Experimental Apparatus Finding the Specific Heat of a Ceramic Mug

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Introduction

The goal of this experiment is to design an experimental apparatus capable of measuring the specific heat of a ceramic coffee mug. The constraints presented require that the uncertainty of the measured value not exceed ±7% total uncertainty for the specific heat and that the liquid volume be twice that of the solid. The range of specific heat values is between 0.6 and 0.9 $\frac{kJ}{kgK}$.

Experimental Apparatus and Procedure

Experimental Apparatus

The experimental setup consists of three beakers labeled A, B, and C, each containing water. Beakers A and B are maintained at room temperature, while beaker C is heated to an elevated, controlled temperature Figure 1. Temperature measurements are obtained using calibrated thermometers. The mass measurements were conducted using a food weigh scale.

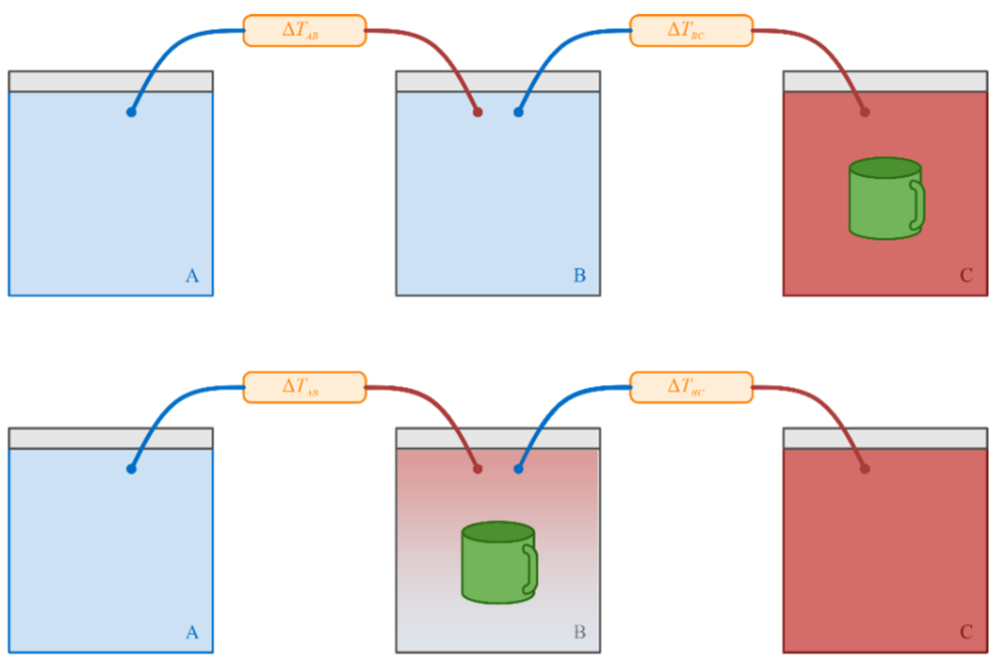

Figure 1: Apparatus used for the experiment and procedure to find the specific heat of the ceramic coffee mug.

Procedure

Prior to the experiment, the mass of the solid specimen and an empty beaker B is measured using the balance. The mass of the water in beaker B is also measured and recorded taking care to subtract the mass of the empty beaker. The masses of the water in beakers A and C are not required for analysis. Care is taken to ensure that beaker B contains sufficient water to fully submerge the solid specimen; specifically, the liquid volume must be at least twice the volume of the solid to guarantee complete immersion.

The solid specimen is first immersed in beaker C and allowed to remain there for an extended duration to ensure thermal equilibrium. At this stage, it is assumed that the initial temperature of the solid, T_{1s}, is equal to the temperature of beaker C, T_C. The solid is then removed from beaker C using tongs and immediately transferred into beaker B. The specimen is fully submerged in the water in beaker B.

The temperature of beaker B is monitored until it stabilizes, indicating that thermal equilibrium has been reached between the solid and the water. Once stabilization occurs, the temperature differences $\Delta T_{AB}$and ΔT_{BC} are recorded. After equilibrium in beaker B is established, it is assumed that the final temperature of the solid, T_{2s}, is equal to the temperature of beaker B, T_B. It is further assumed that the temperatures of beakers A and C experience negligible change over the course of the experiment.

Data Reduction

The specific heat of a ceramic mug, C_s, is determined from an energy balance using the measured temperature change of the water in beaker B. The analysis is based on the finite (integral) conservation of energy for a closed system:

$$\Delta E_{sys} = Q_{net,in} - W_{net,out} \tag{1}$$

The system is defined as the water in beaker B and the ceramic mug. ΔE_{sys} is composed of two states, the final and the initial. The initial state will be defined when the ceramic mug is in beaker C at equilibrium. The final state will be defined when the ceramic mug reaches equilibrium in beaker B. The following assumptions are applied:

