Gökberk Altıparmak

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CS202-3

Homework 1

Question 1

a) Prove $F(n) = 8n^4 + 5n^2 + 7$ is $O(n^5)$

$F(n) <= cn^5$ for some $n >= n_0$

If, $8n^4 + 5n^3 + 7 <= cn^5$ then, $8/n + 5/n^2 + 7/n^5 <= c$

Thus, big O notation only holds for $n >= (n_0 = 1)$ and $c >= 20(8 + 5 + 7)$

b) Selection Sort

Note: Left side of the sign "*" is the unsorted and right side is sorted part. "+" on the left top of the number is symbolize the biggest number in the unsorted array (ex: $number^+$)

Initial Array = [22 8 49^+ 25 18 30 20 15 35 27^*]

After First Step = [22 8 25 18 30 20 15 35^+ 27^* 49]

After Second Step = [22 8 25 18 30^+ 20 15 27^* 35 49]

After Third Step = [22 8 25 18 20 15 27^{+*} 30 35 49]

After Fourth Step = [22 8 25^+ 18 20 15^* 27 30 35 49]

After Fifth Step = [22^+ 8 18 20 15^* 25 27 30 35 49]

After Sixth Step = [8 18 20^+ 15^* 22 25 27 30 35 49]

After Seventh Step = [8 18^+ 15^* 20 22 25 27 30 35 49]

After Eighth Step = [8 15^{+*} 18 20 22 25 27 30 35 49]

After Ninth Step = [8^{+*} 15 18 20 22 25 27 30 35 49]

After Tenth Step = [*8 15 18 20 22 25 27 30 35 49]

Bubble Sort

Note: “()” indicates where the swap action will be.

Initial Array = [(22 8) 49 25 18 30 20 15 35 27*]

First Pass:

=> [8 (22 49) 25 18 30 20 15 35 27*]

=> [8 22 (49 25) 18 30 20 15 35 27*]

=> [8 22 25 (49 18) 30 20 15 35 27*]

=> [8 22 25 18 (49 30) 20 15 35 27*]

=> [8 22 25 18 30 (49 20) 15 35 27*]

=> [8 22 25 18 30 20 (49 15) 35 27*]

=> [8 22 25 18 30 20 15 (49 35) 27*]

=> [8 22 25 18 30 20 15 35 (49 27)*]

=> [8 22 25 18 30 20 15 35 27*49]

In the same way

Second Pass:

=> [8 22 18 25 20 15 30 27*35 49]

Third Pass:

=> [8 18 22 20 15 25 27*30 35 49]

Fourth Pass:

=> [8 18 20 15 22 25*27 30 35 49]

Fifth Pass:

=> [8 18 15 20 22*25 27 30 35 49]

Sixth Pass:

=> [8 15 18 20*22 25 27 30 35 49]

Seventh Pass:

=> [8 15 18*20 22 25 27 30 35 49]

Eight Pass:

=> [8 15*18 20 22 25 27 30 35 49]

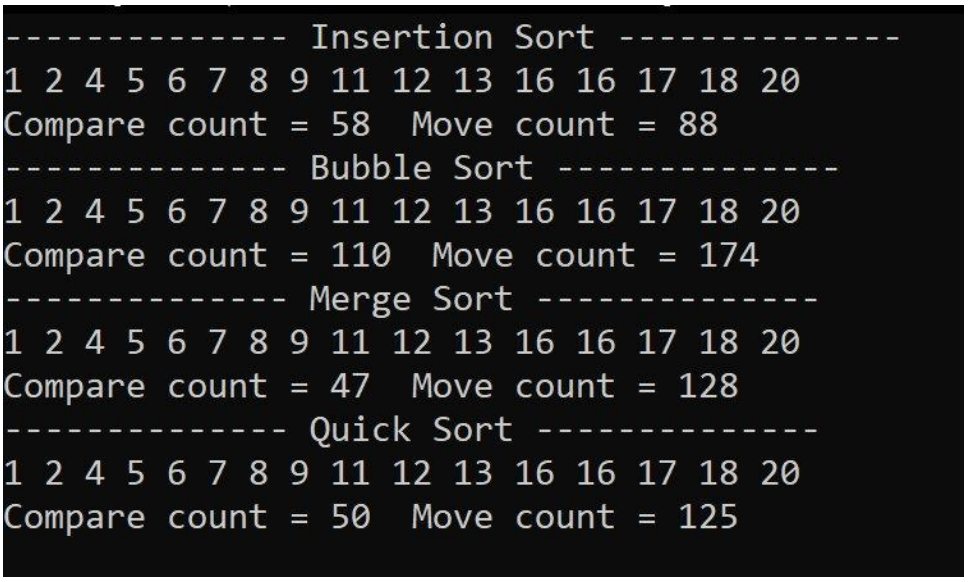
Ninth Pass;

