Politecnico di Milano

Prova finale: Introduzione all'analisi di missioni spaziali
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Docente:
Colombo

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Autori:

Cod. Persona	Cognome	Nome
10742821	Ferrantino	Michael Pio
10719352	Fiume	Elisa
10766758	Gallo	Emanuele

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1. Introduction

The assignment aims at analysing the standard transfer manoeuvre between two MEO orbits and at designing innovative and convenient ways to accomplish the same purpose. The activity was performed by developing and testing MATLAB® algorithms. Thanks to them it was possible to compare and optimise different strategies, in order to find the most convenient ones in term of cost and time.

2. Initial orbit characterization

2.1. Initial orbital parameters from given position and velocity

The initial conditions were given in terms of position and velocity vectors:

$$\boldsymbol{r_i} = [5557.9661, -6686.4672, 2797.4969]^T km$$

$$\boldsymbol{v_i} = [4.3870, 4.1670, -2.1050]^T \frac{km}{s}$$

The corresponding orbital parameters were determined thanks to the implemented MATLAB® function *car2kep.m*:

Symbols	Parameters	Values
$\boldsymbol{a_i}$	**Semimajor axis** $[\boldsymbol{km}]$	8620.2746
$\boldsymbol{e_i}$	**Eccentricity** $[-]$	0.0724
$\boldsymbol{i_i}$	**Inclination** $[\boldsymbol{deg}]$	26.1196
$\boldsymbol{\Omega_i}$	**RAAN** $[\boldsymbol{deg}]$	88.7241
$\boldsymbol{\omega_i}$	**Argument of periapsis** $[\boldsymbol{deg}]$	76.3963
$\boldsymbol{\theta_i}$	**True anomaly** $[\boldsymbol{deg}]$	147.6870

2.2. Results and relevant data

Thanks to orbital parameters apoapsis and periapsis heights from Earth surface could be calculated:

$$h_{ai} = r_{ai} - r_E = 2866.12\ km$$

$$h_{pi} = r_{pi} - r_E = 1618.43\ km$$

Since those values are included in $[5000, 20000]km$ range, the orbit could be classified as Medium Earth Orbit. Given that $0° < i_i < 90°$, the orbit is prograde. Further observations could be done basing on the value of eccentricity suggests that orbit is almost circular. Moreover, the orbit period, specific energy and semi latus rectum are respectively $T_i \cong 2h\ 12min\ 45s$, $E_i = -23.1199\ km^2/s^2$ and $p_i = 8575.1268\ km$.

2.3. Graphic representation

The graphic representation of the initial orbit follows:

3. Final orbit characterisation

3.1. Final position and velocity from assigned final orbital parameters

Through the given final orbital parameters:

Symbols	Parameters	Values
$\boldsymbol{a_f}$	**Semimajor axis** $[\boldsymbol{km}]$	12770
$\boldsymbol{e_f}$	**Eccentricity** $[-]$	0.2089
$\boldsymbol{i_f}$	**Inclination** $[\boldsymbol{deg}]$	45.1949
$\boldsymbol{\Omega_f}$	**RAAN** $[\boldsymbol{deg}]$	66.4631
$\boldsymbol{\omega_f}$	**Argument of periapsis** $[\boldsymbol{deg}]$	133.9575
$\boldsymbol{\theta_f}$	**anomaly** $[\boldsymbol{deg}]$	62.4524

final position and velocity could be determined using the implemented MATLAB® function *kep2car.m:*

$$\boldsymbol{r_i} = [-2233.5129, -10679.7003, -2232.2711]^T km$$

$$\boldsymbol{v_i} = [4.3773, -1.0833, -4.4760]^T \frac{km}{s}$$

3.2. Results and relevant data

Computing the periapsis and apoapsis height of the final orbit:

$$h_{af} = 9059.6530\ km$$

$$h_{pf} = 3724.3470\ km$$

So, it could be observed that even the final orbit could be considered as MEO. In this case the value of inclination $0° < i_f < 90°$, so, the orbit is prograde. The value of e_f is almost tripled in comparison to initial one. In addition to that, the period is $T_f \cong 3h\ 59min\ 21s$, the specific energy corresponds to $E_f \cong -15.6069 km^2/s^2$ and semi latus rectum $p_f = 12212.7272\ km$.

3.3. Graphic representation

The graphic representation of the final orbit follows:

4. Transfer trajectory definition and analysis

4.1. Ways the final position and velocity can be achieved, starting from the initial orbit

The final position and velocity could be reached by changing shape, size, orbital plane and periapsis argument. In order to do this, different strategies could be developed:

1. 0Strategy 1 (or standard strategy): composed by plane change, change of periapsis argument, periapsis to apoapsis bitangent transfer.
2. Strategy 2 (or direct transfer): composed by a two impulses single manoeuvre that connects directly the initial and final orbits in the most convenient points.
3. Strategy 3: composed by an initial change of eccentricity in a strategical point, to reduce the cost of plane change, then plane change, change of periapsis argument, periapsis to apoapsis bitangent transfer.
4. Strategy 4: composed by an initial shape and size changing transfer to convenient values, then plane change, circularization in a proper point and bitangent transfer to the final orbit.

