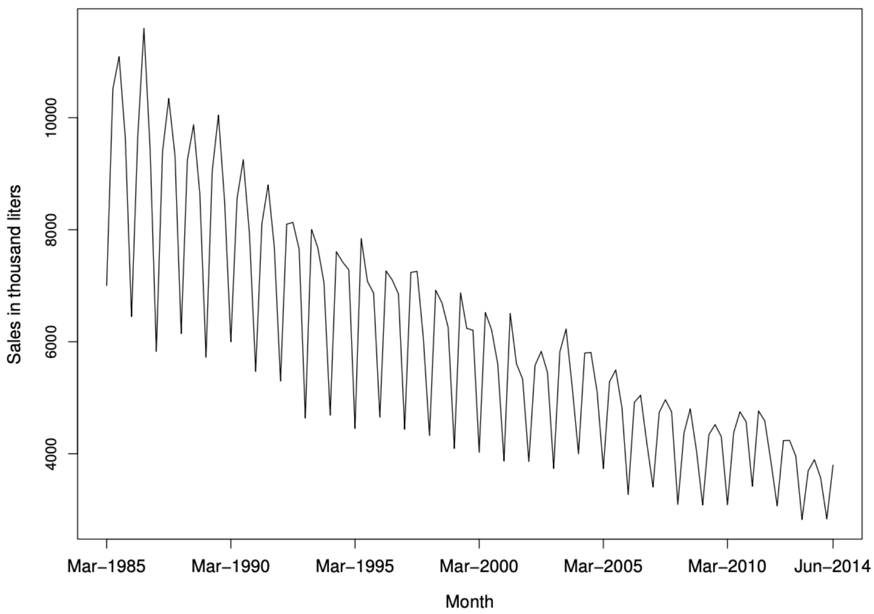


Figure 1.4: Domestic sales (unit: thousand liters) of Australian fortified wine by winemakers in successive quarters from March 1985 to June 2014.

Note the trend and seasonal variation above.

Demographic time series

Various time series occur in the study of population change. Examples include the total population of Canada measured annually, and monthly birth totals in England. Fig. 1.5 shows the total population and crude birth rate (per 1000 people) for the UNited States from 1965 to 2015. The data are available at the US Federal Reserve Bank of St. Louis.

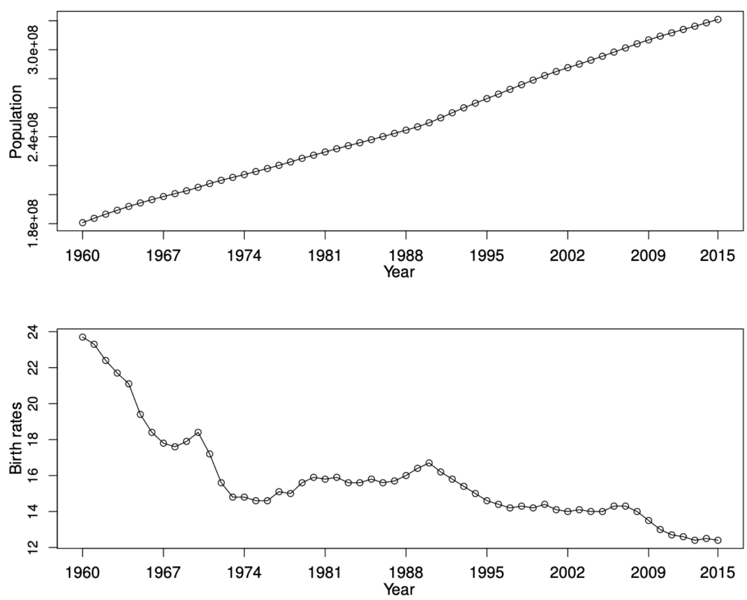


Figure 1.5: Total population and birth rate (per 1000 people) for the United States from 1965 to 2015.