1. The process occurs over a short time interval such that heat exchange with the surroundings is negligible, so

$$Q_{net,in} \approx 0 \tag{2}$$

2. No shaft work or boundary work is performed, so

$$W_{net,out} \approx 0 \tag{3}$$

3. Changes in kinetic and potential energy are negligible, and the beaker and thermometer have negligible heat capacity.

Under these conditions,

$$\Delta E_{sys} = \Delta U_{sys} \approx 0 \tag{4}$$

The internal energy change of the system is the sum of the internal energy changes of the water and the mug:

$$\Delta U_{water} + \Delta U_{mug} = 0 \tag{5}$$

Assuming constant specific heats over the measured temperature range, each term is modeled as

$$\Delta U = mC\left(T_f - T_i\right) \tag{6}$$

Let the water in beaker B initially be at T_A, the mug initially be at T_C, and the final equilibrium temperature be T_B. Substituting gives

$$M_l C_l (T_B - T_A) + M_s C_s (T_B - T_C) = 0 \tag{7}$$

Where M_L is the mass of water, C_L is the specific heat capacity of water, M_S is the mass of the ceramic mug, and C_S is the specific heat capacity of the mug. Defining the measured temperature changes as

$$\Delta T_{AB} = T_B - T_A \text{ (water temperature rises)} \tag{8}$$

$$\Delta T_{BC} = T_C - T_B \text{ (mug temperature drops)} \tag{9}$$

Where in Equation 8, T_B and T_A are the temperatures in Beaker A and Beaker B. In Equation 9 T_C is the temperature in Beaker C. Substituting (8) & (9) into (7) gives

$$M_L C_L \Delta T_{AB} = M_S C_S \Delta T_{BC} \tag{10}$$

Where ΔT_{AB} and ΔT_{BC} are the temperature differences simplified. This gives us our final DRE equation where the specific heat of the mug is

$$C_s = \frac{M_L C_L \Delta T_{AB}}{M_S \Delta T_{BC}} \tag{11}$$

Uncertainty Budget

The Data Reduction Equation was used to find both uncertainty tables. The following tables show the final relative uncertainty of the minimum and maximum ranges of C_s, 0.6 $\frac{kJ}{kgK}$ and 0.9 $\frac{kJ}{kgK}$. To see the original uncertainty budget and values, see Appendix B. Both Table 1 and Table 2 show that the selected measurands will provide total relative uncertainty below the ±7% threshold.

Table 1 summarizes the representative values, systematic uncertainties, relative uncertainties, and the uncertainty percent contributions for the minimum range of 0.6 $\frac{kJ}{kgK}$.

Table 1: The greatest source of uncertainty is ΔT_{AB}. The selected result of 0.6 $\frac{kJ}{kgK}$ fits under the chosen relative uncertainty of ±7%.

Parameter	Units	Representative Value	Uncertainty	Relative Uncertainty (%)	UMF	RSSC (%)	UPC (%)
ΔT_{AB}	K	7.4	0.4	5.41	1	5.41	85.0
ω_{random}	$\frac{kJ}{kgK}$	[redacted]	[redacted]	2	1	2	11.6
ΔT_{BC}	K	51.6	0.4	0.78	1	0.78	1.74
M_S	kg	0.25	0.0013	0.52	1	0.52	0.787
M_L	kg	0.25	0.0013	0.52	1	0.52	0.787
C_L	$\frac{kJ}{kgK}$	4.182	0.0042	0.10	1	0.1	0.0291
C_S	$\frac{kJ}{kgK}$	0.6	[redacted]	5.86	[redacted]	[redacted]	100

As shown in Table 1, most of the measurands exhibit small relative uncertainties; however, ΔT_{AB} stands out with a relative uncertainty of 5.41%. This is because ΔT_{AB} was chosen as the temperature difference that needed a higher relative uncertainty. The reasoning for feeding ΔT_{AB} is because the difference in temperature is assumed to be greater than in ΔT_{BC}. This resulted in a relative uncertainty of C_s that is well under the maximum relative uncertainty. The total relative uncertainty of 5.86%.

Table 2 is designed for $C_s = 0.9 \frac{kJ}{kgK}$. Most of the parameters exhibit relatively small percentage uncertainties.

Table 2: The greatest source of uncertainty is ΔT_{AB}. The selected result of $0.9 \frac{kJ}{kgK}$ fits under the chosen relative uncertainty of ±7%.

Parameter	Units	Representative Value	Uncertainty	Relative Uncertainty (%)	UMF	RSSC (%)	UPC (%)
ΔT_{AB}	K	10.4	0.4	3.85	1	3.85	73.9
ω_{random}	$\frac{kJ}{kgK}$	[redacted]		2	1	2	19.9
ΔT_{AB}	K	48.6	0.4	0.82	1	0.82	3.38
M_S	kg	0.25	0.0013	0.52	1	0.52	1.35
M_L	kg	0.25	0.0013	0.52	1	0.52	1.35
C_L	$\frac{kJ}{kgK}$	4.182	0.0042	0.10	1	0.1	0.0499
C_S	$\frac{kJ}{kgK}$	0.9	[redacted]	4.47	[redacted]		100

However, ΔT_{AB} stands out yet again with a relative uncertainty value of 3.85%. The relative uncertainty of C_S is well under the maximum relative uncertainty with a value of 4.47%. The reasoning for ΔT_{AB} having a larger uncertainty is the same as in Table 1. Notice that in both Table 1 and Table 2, the C_L has the lowest uncertainty value. The reason for the low uncertainty value can be seen in Table 4. The C_L uncertainty value is taken from an online resource, and therefore not measured with a device (W. Wagner, 1995).