4.2. Comments and analysis of the chosen strategies

Before truly analysing the chosen strategies, it's important to provide the following details. In all the subsequent strategies those important simplifications are made:

a. The bielliptic transfer is never considered because of its inconvenience in every manoeuvre, as demonstrated in the MATLAB® code *BitangentBiellipticComparison*.m and the proper strategies codes. Define r_A as the apoapsis of initial orbit, r_B as the apoapsis of both elliptic orbits in bielliptic manoeuvre and r_C as the apoapsis of final orbit. Plotting the curves of the cost of bielliptic and bitangent manoeuvres for different values of $x = r_C/r_A$, it could be noticed that the plane is divided into two areas, where one transfer must be preferred above the other. Note that, repeating this argument for different bielliptic curves, selected for different values of $y = r_B/r_A$, the identified intersection point between the two curves moves to the left for increasing value of y. Thus, the minimum value of x ratio could be identified studying the $y \to \infty$ case, whereas the maximum is the point of maximum cost value of bitangent transfer, fixed initial and final orbits. The interval of x values in which those points are included is $I = [14.53, 17.61]$, there the preference for bielliptic above bitangent or vice versa must be analysed case by case, basing on the y value. Graphing what has just been shown:

TABELLA BITANGENTBIELLIPTICCOMPARISON

In the proper codes of the manoeuvres, it's determined the value of x ratio, demonstrating, where necessary, that bielliptic transfer it's not convenient.

b. The bielliptic plane change is never evaluated because it's not convenient compared to one-impulse plane change. From theory it's know that the first one must be preferred in terms of cost above the second one if the vertex angle of spherical triangle $\alpha > 38.94°$. In every case this inequality is not true, as demonstrated code by code.

4.2.1 Strategy 1 (or standard strategy)

In this strategy the satellite is transferred spending Δt_1 time frame from the initial point to the point of plane change, where it's performed. This first impulsive manoeuvre changes $\{i_i \to i_f, \Omega_i \to \Omega_f, \omega_i \to \omega_2\}$. After waiting Δt_2 to get to the point where the second impulse is burnt from the final point of previous transfer, a change of periapsis occurs so that $\{\omega_2 \to \omega_f, \theta_2 \to \theta_3\}$. Awaiting the proper Δt_3 time frame from the final point of the second manoeuvre to the periapsis, a bitangent transfer is performed in Δt_4 so that $\{a_i \to a_f, e_i \to e_f, \theta_{peri} \to \theta_{apo}\}$. This brings to the apoapsis of the final orbit, in which a Δt_5 is necessary to get to the final point.

	$\boldsymbol{\Delta t}$ $[\boldsymbol{s}]$	$\boldsymbol{\Delta v}$ $[\boldsymbol{km/s}]$	Initial position $[\boldsymbol{deg}]$	Final position $[\boldsymbol{deg}]$
Manoeuvre 1 (Trajectory 1)	$\Delta t_1 = 2301.4774$	$\Delta v_1 = 2{,}5977$	$\theta_i = 147.6870$	$\theta_1 = 239.7064$
Manoeuvre 2 (Trajectory 2)	$\Delta t_2 = 2873.3753$	$\Delta v_2 = 0.3306$	$\theta_2 = 239.7064$	$\theta_{2a} = 239.7064$

Trajectory 3	$\Delta t_3 = 374.6061$	$-$	$\theta_{3a} = 340.4269$	$\theta_p = 0$
Manoeuvre 3 (Trajectory 4)	$\Delta t_4 = 6311.1403$	$\Delta v_3 = 1.1146$	$\theta_p = 0$	$\theta_a = 180$
Trajectory 5	$\Delta t_5 = 8887.524781$	$-$	$\theta_a = 180$	$\theta_f = 62.4524$
Total	$\Delta t_{TOT} = 20748.1238$	$\Delta v_{TOT} = 4.04287$	$\theta_i = 147.6870$	$\theta_f = 62.4524$

4.2.2 Strategy 2 (or direct transfer)

In this strategy the satellite is transferred spending Δt_1 time frame from the initial point θ_i to a second point θ_1 in the initial orbit, chosen basing on the convenience of the entire strategy. After this, one impulse $\{a_i \to a_t, e_i \to e_t, i_i \to i_t, \Omega_i \to \Omega_t, \omega_i \to \omega_t\}$ is burnt to get to a transfer orbit, which intersect the final orbit in the point θ_2, spending Δt_2. In θ_2 another impulse $\{a_t \to a_f, e_t \to e_f, i_t \to i_t, \Omega_t \to \Omega_f, \omega_t \to \omega_f\}$ is burnt to merge onto the final orbit. Thus, covering the trajectory on the final orbit to the final point θ_f in a Δt_3 time frame. Notice that the two impulses direct transfer could be realized in multiple ways, however the analysis is limited to elliptic orbits, leaving out unpractical and time spending parabolic and hyperbolic manoeuvres.