We can reproduce Fig. 1.5 using the following piece of code:

```
> pop<-read.csv("mydata/US\_pop\_birthrate.csv", header=T)
> x.pos<-c(seq(1, 56, 7), 56)
> x.label<-c(seq(1960, 2009, by=7), 2015)
```

```
> par(mfrow=c(2,1), mar=c(3,4,3,4))
> plot(pop[,2], type="l", xlab="", ylab="", xaxt="n")
> points(pop[,2])
> axis(1, x.pos, x.label, cex.axis=1.2)
> title(xlab="Year", ylab="Population", line=2, cex.lab=1.2)

> plot(pop[,3], type="l", xlab="", ylab="", xaxt="n")
> points(pop[,3])
> axis(1, x.pos, x.label, cex.axis=1.2)
> title(xlab="Year", ylab="Birth rates", line=2, cex.lab=1.2)
```

Process control data

In process control, a problem is to detect changes in the performance of a manufacturing process by measuring a variable, which shows the quality of the process. These measurements can be plotted against time, as shown in Fig. 1.6 below.

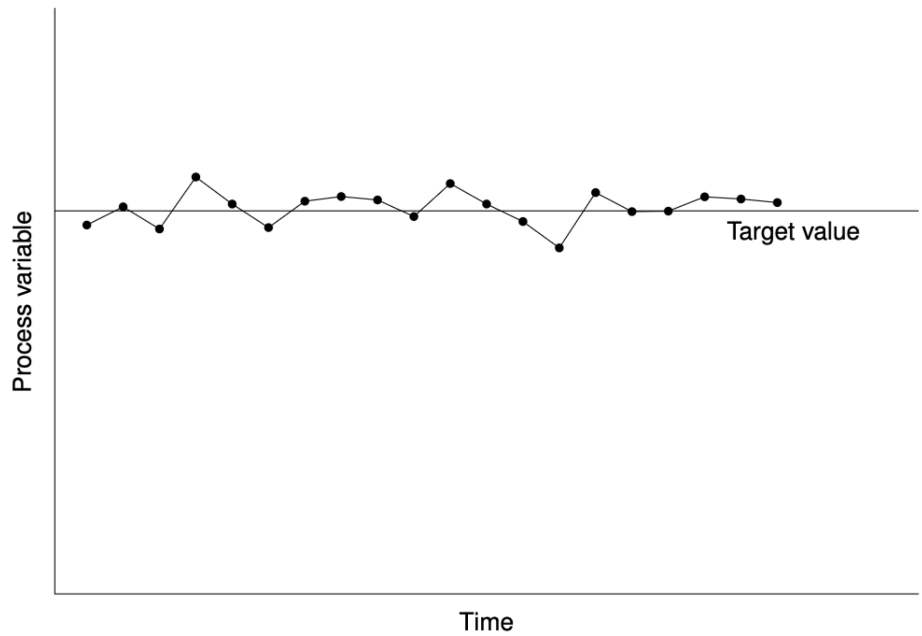


Figure 1.6: A process control chart.

Special techniques have been developed for this type of time series problems, namely statistical quality control (Montgomery, 1996).

Binary processes A special type of time series arisis when observations can only take one of only two values, usually denoted by 0 and 1 (see Fig. 1.7). For example, in computer science, the position of a switch could be recorded as 1 or 0 respectively.

Time series of this type occur in many situations, including the study of communication theory. A problem here is to predict when the process will take a different value.

Point processes We can consider a series of events occuring 'randomly' through time, such as the dates of major railway disasters. A series of events of this type is usualled called a point process. As an example, Fig. 1.8 shows the intraday transaction data of the International Business Machines (IBM) Corporation from 9:35:00 to 9:38:00 on January 4th, 2010. When a trade event occurs, the corresponding trading price and trading volume are observed.

Methods of analyzing point process data are generally very different from those used for analyzing standard time series data. The text by Cox and Isham (1980) is recommended.

