Intermediate Microeconomics. Lecture 4

Utility

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Utility function

A utility function is a way of assigning a number to every possible consumption bundle such that more-preferred bundles get assigned larger numbers than less-preferred bundles

$$(x_1, x_2) \succ (y_1, y_2) \Leftrightarrow u(x_1, x_2) > u(y_1, y_2)$$

The only property of a utility assignment that is important is how it orders the bundles of goods

Perfect Substitutes

Remember the red pencil and blue pencil example? All that mattered to the consumer was the total number of pencils. Thus it is natural to measure utility by the total number of pencils

$$u(x_1, x_2) = x_1 + x_2$$

Suppose now that the consumer would require two units of good 2 to compensate him for giving up one unit of good 1. This means that good 1 is twice as valuable to the consumer as good 2

$$u(x_1, x_2) = 2x_1 + x_2$$

Perfect Substitutes

In general, preferences for perfect substitutes can be represented by a utility function of the form

$$u(x_1, x_2) = ax_1 + bx_2$$

where a and b are some positive numbers that measure the "value" of goods 1 and 2 to the consumer

$$MRS_{12} = \frac{a}{b}$$

Perfect Complements

- This is the left shoe–right shoe case. In these preferences the consumer only cares about the number of pairs of shoes he has, so it is natural to choose the number of pairs of shoes as the utility function
- The number of complete pairs of shoes that you have is the minimum of the number of right shoes you have, x_1, and the number of left shoes you have, x_2

$$u(x_1, x_2) = min\{x_1, x_2\}$$

Perfect Complements

What about the consumer who always uses 2 teaspoons of sugar with each cup of tea? If x_1 is the number of cups of tea available and x_2 is the number of teaspoons of sugar available

$$u(x_1, x_2) = min\{x_1, \frac{1}{2}x_2\}$$

Any monotonic transformation of this utility function will describe the same preferences

$$u(x_1, x_2) = min\{2x_1, x_2\}$$

Perfect Complements

In general, a utility function that describes perfect-complement preferences is given by

$$u(x_1, x_2) = min\{ax_1, bx_2\}$$

where a and b are positive numbers that indicate the proportions in which the goods are consumed

Quasilinear Preferences

Suppose that a consumer has indifference curves that are vertically “shifted” versions of one indifference curve.

$$u(x_1, x_2) = v(x_1) + x_2$$

In this case the utility function is linear in good 2, but nonlinear in good 1. Hence the name quasilinear utility, means “partly linear” utility.

Quasilinear Preferences

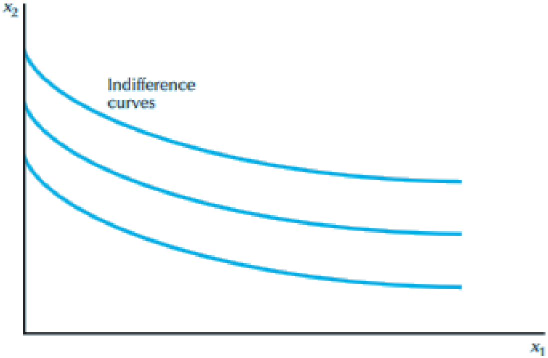

Figure: Indifference Curves for Quasilinear Preferences (Source: Varian, Intermediate Microeconomics 8e, 2010)

Quasilinear Preferences

Specific examples of quasilinear utility would be

$$u(x_1, x_2) = \sqrt{x_1} + x_2$$

$$u(x_1, x_2) = ln(x_1) + x_2$$

Cobb-Douglas Preferences

One of the most commonly used utility functions is the Cobb-Douglas utility function

$$u(x_1, x_2) = x_1^c x_2^d$$

where c and d are positive numbers that describe the preferences of the consume

Cobb-Douglas Preferences

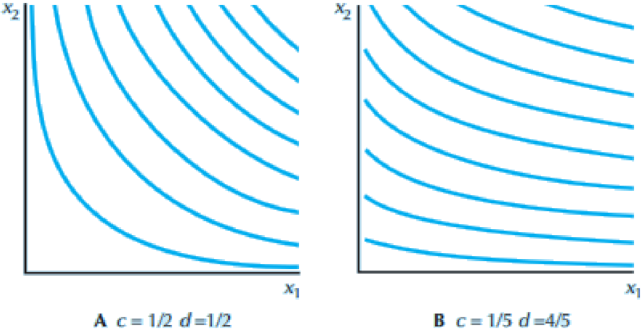

Figure: Indifference Curves for Cobb-Douglas Preferences (Source: Varian, Intermediate Microeconomics 8e, 2010)

Cobb-Douglas Preferences

Let us consider a monotonic transformation of the Cobb-Douglas utility function by raising to the $\frac{1}{c+d}$ power

$$u(x_1, x_2) = x_1^{\frac{c}{c+d}} x_2^{\frac{d}{c+d}}$$

Define $a = \frac{c}{c+d}$. We can now write our utility function as

$$u(x_1, x_2) = x_1^a x_2^{1-a}$$

This is a Cobb-Douglas utility function where the exponents sum to 1

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Marginal Utility

Consider a consumer who is consuming some bundle of goods, (x_1, x_2). How does this consumer's utility change as we give him or her a little more of good 1? This rate of change is called the marginal utility with respect to good 1

$$MU_{x_1} = \frac{\partial u(x_1, x_2)}{\partial x_1}$$

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Marginal Rate of Substitution

- The MRS measures the slope of the indifference curve at a given bundle of goods
- It can be interpreted as the rate at which a consumer is just willing to substitute a small amount of good 2 for good 1

$$MRS_{12} = \frac{MU_{x_1}}{MU_{x_2}}$$

Example: Quasilinear Preferences

Let us compute the MRS with quasilinear preferences

$$u(x_1, x_2) = \sqrt{x_1} + x_2 = x_1^{\frac{1}{2}} + x_2$$

First step, compute the marginal utility for each good:

$$MU_{x_1} = \frac{\partial u(x_1, x_2)}{\partial x_1} = \frac{1}{2} x_1^{-\frac{1}{2}}$$

$$MU_{x_2} = \frac{\partial u(x_1, x_2)}{\partial x_2} = 1$$

Example: Quasilinear Preferences

Second step, use the formula of MRS

$$MRS_{12} = \frac{MU_{x_1}}{MU_{x_2}} = \frac{\frac{1}{2}x_1^{-\frac{1}{2}}}{1}$$

$$MRS_{12} = \frac{1}{2}\frac{x_1^{-\frac{1}{2}}}{1} = \frac{1}{2}\frac{1}{x_1^{\frac{1}{2}}} = \frac{1}{2x_1^{\frac{1}{2}}}$$

Example: Cobb-Douglas Preferences

Let us compute the MRS with Cobb-Douglas preferences

$$u(x_1, x_2) = x_1^a x_2^b$$

First step, compute the marginal utility for each good:

$$MU_{x_1} = a x_1^{a-1} x_2^b$$

$$MU_{x_1} = b x_1^a x_2^{b-1}$$

Example: Cobb-Douglas Preferences

Second step, use the formula of MRS

$$MRS_{12} = \frac{MU_{x_1}}{MU_{x_2}} = \frac{a x_1^{a-1} x_2^b}{b x_1^a x_2^{b-1}}$$

$$MRS_{12} = \frac{a}{b} \frac{x_1^a x_2^b x_2}{x_1^a x_2^b x_1}$$

$$MRS_{12} = \frac{a}{b} \frac{x_2}{x_1}$$