Design Minimums & Practical Maxes

From the beginning of our process, we assumed the random uncertainty of our experiment to be 2% and calculated a 3% uniform relative uncertainty for each measurand. To find this blanket value, we utilized the following Equation 12

$$\omega_{C_S,rel}^2 = \omega_{C_L,rel}^2 + \omega_{M_S,rel}^2 + \omega_{M_L,rel}^2 + \omega_{\Delta T_{AB},rel}^2 + \omega_{\Delta T_{BC},rel}^2 + \omega_{Random}^2 \tag{12}$$

Where the $\omega_{C_S,rel}$ is the total relative uncertainty. $\omega_{C_L,rel}$ is the relative uncertainty of the coefficient of specific heat for the water. The $\omega_{M_S,rel}$ and $\omega_{M_L,rel}$ variables are the relative uncertainty of the masses, and $\omega_{\Delta T_{AB},rel}$ and $\omega_{\Delta T_{BC},rel}$ are the relative uncertainties of the temperature differences. To solve for the total relative uncertainty $\omega_{relative\ total}$ Equation 12 was simplified to Equation 13

$$\omega_{C_S,rel}^2 = 5\omega_{relative\ total}^2 + \omega_{random}^2 \tag{13}$$

With the 3% uniform relative uncertainty, each measurand's design minimum value was found using the following Equation 14

$$x = \frac{\omega_{sys}}{3\%} \tag{14}$$

Where x is the design minimum value found and ω_{sys} is the systematic uncertainty.

With the design minimum found, the maximum value for each measurand was a practical maximum that was set either by the measuring device limitations or physical limitations. For further details, see Appendix B.

By using a uniform relative uncertainty, the minimum and maximum of every measurand was calculated but when the resultant was calculated it did not fall within the desired range. For further details see Appendix B. The process of using a blanket relative uncertainty was scraped and instead more relative uncertainty was placed on ΔT_{AB} while the other measurands received less relative uncertainty.

Shown in Table 3 are the new relative uncertainties along with the selected values of our measurands. The instruments we determine that we would utilize in our experiment are also mentioned in Table 3.

Table 3: The minimums and maximums of each measurand along with their respected instrument used to measure them and the uncertainty in those instruments for the final relative uncertainties.

Measurands	Min	Max	Instrument	Uncertainty
M_S	0.08 kg	0.4 kg	Weigh Scale	0.001
M_L	0.08 kg	0.4 kg	Weigh Scale	0.001
C_L	4.182 $\frac{kJ}{kgK}$.	4.182 $\frac{kJ}{kgK}$.	Textbook	0.001
ΔT_{AB}	7.4	10.4	Differential Thermometers	0.4
ΔT_{BC}	48.6	51.6	Differential Thermometers	0.4

As seen above in Table 3, the maximum values did not change for the controlled measurands, but the minimum values did. These values changed in respect to our first minimum values that are seen in Appendix B. Notice in Table 3 that the minimum and maximum for C_L is the same. This is because C_L is a coefficient and not a number we calculate. This does not mean it is a discovery variable but is instead a set value used for this apparatus (W. Wagner, 1995).

Design Space Plots

Design Space Plot One

Referring to Figure 2, notice that the design space plot shows a direct correlation between the constraints placed on the first discovery variable ΔT_{BC} and the range of the other variables. The two black dashed lines represent the minimum and maximum of the discovery variable. All the colored lines represent values that are not the discovery variable, such as the solid lines seen as slopes, being the minimum and maximum resultant ranges. While the colored lines are the maximum and minimum values of the DRE equation with ΔT_{BC} cancelled out.