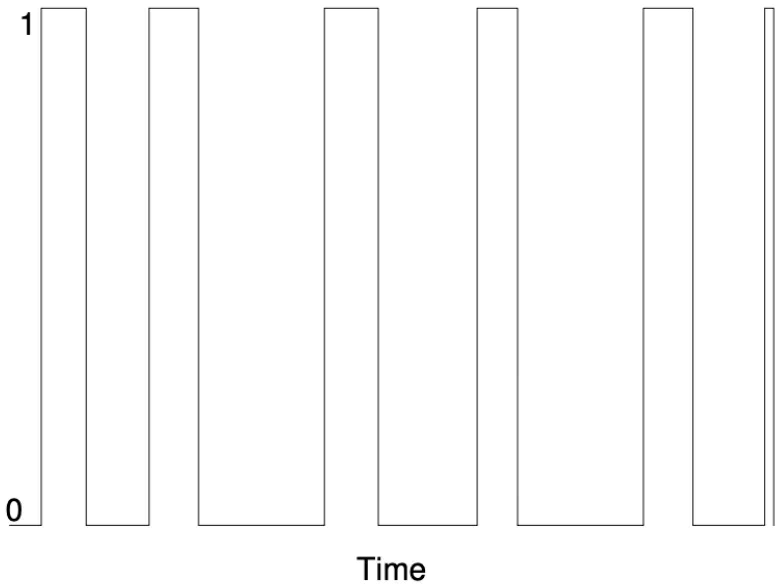


Figure 1.7: A realization of a binary process.

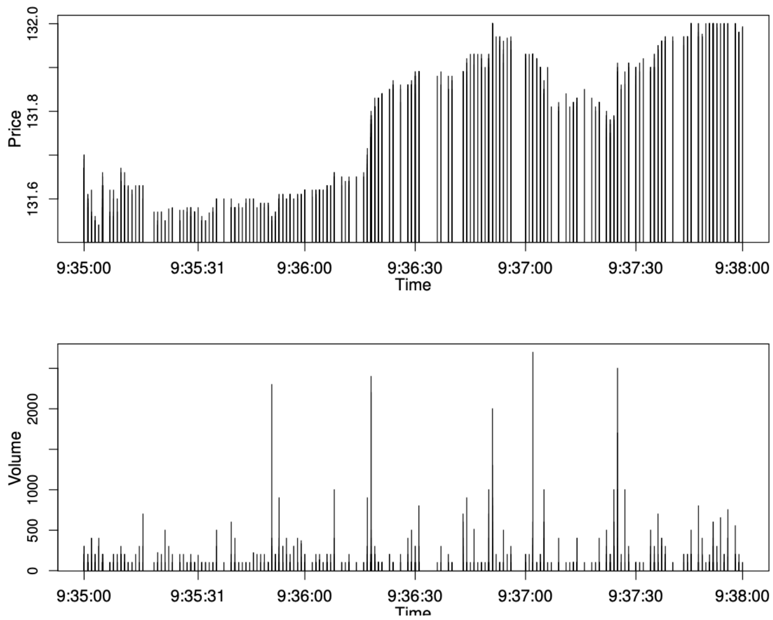


Figure 1.8: Transaction prices and volumes of the IBM stocks from 9:35:00 to 9:38:00 on January 4th, 2010.

To reproduce Fig. 1.8, we can use the following piece of code:

```
> ibm<-read.table("mydata/taq\_trade\_ibm\_100104.txt",
header=T, sep="\t")
> ibm.new<-ibm[,c(1,2,7)]
> ibm[,2]<-as.numeric(as.character(ibm[,2]))

> ### take 9:35:00-9:37:59am trading record
> data<-ibm.new[1458:2371,]
> newtime<-rep(0, nrow(data))
> for (i in 1:nrow(data)){
  min<-as.numeric(substr(as.character(data\$TIME[i]),3,4))
  sec<-as.numeric(substr(as.character(data\$TIME[i]),6,7))
  newtime[i]<- (min-30)*60+sec
}

> x.label<-c("9:35:00", "9:35:31", "9:36:00", "9:36:30",
"9:37:00", "9:37:30", "9:38:00")
> x.pos<-c(1, 139, 249, 485, 619, 776, 914)
> par(mfrow=c(2,1), mar=c(2,4,2,4))

> plot(newtime, data[,2],xlab="",ylab="",xaxt="n",type="h")
> axis(1, newtime[x.pos], x.label, cex.axis=1.2)
> title(xlab="Time", ylab="Price", line=2, cex.lab=1.2)
> plot(newtime, data[,3],xlab="",ylab="",xaxt="n",type="h")
> axis(1, newtime[x.pos], x.label, cex.axis=1.2)
> title(xlab="Time", ylab="Volume", line=2, cex.lab=1.2)
```

1.2 Terminology

Definition. A time series is said to be <u>continuous</u> when observations are made continuously through time.
A time series is said to be <u>discrete</u> when observations are taken only at specific times, usually equally spaced.

