Intermediate Microeconomics. Lecture 14

The firm's optimization problem

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Summer 2021

Contents

Questions from PS3

Consider the following production function

$$f(K,L) = 100\sqrt{KL} = 100(KL)^{0.5}$$

- If currently K is fixed at 9, write down an equation for the short-run production function that shows output as a function of labor only

$$f(K,L) = 300L^{0.5}$$

- Calculate production, MP_L and AP_L for L=1,2,3 and 4.

Questions from PS3

Suppose that a firm's production function is given by

$$f(K,L) = K^{0.33}L^{0.67}$$

Find the marginal product of both inputs

$$MP_K = 0.33K^{-0.67}L^{0.67}$$

$$MP_L = 0.67K^{0.33}L^{-0.33}$$

- As L increases the marginal product of labor decreases
- As K increases the marginal product of labor increases

Questions from PS3

Consider the following production function

$$f(K, L) = 4K^{0.5}L^{0.5}$$

Find the marginal product of both inputs

$$MP_K = 2K^{-0.5}L^{0.5}$$

$$MP_L = 2K^{0.5}L^{-0.5}$$

- Do both labor and capital display diminishing marginal products? **Yes**

Questions from PS3

Find the marginal rate of technical substitution for this production function

$$MRTS = \frac{MP_L}{MP_K} = \frac{2K^{0.5}L^{-0.5}}{2K^{-0.5}L^{0.5}} = \frac{K^{0.5}K^{0.5}}{L^{0.5}L^{0.5}}$$

$$\boxed{MRTS = \frac{K}{L}}$$

- Does this production function exhibit a diminishing marginal rate of substitution? **Yes**

Questions from PS3

Consider the following production function

$$f(K, L) = 4K^{0.75}L^{0.25}$$

Find the marginal product of both inputs

$$MP_K = 3K^{-0.25}L^{0.25}$$

$$MP_L = K^{0.75}L^{-0.75}$$

The isocost function is

$$C = 10K + 2L$$

Questions from PS3

- What ratio of capital to labor minimizes total costs?

$$MRTS = \frac{\omega}{r}$$

$$\frac{1}{3}\frac{K}{L} = \frac{1}{5}$$

$$\frac{K}{L} = \frac{3}{5} \Rightarrow 0.6 : 1$$

Questions from PS3

- How much capital and labor to rent and hire in order to produce 1,000 units

$$f(K, L) = 4K^{0.75}L^{0.25}$$

$$1,000 = 4(\frac{3}{5}L)^{0.75}L^{0.25} \Rightarrow 1,000 = 2.7269L \Rightarrow \boxed{L = 367}$$

$$K = \frac{3}{5}L \Rightarrow K = \frac{3}{5}(367) \Rightarrow \boxed{K = 220}$$

Contents

Cost minimization

Our minimization problem is

$$min \;\; \omega_1 x_1 + \omega_2 x_2$$

st

$$y = f(x_1, x_2)$$

Use FOC to find the marginal products in order to compute MRTS and set this equal to the factor price ratio

$$MRTS = \frac{\omega_1}{\omega_2}$$

Substitute this optimal condition in the production function and solve for the factor demands

Cost minimization

Consider the following minimization problem

$$min \ \ \omega_1 x_1 + \omega_2 x_2$$

st

$$y = x_1^{\alpha} x_2^{1-\alpha}$$

Show that the optimal factor demands are

$$\boxed{x_1^* = \left[\frac{1-\alpha}{\alpha}\frac{\omega_2}{\omega_1}\right]^{\alpha} y}$$

$$\boxed{x_2^* = \left[\frac{\alpha}{1-\alpha}\frac{\omega_1}{\omega_2}\right]^{1-\alpha} y}$$

Contents

Profit maximization

The profits a firm receives, π, can be expressed as

$$\pi = p \cdot f(x_1, ..., x_n) - \sum_{i=1}^{n} \omega_i x_i$$

or, if we only have two goods

$$\pi = p \cdot f(x_1, x_2) - \omega_1 x_1 - \omega_2 x_2$$

where the first term is revenue, and the second term is cost

Profit maximization

The profit-maximization problem of the firm is

$$max \;\; pf(x_1, x_2) - \omega_1 x_1 - \omega_2 x_2$$

FCO

$$p\frac{\partial f(x_1, x_2)}{\partial x_1} - \omega_1 = 0$$

$$p\frac{\partial f(x_1, x_2)}{\partial x_2} - \omega_2 = 0$$

Profit maximization

Let's see an example with a Cobb-Douglas production function. The profit-maximization problem of the firm is

$$max \;\; px_1^a x_2^b - \omega_1 x_1 - \omega_2 x_2$$

FCO

$$pax_1^{a-1} x_2^b - \omega_1 = 0$$

$$pbx_1^a x_2^{b-1} - \omega_2 = 0$$

Profit maximization

Multiply the first equation by x_1 and the second equation by x_2 to get

$$pax_1^a x_2^b - \omega_1 x_1 = 0$$

$$pbx_1^a x_2^b - \omega_2 x_2 = 0$$

Using $y = x_1^a x_2^b$ to denote the level of output of this firm we can rewrite these expressions as

$$pay = \omega_1 x_1$$

$$pby = \omega_2 x_2$$

Profit maximization

Solving for x_1 and x_2 we have

$$\boxed{x_1^* = \frac{pa}{\omega_1} y}$$

$$\boxed{x_2^* = \frac{pb}{\omega_2} y}$$

- This gives us the demands for the two factors as a function of the optimal output choice
- But we still have to solve for the optimal choice of output

Profit maximization

Inserting the optimal factor demands into the Cobb-Douglas production function, we have the expression

$$y = \left[\frac{pa}{\omega_1} y\right]^a \left[\frac{pb}{\omega_2} y\right]^b$$

Factoring out the y gives

$$\boxed{y = \left[\frac{pa}{\omega_1}\right]^{\frac{a}{1-a-b}} \left[\frac{pb}{\omega_2}\right]^{\frac{b}{1-a-b}}}$$

- Along with the factor demand functions derived above it gives us a complete solution to the profit-maximization problem
- Note that when the firm exhibits constant returns to scale (a + b = 1) this supply function is not well defined