NATURAL MONOPOLY

$$f(x) \wedge g(x)$$

$$\frac{d f(x) \cdot g(x)}{dx} = \frac{d f(x)}{dx} \cdot g(x) + f(x) \cdot \frac{d g(x)}{dx}$$

$$f(x) = x$$

$$\Pi = TR(Q) - TC(Q)$$

FOC

$$\underbrace{\frac{dTR(\cdot)}{dQ}}_{MR} - \underbrace{\frac{dTC(\cdot)}{dQ}}_{MC} = 0$$

$$MC = 0 \Rightarrow \boxed{MR = MC}$$

$$Q = 200 - \frac{1}{5}P$$

$$\frac{1}{5}P = 200 - Q$$

$$P = 1,000 - 5Q \quad \longleftarrow \text{Inv. demand}$$

$$TR = PQ = (1,000 - 5Q)*Q$$

$$TR = 1,000Q - 5Q^2 \quad \longleftarrow$$

$$MR = \frac{d\,TR(Q)}{dQ} = \frac{d(1{,}000Q - 5Q^2)}{dQ}$$

$$MR = 1{,}000 - 2(5)Q^{2-1}$$

$$MR = 1{,}000 - 10Q$$

$$Q = 10 - \frac{2}{5}P \qquad\qquad MC = 5Q$$

$$\frac{2}{5}P = 10 - Q \implies P = \frac{5}{2}10 - \frac{5}{2}Q$$

$$\boxed{P = 25 - \frac{5}{2}Q} \qquad TR = P \cdot Q$$

$$TR = P(Q) \cdot Q = \left(25 - \frac{5}{2}Q\right) * Q$$

$$TR = 25Q - \frac{5}{2}Q^2$$

$$MR = \frac{d\,TR(Q)}{dQ} = \frac{d\left(25Q - \frac{5}{2}Q^2\right)}{dQ}$$

$$MR = 25 - 2\left(\frac{5}{2}\right)Q^{2-1} = 25 - 5Q$$

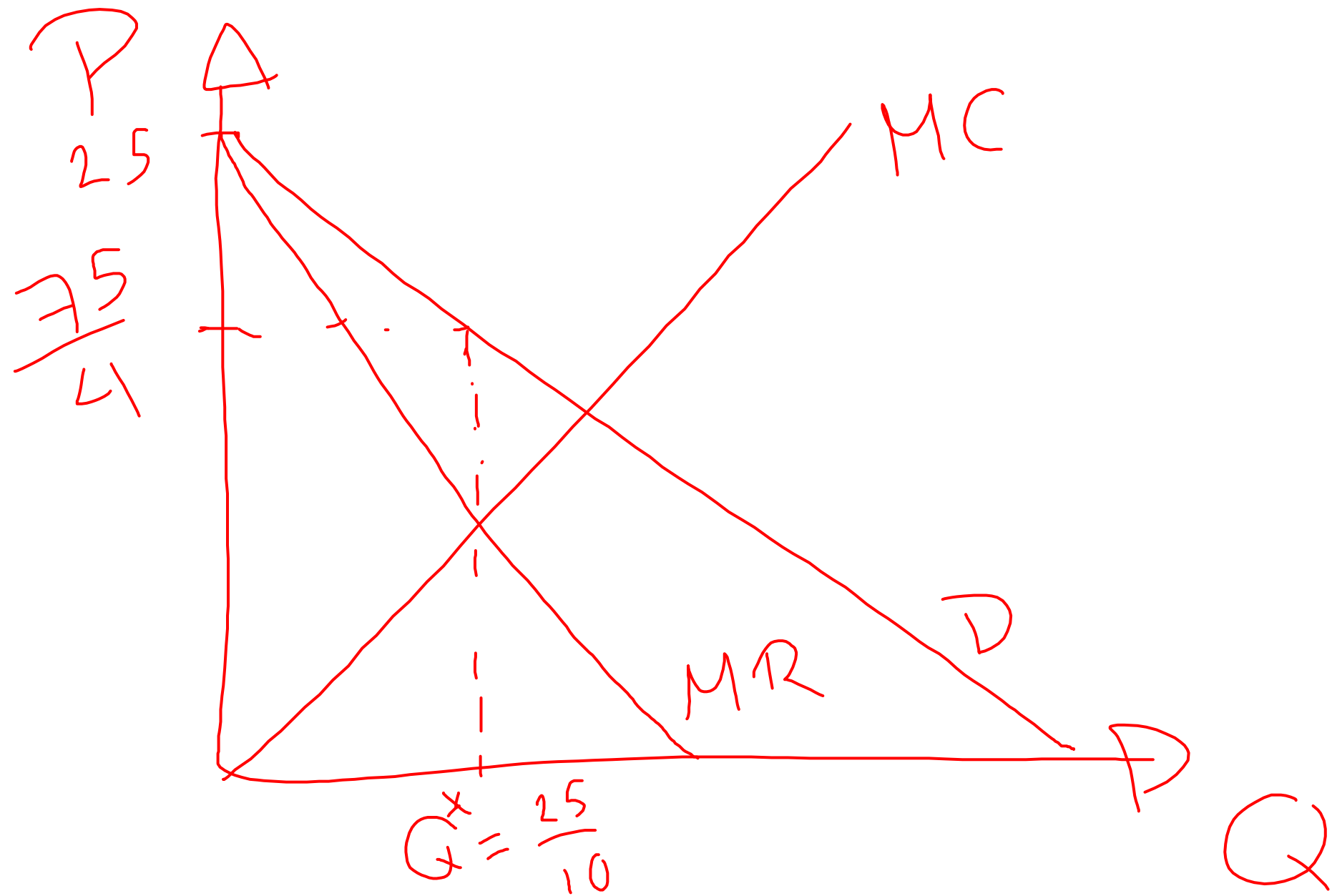$$MR = MC$$