Note that the term 'discrete' is used for series of this type even when the measured variable is continuous. This book mostly concerns with discrete time series.

Discrete time series can arise in several ways. Given a continuous time series, we can read off/digitize the values at equal intervals of time to give a discrete time series, sometimes called sampled series. A different type arises when a variable does not have an instantaneous value but we can aggregate/accumulate the values over equal intervals of time (like monthly exports). Finally, some time series are inherently discrete (dividend paid by a company to shareholders in successive years).

Important: Most statistical theory relies on samples of independent observations. This is not true with time series! Successive observations are usually *not* independent hence we must take the *time order* into account when doing the analysis. If a time series can be predicted exactly, it is said to be deterministic. However, most time series are stochastic, hence future values have a probability distribution that depend on the past.

1.3 Objectives of Time Series Analysis

There are several objectives of analyzing a time series:

(i) Description

When presented with a time series, the firrst step is usually get the time plot from the data. For simpler time series, this is useful, like in Fig. 1.4, where we can see a regular seasonal effect. For more complex models, more sophisticated techniques are required.

The book devotes a greater amount of space to the more advanced techniques, but this does not mean the elementary descriptive techniques are not important. These are very important when dealing with outliers, which is a very complex subject in time series analysis.

Other features to look for in a time plot include sudden or gradual changes in the properties of the series, in which case we may or may not need to use piecewise model to fit parts of the time series one at a time.

(ii) Explanation

When observations are taken on two or more variables, it may be possible to use the variation in one time series to explain the variation in another series. This may lead to a deeper understanding of the mechanism that generated a given time series.

When dealing with many factors, regression is usually not too helpful in handling time series data. We'll use something called a linear system, which will be discussed in Chapter 9, which converts an input series to an output series via a linear operator. Given observations on the input and output to a linear system (see Fig. 1.9), the analyst wants to assess the properties of the linear system. For example, it is of interest to see how sea level is affected by temperature and pressure, and to see how sales are affected by price and economic conditions. A class of models, called transfer function models, enables us to model time series data in an appropriate way.

(iii) Prediction

Figure 1.9: Schematic representation of a linear system.

Given a time series, we may want to predict future values based on the current and past values, for example in sales forecasting. Here we are doing 'prediction' and 'forecasting'.

(iv) Control

Time series are sometimes collected to improve control over some physical or economic system. For example, if a time series is used to keep track of the quality of manufacturing, we would want it to always be at the 'high' level. Control problems are related to predictions, as we can use predictions for error correction.

This topic is only briefly touched on in Section 14.3.

1.4 Approaches to Time Series Analysis

This book describes various approaches to time series:

- Chapter 2: Simple descriptive techniques (plotting, trend and seasonality, etc.)
- Chapter 3: Probability models for time series
- Chapter 4: Fitting models to time series
- Chapter 5: Forecasting procedures
- Chapter 6: Spectral density function
- Chapter 7: Using spectral analysis to estimate the spectral density function
- Chapter 8: Analysis of bivariate time series
- Chapter 9: Using linear systems
- Chapter 10: State-space models + Kalman filter

- Chapter 11: Nonlinear time series analysis
- Chapter 12: Volatility time series model analysis
- Chapter 13: Multivariate time series model analysis
- Chapter 14: More advanced stuff!

1.5 Review of Books on Time Series

This subsections simply concerns other books on time series that may be helpful. The complete list is here:

2 Basic Descriptive Techniques

Statistical techniques for analyzing time series vary from relatively straightforward descriptive techniques to sophisticated inferential techniques. This chapters introduces the former. Descriptive techniques should be tried before attempting more complicated procedures, because they are important in 'cleaning' the data, and then getting a 'feel' for them, before trying to generate ideas as regards to a suitable model.