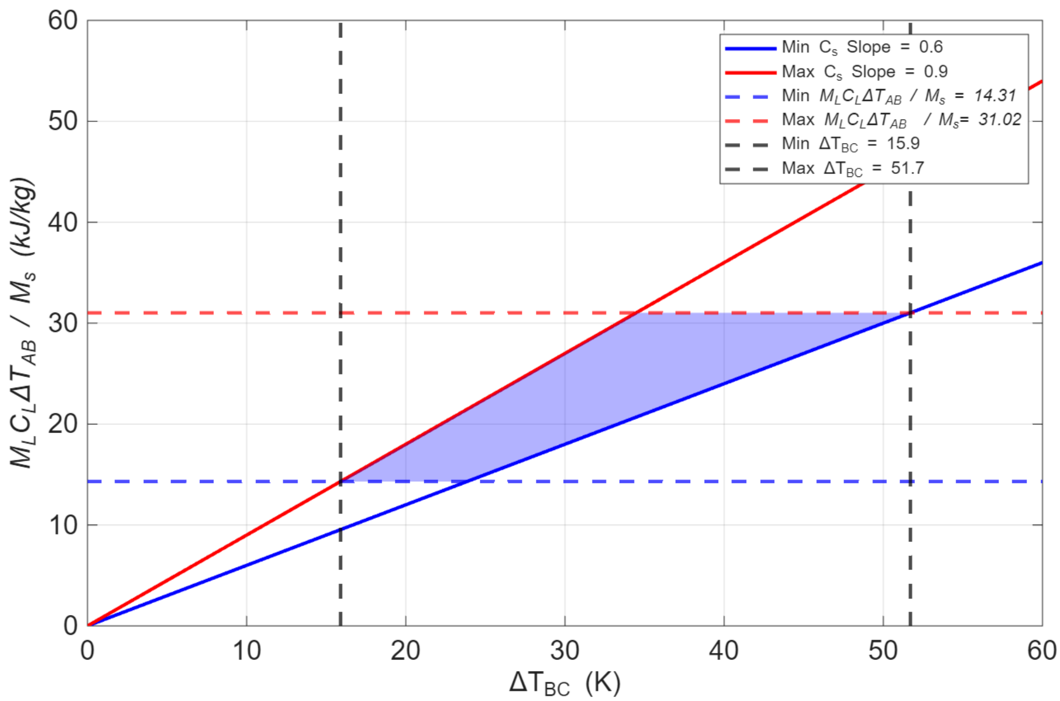

Figure 2: The discovery variable ΔT_{BC} limitations on the x-axis of the plot, find acceptable values for the measurands on the y-axis. This defines a feasible design space.

From Figure 2, see that the correlation between the discovery variable and the altered DRE equation will set points in which the range of the other variables can be in. This will play a role in which values are chosen for our apparatus. However, ΔT_{BC} is not the only discovery variable so another design space plot with a discovery variable will be needed to have the final set points in which the experiment can operate in.

Design Space Plot Two

In Figure 3, there is once again a correlation between the discovery variables, practical maximum and design minimum, and the controlled measurands that will not change during the experimental procedure. The dotted colored lines represent the minimum and maximum values the control measurands can have. The slope for Figure 3 is the minimum and maximum values of the control measurands found from Figure 2. The black dotted lines are the minimum and maximum values of the discovery variable $\frac{1}{\Delta T_{AB}}$.

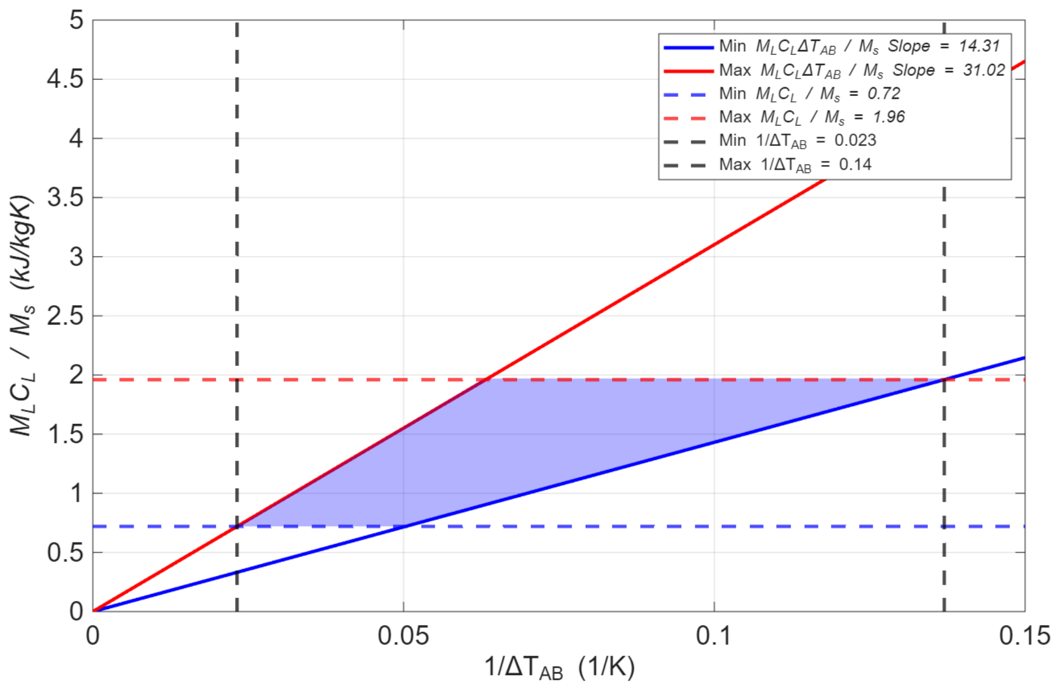

Figure 3: The second discovery variable $\frac{1}{T_{AB}}$ limitations on the x-axis of the plot, find acceptable values for the measurands on the y-axis. This defines a feasible design space.