4.2.3 Strategy 3

	$\boldsymbol{\Delta t}$ $[\boldsymbol{s}]$	$\boldsymbol{\Delta v}$ $[\boldsymbol{km/s}]$	Initial position $[\boldsymbol{deg}]$	Final position $[\boldsymbol{deg}]$
Trajectory 1	$\Delta t_1 = 5967.1747$	$-$	$\theta_i = 147.6870$	$\theta_2 = 59.7064$
Manoeuvre 1 (Trajectory 2)	$\Delta t_2 = 6426.0852$	$\Delta v_1 = 0.9549$	$\theta_2 = 59.7064$	$\theta_3 = 180$
Manoeuvre 2 (Trajectory 3)	$\Delta t_3 = 6238.2291$	$\Delta v_2 = 1.6780$	$\theta_3 = 180$	$\theta_{4i} = 349.7199$
Manoeuvre 3 (Trajectory 4)	$\Delta t_4 = 12664.3143$	$\Delta v_3 = 0.6564$	$\theta_{4i} = 349.7199$	$\theta_{4f} = 10.2801$
Manoeuvre 4 (Trajectory 5)	$\Delta t_5 = 6423.2426$	$\Delta v_4 = 0.2748$	$\theta_{4f} = 10.2801$	$\theta_a = 180$
Trajectory 6	$\Delta t_6 = 8887.5248$	$-$	$\theta_a = 180$	$\theta_f = 62.4524$
Total	$\Delta t_{TOT} = 46606.5706$	$\Delta v_{TOT} = 3.5647$	$\theta_i = 147.6870$	$\theta_f = 62.4524$

4.2.4 Strategy 4

This strategy consists of spending Δt_{0i} time frame to get from θ_i to the point of the chosen bitangent transfer point θ_{0i}. There the bitangent transfer $\{a_i \to a_{opt}, e_i \to e_{opt}, \theta_{0i} \to \theta_{0f}\}$ is performed in Δt_{0f} between the two most convenient points chosen by the proper algorithm. After a Δt_1 waiting, the plane change $\{i_i \to i_f, \Omega_i \to \Omega_f, \omega_i \to \omega_2\}$ is done. After waiting for Δt_{2i} from the point of plane change θ_{f1} to the proper point for the next manoeuvre θ_{circ}, which is found to be the apoapsis, the circularization $\{a_{opt} \to r_{circ}, e_{opt} \to 0\}$ is done, spending on the circular orbit the time frame Δt_{2f} to arrive at the point of the next manoeuvre θ_{3i}. Finally, in order to merge onto the final orbit, the last bitangent transfer $\{r_{circ} \to a_f, 0 \to e_f, \theta_{3i} \to \theta_{3f}\}$ is completed in Δt_3. In the final orbit it's spent Δt_4 time frame to get to the final point θ_f.

	$\boldsymbol{\Delta t}$ $[\boldsymbol{s}]$	$\boldsymbol{\Delta v}$ $[\boldsymbol{km/s}]$	Initial position $[\boldsymbol{deg}]$	Final position $[\boldsymbol{deg}]$
Trajectory 1	$\Delta t_{0i} = 4800.2466$	$-$	$\theta_i = 147.6870$	$\theta_{0i} = 0$
Manoeuvre 0 (Trajectory 2)	$\Delta t_{0f} = 4283.2320$	$\Delta v_0 = 0.3658$	$\theta_{0i} = 0$	$\theta_{0f} = 180$
Manoeuvre 1 (Trajectory 3)	$\Delta t_1 = 1688.9258$	$\Delta v_1 = 2.4844$	$\theta_{0f} = 180$	$\theta_{1f} = 239.7064$
Manoeuvre 2 (Trajectory 4)	$\Delta t_2 = 7701.6494$	$\Delta v_2 = 0.1591$	$\theta_{1f} = 239.7064$	$\theta_{circ} = 180$
Trajectory 5	$\Delta t_{3i} = 6150.4576$	$-$	$\theta_{circ} = 180$	$\theta_{3i} = 0$

Manoeuvre 3 (Trajectory 6)	$\Delta t_{3f} = 7180.2627$	$\Delta v_3 = 0.6253$	$\theta_{3i} = 0$	$\theta_{3f} = 180$
Trajectory 7	$\Delta t_4 = 8887.5248$	$-$	$\theta_{3f} = 180$	$\theta_f = 62.4524$
Total	$\Delta t_{TOT} = 40692.2987$	$\Delta v_{TOT} = 3.6346$	$\theta_i = 147.6870$	$\theta_f = 62.4524$

Discuss and describe the possible transfer strategies, motivate the selection of one orbit transfer strategy and calculate the transfer trajectory, the manoeuvres v and transfer time.