=> [8*15 18 20 22 25 27 30 35 49]

Tenth Pass:

=> [*8 15 18 20 22 25 27 30 35 49]

Question 2



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------------Random Created Arrays------------
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Analysis of insertion sort
Array size        Elapsed Time          compCount          moveCount
5000                15.690000ms          6392320          6402318

10000                58.954600ms          25077951          25097949

15000                127.219100ms          56079024          56109022

20000                228.269800ms          99988509          100028507

25000                354.940600ms          155623484          155673482

30000                503.189300ms          225522423          225582421

35000                697.589100ms          304278999          304348997

40000                910.700200ms          398632716          398712714

------------------------------
Analysis of bubble sort
Array size        Elapsed Time          compCount          moveCount
5000                62.321900ms          12486175          19176960

10000                273.457700ms          49966080          75233853

15000                615.359500ms          112489944          168237072

20000                1143.577100ms          199948959          299965527

25000                1776.993500ms          312473639          466870452

30000                2684.540100ms          449935230          676567269

35000                3546.019500ms          612480847          912836997

40000                4799.646200ms          799950597          1195898148

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Analysis of merge sort
Array size        Elapsed Time          compCount          moveCount
5000                0.884200ms          55177          123616

10000                1.875800ms          120491          267232

15000                2.612000ms          189261          417232

20000                3.674700ms          260957          574464

25000                4.875200ms          333959          734464

30000                6.776500ms          408584          894464

35000                6.237200ms          484377          1058928

40000                7.455400ms          561802          1228928

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Analysis of quick sort
Array size        Elapsed Time          compCount          moveCount
5000                0.476500ms          75892          116775

10000                1.095000ms          154289          237460

15000                1.574900ms          235751          345057

20000                2.077400ms          326930          518112

25000                2.700500ms          422066          701546

30000                4.286700ms          567272          939524

35000                3.835900ms          630932          968971

40000                4.496000ms          718896          1064699
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------------Almost Sorted Arrays------------
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Analysis of insertion sort
Array size        Elapsed Time          compCount          moveCount
5000                0.127600ms          47351          57349

10000                0.241300ms          94851          114849

15000                0.347800ms          142351          172349

20000                0.528100ms          209829          249827

25000                0.704800ms          262329          312327

30000                0.736300ms          284851          344849

35000                0.995900ms          367329          437327

40000                1.107600ms          419829          499827

------------------------------
Analysis of bubble sort
Array size        Elapsed Time          compCount          moveCount
5000                24.932500ms          12461989          142053

10000                97.753200ms          49994790          314487

15000                220.364600ms          111344130          384666

20000                392.388200ms          199473364          569553

25000                608.244800ms          311685489          712053

30000                883.188600ms          448835114          854553

35000                1166.815700ms          610922239          997053

40000                1664.717000ms          799979790          1259487

------------------------------
Analysis of merge sort
Array size        Elapsed Time          compCount          moveCount
5000                0.625800ms          32103          123616

10000                1.229100ms          69101          267232

15000                2.410300ms          106480          417232

20000                2.528800ms          185611          574464

25000                3.129300ms          234666          734464

30000                3.755600ms          281882          894464

35000                4.223800ms          269474          1058928

40000                5.308500ms          316134          1228928

------------------------------
Analysis of quick sort
Array size        Elapsed Time          compCount          moveCount
5000                1.637400ms          1024925          94358