$$25 - 5Q = 5Q \implies 10Q = 25$$

$$\boxed{Q^* = \frac{25}{10} = 2.5}$$

$$\boxed{Q^* = \frac{25}{10}}$$

$$P(Q) = 25 - \frac{5}{2}Q$$

$$P = 25 - \frac{5}{2}\left(\frac{25}{10}\right) = 25 - \frac{25}{4}$$

$$P = \frac{4(25)}{4} - \frac{25}{4} = \frac{75}{4} = 18.75$$

$$P + \left(\frac{1}{E^D}\right) P = MC$$

$$\frac{P}{P} + \left(\frac{1}{E^D}\right)\frac{P}{P} = \frac{MC}{P}$$

$$\frac{P}{P} - \frac{MC}{P} = -\frac{1}{E^D}$$

$$\boxed{\frac{P - MC}{P} = -\frac{1}{E^D}}$$

$$P = 1,000 - 5Q \qquad MC = 250$$

$$TR = P(Q) * Q = (1,000 - 5Q) * Q = 1,000Q - 5Q^2$$

$$MR = \frac{d\,TR(\cdot)}{dQ} = 1,000 - 10Q$$

$$MR = MC$$

$$1,000 - 10Q = 250$$

$$10Q = 750 \implies \boxed{Q^* = 75}$$

$$P = 1,000 - 5(75) = 1000 - 375$$

$$\boxed{P^* = 625}$$

$$P = 1,400 - 5Q \qquad MC = 200$$

$$TR = P(Q) * Q = (1400 - 5Q)Q = 1400Q - 5Q^2$$

$$MR = \frac{dTR(\cdot)}{dQ} = 1400 - 10Q$$

$$MR = MC \implies 1400 - 10Q = 200$$

$$10Q = 1,200 \implies \boxed{Q^* = 120}$$

$$P = 1,400 - 5(120) = 1,400 - 600$$

$$\boxed{P^* = 800}$$