For the second discovery variable from Figure 3, notice that we are once again constraining the control measurands on the y-axis to allow us to pick values that will keep us under the ±7% total uncertainty. A key difference between Figure 2 and Figure 3 is that the slope of Figure 3 is not the resultant slope, but instead the max and minimum values of the control measurands found from design space plot one.

Design Space Plot Three

Figure 4 represents the design space plot for the measurand M_S on the x-axis and the remaining measurands on the y-axis. M_S has a minimum design and practical maximum represented on Figure 4 as the vertical black dotted lines. The slopes represented in solid red and blue represent the set points from Figure 3. The colored horizontal dotted lines represent the minimum and max values that the remaining measurands can have. All this together forms the workable area of the design space.

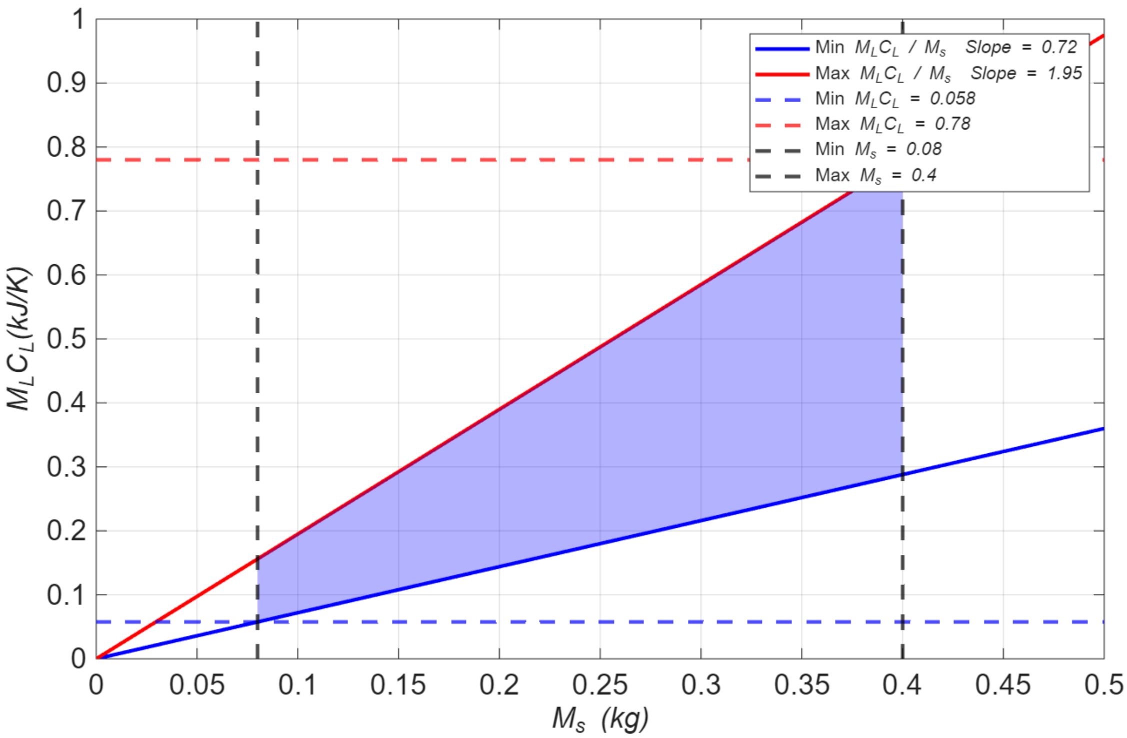

Figure 4: The isolated measurand M_S sets limits on the remaining measurands. The design space shows the appropriate values that the other controlled measurands must fit into.

In Figure 4 there is a subtle difference in the design space plot. This affects the measurands selected. Instead of the design space fitting between where the lower slope intersects with the maximum y value and the higher slope intersects with the minimum y value, it instead flips. The intersections between the two minimums and two maximums now define the design space. This is an important difference as it signifies that both axes are now being defined by control measurands instead of having discovery variables.

Design Space Plot Four

For the final design space plot seen in Figure 5, the same representation of the measurands is used as in Figure 4. The slopes represent the max and minimum values found from Figure 4's design space plot.