10000                16.390100ms          11298869          76575

15000                35.437100ms          23822857          119633

20000                141.276200ms          53441341          61380052

25000                194.119800ms          80861369          83022327

30000                307.245800ms          123197497          129768973

35000                442.018900ms          168869522          207810193

40000                273.308500ms          177164743          311436
```

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------------Almost Unsorted Arrays------------
------------------------------
Analysis of insertion sort
Array size        Elapsed Time          compCount          moveCount
5000                7.859000ms          3392134          3402132

10000               29.115300ms          12059419          12079417

15000               48.795600ms          21555156          21585154

20000               104.093800ms          44920941          44960939

25000               160.159400ms          70036689          70086687

30000               235.810600ms          100111899          100171897

35000               313.557100ms          140620268          140690266

40000               371.211400ms          155955541          156035539

------------------------------
Analysis of bubble sort
Array size        Elapsed Time          compCount          moveCount
5000                35.056100ms          11558365          11032224

10000               130.347400ms          45228672          36568269

15000               265.941600ms          97947579          70244541

20000               485.903500ms          176655304          129985389

25000               765.867500ms          277026669          206094639

30000               1118.751400ms          399869934          299701797

35000               1470.396100ms          545231497          410520897

40000               2124.192900ms          715174224          547046244

------------------------------
Analysis of merge sort
Array size        Elapsed Time          compCount          moveCount
5000                0.686300ms          44707          123616

10000               1.369900ms          89753          267232

15000               1.930500ms          139415          417232

20000               2.761700ms          230288          574464

25000               3.294200ms          293659          734464

30000               4.005900ms          358682          894464

35000               4.877200ms          351873          1058928

40000               5.719900ms          409023          1228928

------------------------------
Analysis of quick sort
Array size        Elapsed Time          compCount          moveCount
5000                2.399300ms          1442664          72830

10000               8.042900ms          5319642          185890

15000               18.780300ms          12394099          260726

20000               82.512000ms          18224217          54335971

25000               150.617000ms          34951428          104329307

30000               164.756500ms          37589803          112033136

35000               265.228200ms          60211051          179937722

40000               130.986300ms          85375552          1017943
```
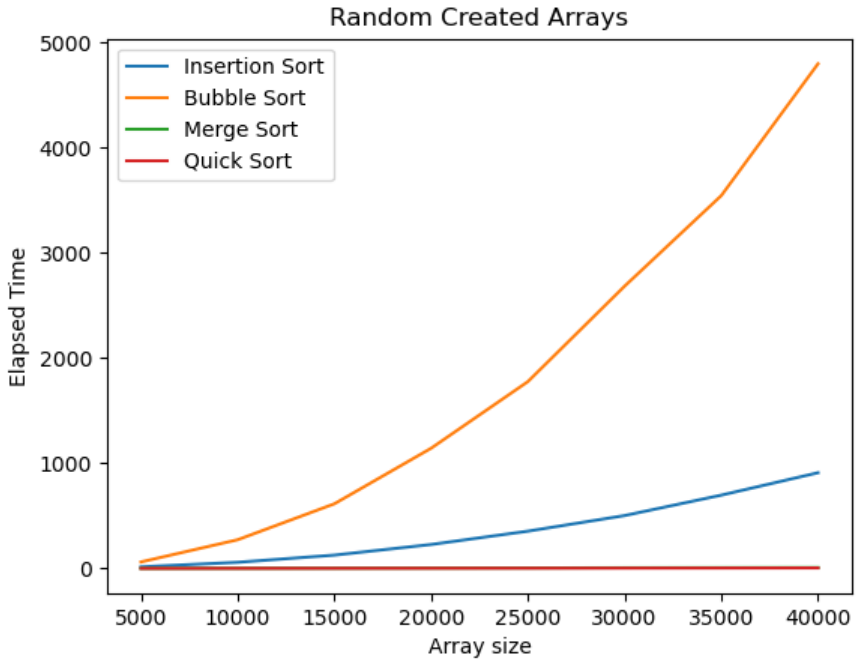
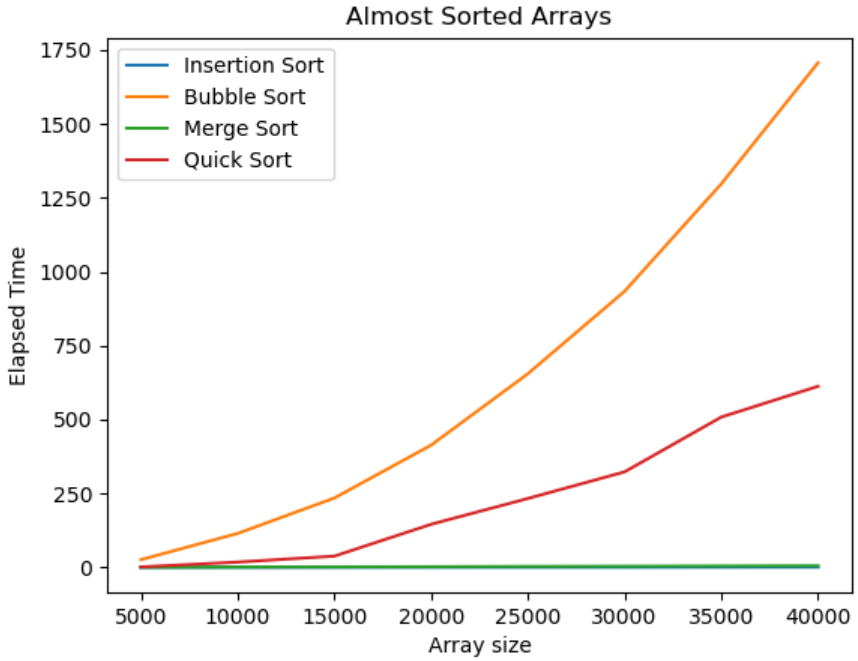
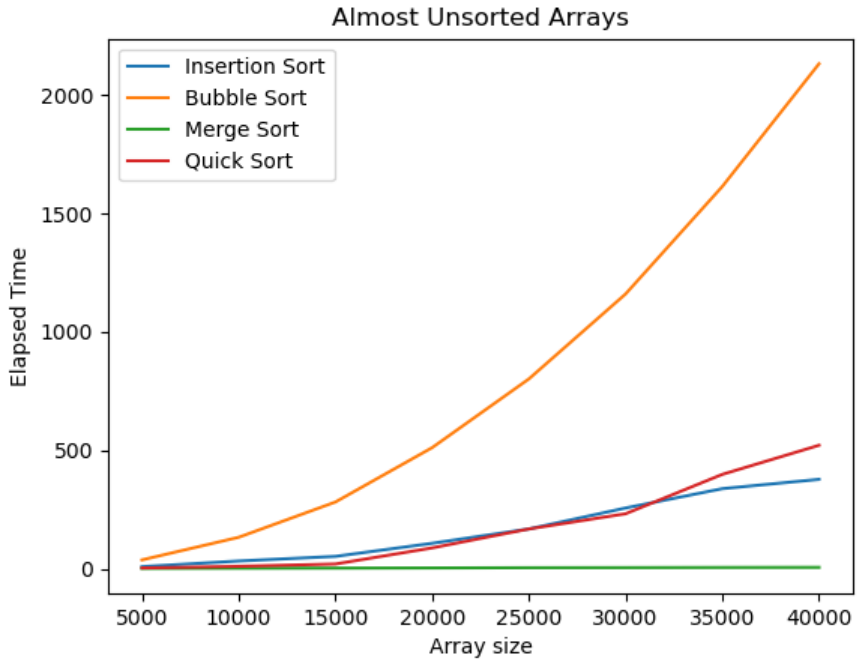