This chapter focuses on ways of understanding typical time-series effects, such as trend, seasonality, and correlations betyween successive observations.

2.1 Types of Variation

Traditional methods of time-series analysis are mainly concerned with decomposing the variation in a series into components representing trend, seasonal variation and other cyclic changes. Any remaining variation is attributed to 'irregular' fluctuations. This approach is not always the best but is particularly valuable when the variation is dominated by trend and seasonality.

Seasonal variation Many time series, such as sales figures and temperature readings, exhibit variation that is annual in period. For example, unemployment is typically 'high' in winter but low in summer. This yearly variation is easy to understand, and can readily be estimated if seasonality is of direct interest. Alternatively, seasonal variation can be removed from the data to give deseasonalized data.

Other cyclic variations Apart from seasonal effects, there are some other variations of time series at a fixed period due to some other physical cause. For example, daily variations in temperature depend on what time of the day it is.

Trend This may be loosely defined as 'long-term change in the mean level'. However, this largely depends how we define the term 'long-term'. It can be a year to even 50 years depending on the span of the time series.

Other irregular flucttutations After trend and cyclic variations have been removed from a set of data, we are left with residuals that may or may not be 'random'. We'll examine later whether this can be explained in terms of probability models, such as the moving average (MA) or autoregressive (AR) models.

2.2 Stationary Time Series

Broadly speaking a time series is said to be stationary if there is no systematic change in mean (no trend), if there is no systematic change in variance and if strictly periodic variations have been removed. In other words, the properties of one section of the data are much like those of any ohter section.

Much of the probability theory of time series is concerned with stationary time series, and for this reason time series analysis often requires one to transform a non-stationary time series to a stationary one so as to use the theory. For example, we can remove the trend and seasonality and model the resuals by a stationary stochastic process. However, sometimes we may be more interested in the non-stationary components too!

2.3 The Time Plot

Ths first, and most important, step in any time-series analysis is to plot the observations against time. This grap, called the time plot, will show up important features of the series such as trend, seasonality, outliers and discontinuities.

Plotting a time series is not as easy as is sounds. The choice of scales, siz eof intercept, way that points are plotted may substantially affect the way the plot looks. Not all computer software plots the time series well, so we'll have to adjust manually sometimes.

2.4 Transformations

Plotting the data may suggest that it is reasonable to transform them by, for example, taking logs or square roots. The three main reasons for making a transformation are as follows:

(i) To stabilize the variance

If there is a trend in the series and the variance appears to increase with the mean, they it may be advisable to transform the data. In particular, if the standard deviation is directly proportional to the mean, a logarithmic transformation is indicated. On the other hand, if the variance changes through time without a trend being present, then a transformation will not help.

(ii) To make the seasonal effect additive

If there is a trend in the series and the size of the seasonal effect appears to increase with the mean, then it may be advisable to transform the data so as to make the seasonal effect constant from year to year. The seasonal effect is then said to be additive. In particular, if the size of the seasonal effect is directly proportional to the mean, then the seasonal effect is said to be multiplicative and a logarithmic transformation is appropriate to make the effect additive. However, this transformation will only stabilize the variance if the error term is also thought to be multiplicative (see Section 2.6), a point that is sometimes overlooked.

(iii) To make the data normally distributed

Model building and forecasting are usually carried out on the assumption that the data are normally distributed. In practice this is not necessarily the case; there may, for example, be evidence of skewness in that there tend to be 'spikes' in the time plot that are all in the same direction (either up or down). This effect can be diffcult to eliminate with a transformation and it may be necessary to model the data using a different 'error' distribution.

The logarithmic and square root transformations above are special cases of a general class of transformations called the Box-Cox transformation. Given an observed time series $\{x_t\}$ and a transformation parameter λ, the transformed series is given by

$$y_t = \begin{cases} (x_t^\lambda - 1)/\lambda & \text{if } \lambda \neq 0, \\ \log(x_t) & \text{if } \lambda = 0. \end{cases}$$

The best value of λ is usually 'guesstimated'.

