$$f(K,L) = 10K + 5L$$

$$MRTS_{LK} = \frac{MP_L}{MP_K} = \frac{5}{10} = \frac{1}{2}$$

$$MP_L = \frac{\partial f(\cdot)}{\partial L} = 5$$

$$MP_K = \frac{\partial f(\cdot)}{\partial K} = 10$$

$f(L,K) = 2K + 15L$

$f(\lambda L, \lambda K)$

$\lambda f(L,K)$

constant returns to scale

$2(\lambda K) + 15(\lambda L)$

$2\lambda K + 15\lambda L$

$\lambda(2K + 15L) = \lambda f(L,K)$

$$f(L,K) = \min\{3K, 4L\}$$

$$f(\lambda L, \lambda K) = \min\{3(\lambda K), 4(\lambda L)\}$$

$$= \min\{3\lambda K, 4\lambda L\}$$

$$= \lambda \min\{3K, 4L\}$$

$$= \lambda f(L,K)$$

CRS

$$f(L,K) = 15K^{0.5}L^{0.4}$$

$$f(\lambda L, \lambda K) = 15(\lambda K)^{0.5}(\lambda L)^{0.4}$$

$$= 15\lambda^{0.5}K^{0.5}\lambda^{0.4}L^{0.4}$$

$$= \lambda^{0.9}\underbrace{15K^{0.5}L^{0.4}}$$

DRS

$$\underbrace{\lambda^{0.9}f(L,K)}_{} < \underbrace{\lambda f(\cdot)}_{}$$