Question 3


Random Created Arrays
Insertion Sort
Bubble Sort
Merge Sort
Quick Sort
Elapsed Time
Array size
5000
4000
3000
2000
1000
0
5000
10000
15000
20000
25000
30000
35000
40000


Almost Sorted Arrays
Insertion Sort
Bubble Sort
Merge Sort
Quick Sort
Elapsed Time
Array size
1750
1500
1250
1000
750
500
250
0
5000
10000
15000
20000
25000
30000
35000
40000



- As expected, merge and quick sort algorithms performed better than bubble and insertion sort algorithms. However, I was expected to see that for each graph quick sort algorithm performed better. I am assuming, taking the first integer in array as a pivot number posed this problem for sorted and unsorted arrays and also it is no longer meaningful to divide the list into half with recursion for quick sort algorithm. Therefore, its time complexity behaves like $O(n^2)$ instead of O(nlog(n)).
- Theoretical results of merge sort are O(nlog(n)) for worst-case, best-case and average-case. Empirical results approve the conditions as it seems on the graphs.
- Theoretical results of bubble sort are $O(n^2)$ for average-case and worst-case, O(n) for best case. Empirical results approve the conditions.
- Theoretical results of insertion sort are $O(n^2)$ for average-case and worst-case, O(n) for best case. Empirical results approve the conditions.