4.3. Graphical representation of initial, final and transfer orbits

4.3.1 Strategy 1 (or standard strategy)

4.3.2 Strategy 2 (or direct transfer)

4.3.3 Strategy 3

4.3.4 Strategy 4

Graphical representation of the initial, final and transfer orbit.

5. Conclusions

Briefly compare and analyse the presented transfer trajectories

6. Appendix

Fill the table below for each transfer presented in Section 4. The first and last row correspond to the given initial and final points, respectively. All the other 2*N rows report the time and the orbital parameters across the N impulsive manoeuvres Δv_i.

Table 1: Strategy 1 (or standard strategy)

$t\ [s]$	$a\ [km]$	e [−]	$i\ [deg]$	$\Omega[deg]$	$\omega\ [deg]$	θ [deg]	$\Delta v\ [km/s]$
0	8620.27464	0.0724	26.1196	88.7241	76.3963	147.687013	–
2301.4774	8620.27464	0.0724	26.1196	88.7241	76.3963	239.7064	2.597715973
	8620.27464	0.0724	45.1949	66.4631	94.8114	239.7064	
5174.8526	8620.27464	0.0724	45.1949	66.4631	94.8114	19.5731	0.330592222
	8620.27464	0.0724	45.1949	66.4631	133.9575	340.4269	
5549.4587	8620.27464	0.0724	45.1949	66.4631	133.9575	0	0.792782412
	11717.0396	0.3175	45.1949	66.4631	133.9575	0	
11860.599	11717.0396	0.3175	45.1949	66.4631	133.9575	180	0.32178023
	12770	0.2089	45.1949	66.4631	133.9575	180	
20748.1238	12770	0.2089	45.1949	66.4631	133.9575	62.4524	

Table 2: Strategy 2 (or direct transfer)

Table 3: Strategy 3

$t\ [s]$	$a\ [km]$	e [−]	$i\ [deg]$	$\Omega[deg]$	$\omega\ [deg]$	θ [deg]	$\Delta v\ [km/s]$
0	8620.27464	0.0724	26.1196	88.7241	76.3963	147.687013	–
5967.1747	8620.27464	0.0724	26.1196	88.7241	76.3963	59.7064	0.9549
	11858.8795	0.3023	26.1196	88.7241	136.1026	180	
12393.2599	11858.8795	0.3023	45.1949	66.4631	136.1026	180	1.6780
	11858.8795	0.3023	45.1949	66.4631	154.5178	180	
18631.4890	11858.8795	0.3023	45.1949	66.4631	154.5177	349.7199	0.6564
	11858.8795	0.3023	45.1949	66.4631	133.9575	10.2801	
31295.8033	11858.8795	0.3023	45.1949	66.4631	133.9575	0	0.006
	11855.3821	0.3021	45.1949	66.4631	133.9575	0	
37719.0459	11855.3821	0.3022	45.1949	66.4631	133.9575	180	0.2748
	12770	0.2089	45.1949	66.4631	133.9575	180	
46606.5706	12770	0.2089	45.1949	66.4631	133.9575	62.4524	

Table 4: Strategy 4

$t\ [s]$	$a\ [km]$	e [−]	$i\ [deg]$	$\Omega[deg]$	$\omega\ [deg]$	θ [deg]	$\Delta v\ [km/s]$
0	8620.27464	0.0724	26.1196	88.7241	76.3963	147.687013	–
4800.2466	8620.27464	0.0724	26.1196	88.7241	76.3963	0	0.1483
	9048.8572	0.1163	26.1196	88.7241	76.3963	0	
9083.4785	9048.8572	0.1163	26.1196	88.7241	76.3963	180	0.2175
	9620.2746	0.0500	26.1196	88.7241	76.3963	180	
10772.4043	9620.2746	0.3023	26.1196	88.7241	76.3963	239.7063	2.4844
	9620.2746	0.3023	45.1949	66.4631	94.8114	239.7063	
18474.0536	9620.2746	0.3023	45.1949	66.4631	94.8114	180	0.1591
	10101.2883	0	45.1949	66.4631	–	180	
24624.5113	10101.2883	0	45.1949	66.4631	–	0	0.6251
	12769.4707	0.2090	45.1949	66.4631	133.9575	0	

31804.7739	12769.4707	0.2090	45.1949	66.4631	133.9575	180	0.0001
	12770	0.2089	45.1949	66.4631	133.9575	180	
40692.2987	12770	0.2089	45.1949	66.4631	133.9575	62.4524	