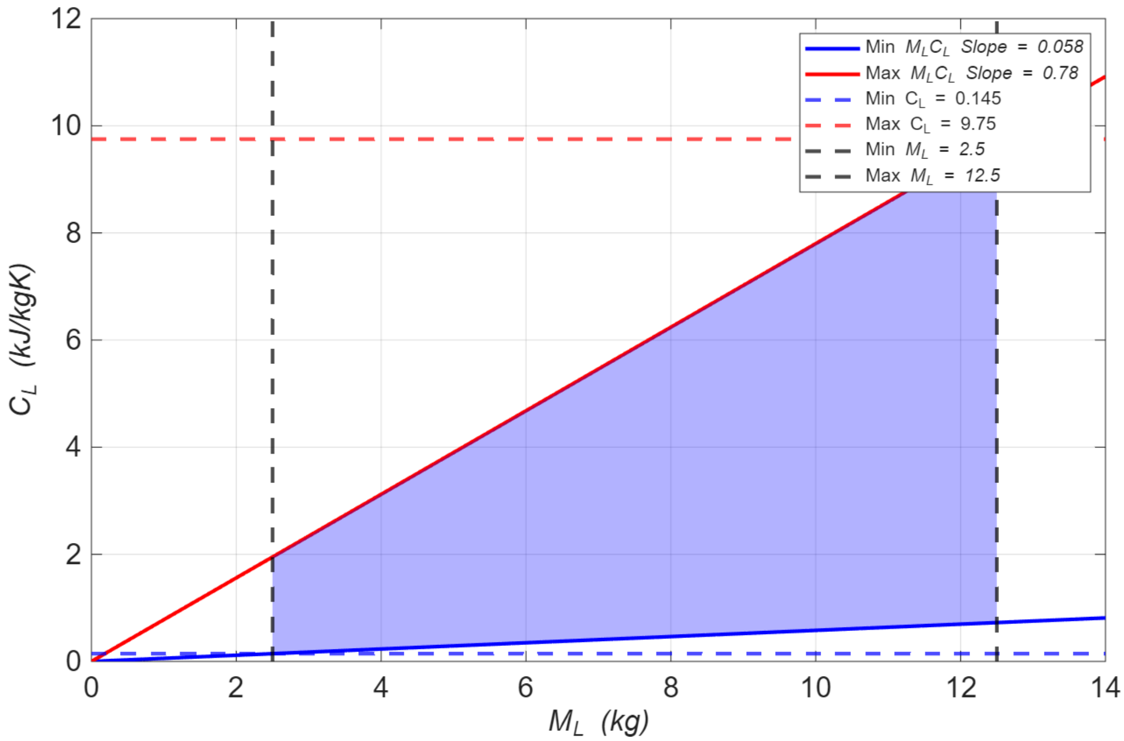

Figure 5: The remaining measurands set limits for each other. This design space plot is where selected values are first chosen.

The design space plot from Figure 5 is the last design space plot. The values of our measurands will first be selected from here. Two things to note are the low minimum slope and minimum y value. The reason behind this is that the small values chosen for both masses.

Using all four of these design space plots, we confirm that the values we selected in Table 1 and Table 2 are valid for our apparatus. For more information on the creation of these plots, refer to Appendix A.

Conclusion

The objective of this project was to design an experimental apparatus capable of measuring the specific heat of a ceramic coffee mug while maintaining a total relative uncertainty within ±7%. Using the derived DRE, uncertainty propagation, and four design space plots, we developed an uncertainty budget that satisfies this constraint across the expected specific heat range of 0.6–0.9 $\frac{kJ}{kgK}$.

For the lower bound of 0.6 $\frac{kJ}{kgK}$, the total relative uncertainty was calculated to be 5.86%, and for the upper bound of 0.9 $\frac{kJ}{kgK}$, the total relative uncertainty was 4.47%. Both values fall below the ±7% design requirement. Additionally, the experimental design satisfies the imposed physical constraint that the liquid volume be at least twice that of the solid.

Overall, the finalized apparatus design meets all performance and physical constraints, demonstrating that the selected measurands, uncertainties, and design limits are sufficient to achieve the project objective.

Appendix

Appendix A- MATLAB Code

Design Plot 1

```
       % Tempratio X-axis
tempRatio_x = linspace(0, 60, 1000);
% Slopes
Cs_low_slope = 0.6*tempRatio_x;
Cs_high_slope = 0.9*tempRatio_x;
% First Discovery Variable Temperature Ratio
difftemp_min = 15.9; % uncertainty / relative of 3%
difftemp_max = 51.7;

% Creating 1st Design Plot
% Design plot 1s y-axis min and max
DP1_ymin = 0.6*difftemp_max;
DP1_ymax = 0.9*difftemp_min;
% HOW I MADE THE X AND Y SHADING
x_rhombus = [15.9  34.5435    51.7  23.85];
y_rhombus = [14.31  31.02    31.02  14.31];
% Plotting Design Plot 1
figure(1)
plot(tempRatio_x, Cs_low_slope, 'b-','LineWidth', 2) % LineWidth adds thickness to line)
hold on
plot(tempRatio_x,Cs_high_slope,'r-','LineWidth', 2) % LineWidth adds thickness to line)
hold on
yline(DP1_ymax,'b--','LineWidth', 2) % LineWidth adds thickness to line)
hold on
yline(DP1_ymin,'r--','LineWidth', 2) % LineWidth adds thickness to line)
hold on
xline(15.9, 'k--','LineWidth', 2) % LineWidth adds thickness to line)
hold on
xline(51.7,'k--','LineWidth', 2) % LineWidth adds thickness to line)
hold on
% Fill command for appropriate values
fill(x_rhombus, y_rhombus, 'b', 'FaceAlpha', 0.3, 'EdgeColor', 'none')
hold off
grid on
legend("Min C_s Slope = 0.6", "Max C_s Slope = 0.9","Min \it{M}_LC_L\DeltaT_{AB} / \it{M}_s =
14.31", "Max \it{M}_LC_L\DeltaT_{AB}  / \it{M}_s= 31.02", 'Min \DeltaT_{BC} = 15.9', "Max
\DeltaT_{BC} = 51.7")
xlabel("\DeltaT_{BC} (K)")
ylabel("\it{M}_LC_L\DeltaT_{AB} / \it{M}_s (kJ/kg)")
```

```
fontsize(16,"points")
fontsize(legend, 10, 'points')
```