However, Nelson and Granger (1979) found that there is little improvement in forecast performance when a general Box–Cox transformation was tried on a number of series. There are cases where the transformation fails to stabilize the variance. Usually, transformations should be avoided wherever possible except where the transformed variable has a direct physical interpretation. For example, when percentage increases are of interest, then taking logarithms makes sense.

2.5 Analyzing Series that Contain a Trend and No Seasonal Variation

The simplest type of trend is the familiar 'linear trend + noise', for which the observation at time t is a random variable X_t given by

$$X_t = \alpha + \beta t + \varepsilon_t,$$

where α, β are constants and ε_t denotes a random error term with zero mean. The mean level at time t is given by $m_t = \alpha + \beta t$; which is some times called the 'trend term'. However, some authors denote β as the trend so it depends on the context.

The trend above is a deterministic function of time and is sometimes called a global linear trend. In practice, this generally provides an unrealistic model, and nowadays there is more emphasis on models that allow for local linear trends. One possibility is to fit a piecewise linear model where the trend line is locally linear but with change points where the slope and intercept change (abruptly). We can also assume that α, β evolve stochastically, giving rise to a stochastic trend.

Now we describe some methods to describing the trend.

2.5.1 Curve Fitting

A traditional method of dealing with non-seasonal data with a trend is to fit a simple function such as a polynomial, a Gompertz curve, or a logistic curve. The Gompertz curve can be written in the form

$$\log x_t = a + br^t$$

where a, b, r are parameters with $0 < r < 1$, or in the alternative form of

$$x_t = \alpha \exp\left[\beta \exp\left(-\gamma t\right)\right],$$

which is equivalent as long as $\gamma > 0$. The logistic curve is given by

$$x_t = a/(1 + be^{-ct}).$$

For curves of this type, the fitted function provides a measure of the trend, and the residuals provide an estimate of local fluctuations.

2.5.2 Filtering

A second procedure for dealing with a trend is by using a linear filter, which converts a time series $\{x_t\}$ into another time series $\{y_t\}$ by the linear operation

$$y_t = \sum_{r=-q}^{s} a_r x_{t+r},$$

where $\{a_r\}$ are a set of weights. In order to smooth out local fluctuations and estimate the local mean, we should clearly choose the weights so that $\Sigma a_r = 1$, and the operation is referred as the moving average.

Moving averages are often symmetric with $s = q$ and $a_j = a_{-j}$. The simplest example of a moving average is a symmetric moving average, defined by

$$\mathrm{Sm}(x_t) = \frac{1}{2q+1} \sum_{r=-q}^{+q} x_{t+r}.$$

The simple moving average is not generally recommended by itself for measuring trend, although it can be useful for removing seasonal variation.

Another example is to take $\{a_r\}$ to be successive terms in the expansion of $(\frac{1}{2} + \frac{1}{2})^{2q}$. As q gets large, the weights approximate a normal curve.

A third example is Spencer's 15-point moving average, which is used for smoothing mortality statistics to get life tables. This covers 15 consecutive points with $q = 7$, and the symmetric weights are

$$\frac{1}{320}[-3, -6, -5, 3, 21, 46, 67, 74, 67, 46, 21, \ldots]$$

A fourth example is the Henderson moving average. This moving average aims to follow a cubic polynomial trend without distortion, and the choice of q depends on the degree of irregularity. The symmetric nine-term moving average, for example, is given by

$$[-0.041, -0.010, 0.119, 0.267, 0.330, \ldots]$$

To demonstrate this effect of moving averages, we use the Beveridge wheat price annual index series from 1500 to 1869 as an example (Figure 1.1 link).

3 Some Linear Time Series Models

4 Fitting Time Series Models in the Time Domain

5 Forecasting

6 Stationary Processes in the Frequency Domain

7 Spectral Analysis

8 Bivariate Processes

9 Linear Systems

10 State-Space Models and the Kalman Filter

11 Non-Linear Models

12 Volatility Models

13 Multivariate Time Series Modeling

14 Some More Advanced Topics