Design Plot 2

```
% Second Discovery Variable Temperature Ratio
difftemp_min2 = 1/43.1;
difftemp_max2 = 1/7.3;

% Tempratio X-axis
tempRatio_x2 = linspace(0, 0.15, 100);

% Creating 2nd Design Plot

% Slopes
DP2_low_slope = 14.31*tempRatio_x2;
DP2_high_slope = 31.02*tempRatio_x2;

% Design plot 1s y-axis min and max
DP2_ymin = 14.31*difftemp_max2;
DP2_ymax = 31.02*difftemp_min2;

% HOW I MADE THE X AND Y SHADING
x_rhombus = [1/43.1  0.063    1/7.3  0.05];
y_rhombus = [0.72  1.97    1.97  0.72];

% Plotting Design Plot 2
figure(2)
plot(tempRatio_x2, DP2_low_slope, 'b-','LineWidth', 2) % LineWidth adds thickness to line)
hold on
plot(tempRatio_x2,DP2_high_slope,'r-','LineWidth', 2) % LineWidth adds thickness to line)
hold on
yline(DP2_ymax,'b--','LineWidth', 2) % LineWidth adds thickness to line)
hold on
yline(DP2_ymin,'r--','LineWidth', 2) % LineWidth adds thickness to line)
hold on
xline(difftemp_max2,'k--','LineWidth', 2) % LineWidth adds thickness to line
hold on
xline(difftemp_min2,'k--','LineWidth', 2) % LineWidth adds thickness to line
fill(x_rhombus, y_rhombus, 'b', 'FaceAlpha', 0.3, 'EdgeColor', 'none')
grid on
legend("Min \DeltaT_{BC} Slope = 14.31", "Max \DeltaT_{BC} Slope = 31.02", "Min \it{M}_LC_L /
\it{M}_s = 0.72", "Max \it{M}_LC_L / \it{M}_s = 1.96", "Min 1/\DeltaT_{AB} = 0.023", "Max
1/\DeltaT_{AB} = 0.14")
xlabel("1/\DeltaT_{AB} (1/K)")
ylabel("\it{M}_LC_L / {\it{M}_s }(kJ/kgK)")
fontsize(16,"points")
fontsize(legend, 10, 'points')
```

Design Plot 3

```
% 1st Regular Variable (Mass of Solid)
mass_solid_min = 0.08; % kg
mass_solid_max = 0.4;

% Mass of Solid X-axis
mass_solid_x = linspace(0, 0.5, 100);

% Creating 3rd Design Plot

% Slopes
DP3_low_slope = 0.72*mass_solid_x;
DP3_high_slope = 1.95*mass_solid_x;

% Design plot 3s y-axis min and max
DP3_ymin = 0.72*mass_solid_min;
DP3_ymax = 1.95*mass_solid_max;

% HOW I MADE THE X AND Y SHADING
x_rhombus = [0.08  0.08    0.4  0.4];
y_rhombus = [0.058  0.157    0.78  0.288];

% Plotting Design Plot 3
figure(3)
plot(mass_solid_x,DP3_low_slope, 'b-','LineWidth', 2) % LineWidth adds thickness to line)
hold on
plot(mass_solid_x,DP3_high_slope,'r-','LineWidth', 2) % LineWidth adds thickness to line)
hold on
yline(DP3_ymin,'b--','LineWidth', 2) % LineWidth adds thickness to line)
hold on
yline(DP3_ymax,'r--','LineWidth', 2) % LineWidth adds thickness to line)
hold on
xline(mass_solid_min,'k--','LineWidth', 2) % LineWidth adds thickness to line
hold on
xline(mass_solid_max,'k--','LineWidth', 2) % LineWidth adds thickness to line
fill(x_rhombus, y_rhombus, 'b', 'FaceAlpha', 0.3, 'EdgeColor', 'none')
grid on
legend("Min M_s Slope = 0.72", "Max M_s Slope = 1.95", "Min \it{M}_LC_L = 0.058", "Max
\it{M}_LC_L = 0.78", 'Min \it{M}_s = 0.08', 'Max \it{M}_s = 0.4')
xlabel("\it{M}_s (kg)")
ylabel("\it{M}_LC_L(kJ/K)")
fontsize(16,"points")
fontsize(legend, 10, 'points')
```

Design Plot 4

```
% 2nd Regular Variable (Mass of Liquid)
mass_liq_min = 1/0.4; % kg
mass_liq_max = 1/0.08;

% Mass of Solid X-axis
mass_liq_x = linspace(0, 14, 100);

% Creating 4th Design Plot

% Slopes
DP4_low_slope = 0.058*mass_liq_x;
DP4_high_slope = 0.78*mass_liq_x;

% Design plot 4s y-axis min and max
DP4_ymax = 0.058*mass_liq_min;
DP4_ymin = 0.78*mass_liq_max;

% HOW I MADE THE X AND Y SHADING
x_rhombus = [2.5  2.5    12.5  12.5];
y_rhombus = [0.145  1.95    9.75  0.725];

% Plotting Design Plot 4
figure(4)
plot(mass_liq_x, DP4_low_slope, 'b-','LineWidth', 2) % LineWidth adds thickness to line)
hold on
plot(mass_liq_x,DP4_high_slope,'r-','LineWidth', 2) % LineWidth adds thickness to line)
hold on
yline(DP4_ymax,'b--','LineWidth', 2) % LineWidth adds thickness to line)
hold on
yline(DP4_ymin,'r--','LineWidth', 2) % LineWidth adds thickness to line)
hold on
xline(mass_liq_min,'k--','LineWidth', 2) % LineWidth adds thickness to line)
hold on
xline(mass_liq_max,'k--','LineWidth', 2) % LineWidth adds thickness to line)
hold on
fill(x_rhombus, y_rhombus, 'b', 'FaceAlpha', 0.3, 'EdgeColor', 'none')
grid on
legend("Min M_L Slope = 0.058", "Max M_L Slope = 0.78", "Min C_L = 0.145", "Max C_L = 9.75", "Min
\it{M}_L = 2.5",'Max \it{M}_L = 12.5')
xlabel("\it{M}_L (kg)")
ylabel("\it{C}_L (kJ/kgK)")
fontsize(16,"points")
fontsize(legend, 10, 'points')
```

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Appendix B- First Uncertainty Budget & First Min/Max

The first integration of calculating the discovery variables with our selected measurands utilizing a blanketed 3% relative uncertainty shown in Table 4.

Table 4: The 3% distribution of allowable uncertainty for each measurand and the 2% for random uncertainty are shown below.

$\omega^2_{C_S,rel}$	$\omega^2_{C_L,rel}$	$\omega^2_{M_S,rel}$	$\omega^2_{M_L,rel}$	$\omega^2_{\Delta T_{AB},rel}$	$\omega^2_{\Delta T_{BC},rel}$	ω^2_{Random}
$7\%^2$	$3\%^2$	$3\%^2$	$3\%^2$	$3\%^2$	$3\%^2$	$2\%^2$

With utilizing the 3% relative uncertainty the minimum and maximum ranges for the measurands were calculated. The maximum mass of the solid mug was determined by the standard mass of a large coffee mug, while the mass of the water was calculated by using a scale. As well as staying within the constraint that the volume of the water is twice that of the solid.

However, the temperature difference maxima were discovered by recognizing the limitations of liquid water. Water boils at 100°C so we chose a max temperature value of beaker C to be 80 °C to leave a comfortable distance between the temperature of water in our experiment and water's boiling point. We also recognized that for beakers A and B to be at a room temperature of 21°C, that leaves a difference of 59°C between 80°C and 21°C. With the minimum value known for each temperature difference and the total difference the practical maximum for both temperature differences were found to be 45°C. The minimum and maximum for C_l are the same value because C_l is assumed as a constant, and all the minimum and maximum ranges for the measurands are placed in Table 5.

Table 5: The original minimum and maximum values for each measurand using the 3% distribution of allowable uncertainty for each measurand and the 2% for random uncertainty.

Controlled Measurand	Minimum	Maximum	Instrument	Uncertainty
M_S	0.04 kg	0.4 kg	Weigh Scale	0.001
M_L	0.04 kg	0.4 kg	Weigh Scale	0.001
C_L	4.182 $\frac{kJ}{kgK}$.	4.182 $\frac{kJ}{kgK}$.		0.001
ΔT_{AB}	14	45	Differential Thermometers	0.4
ΔT_{BC}	14	45	Differential Thermometers	0.4

When using the minimum and maximum values from the 3% relative uncertainty the resultant value was found to be outside of the goal range for our experiment. To rectify this, we added more relative uncertainty of ΔT_{AB} to lower the minimum range value to be able to calculate the minimum resultant of 0.6. The new relative uncertainty values were calculated for each measurement using Equation 3 and completing multiple iterations until the uncertainty goal was achieved and the ranges for the temperature differentials were correct. Table 6 shows the updated distribution of allowable uncertainty for each measurand that would allow C_s to be within the desired value range.

Table 6: The new distribution of allowable uncertainty for each measurand and the 2% for random uncertainty to expand the minimum and maximum ranges for ΔT_{AB} and ΔT_{AB}

$\omega^2_{C_S,rel}$	$\omega^2_{C_L,rel}$	$\omega^2_{M_S,rel}$	$\omega^2_{M_L,rel}$	$\omega^2_{\Delta T_{AB},rel}$	$\omega^2_{\Delta T_{BC},rel}$	ω^2_{Random}
$7\%^2$	$1.74\%^2$	$1.63\%^2$	$1.63\%^2$	$5.5\%^2$	$2.54\%^2$	$2\%^2$

Bibliography

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