Intermediate Microeconomics. Lecture 13

The Firm's Cost-Minimization Problem

Oscar Gálvez-Soriano[1]

[1]University of Houston
Department of Economics

Summer 2021

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Isoquants

- Isoquants are the curves representing all the combinations of inputs that allow a firm to make a particular quantity of output
- Just as with indifference curves, isoquants further from the origin correspond to higher output levels
- Isoquants cannot cross: if they did, the same quantities of inputs would yield two different quantities of output
- Isoquants are convex to the origin: using a mix of inputs generally lets a firm produce a greater quantity than it could by using an extreme amount of one input and a tiny amount of the other

Isoquants

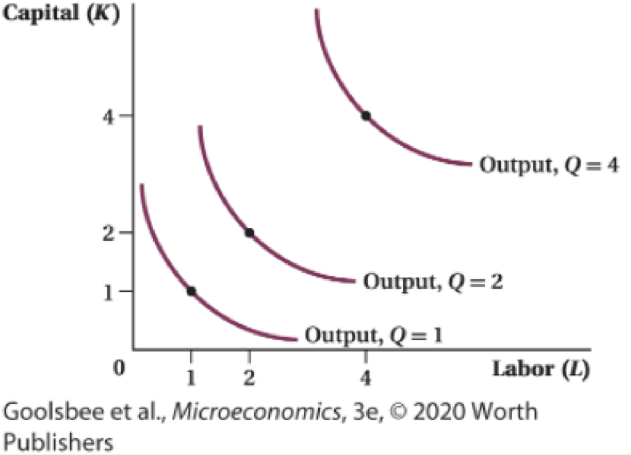

Figure: Isoquants

Marginal Rate of Technical Substitution

- The slope of the isoquant plays a key role in analyzing production decisions because it captures the tradeoff in the productive abilities of capital and labor
- The negative of the slope of the isoquant is called the marginal rate of technical substitution ($MRTS_{XY}$)
- The MRTS is the rate at which the firm can trade input X for input Y, holding output constant

Marginal Rate of Technical Substitution

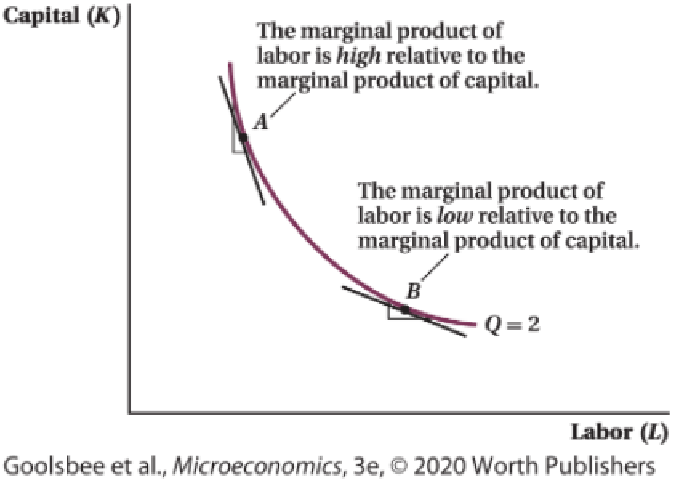

Figure: Marginal Rate of Technical Substitution

Marginal Rate of Technical Substitution

For the most part in this chapter, we will be interested in the MRTS of labor for capital ($MRTS_{LK}$), which is the amount of capital needed to hold output constant if the quantity of labor used by the firm changes

$$MRTS_{LK} = \frac{MP_L}{MP_K}$$

where $MP_L = \frac{\partial f(L,K)}{\partial L}$ and $MP_K = \frac{\partial f(L,K)}{\partial K}$

Substitutability

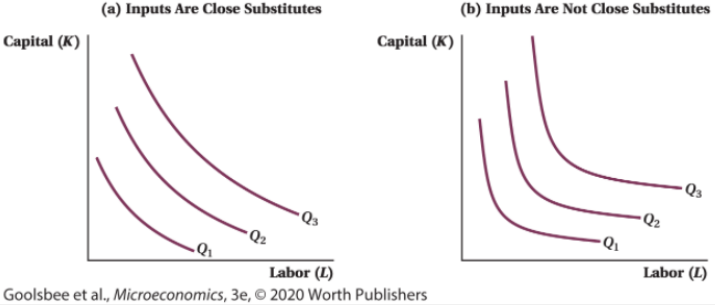

Figure: The Shape of Isoquants Indicates the Substitutability of Inputs

Perfect Substitutes and Perfect Complements in Production

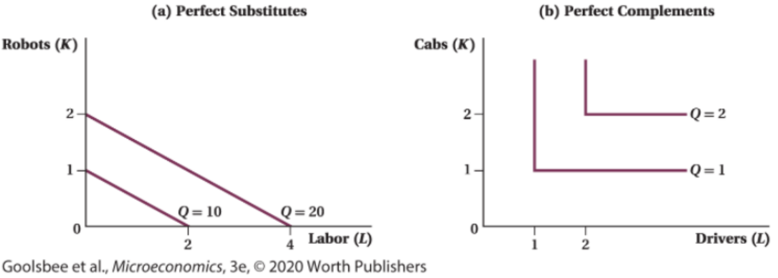

Figure: Perfect Substitutes and Perfect Complements in Production

Examples

An example of a production function where labor and capital are perfect substitutes is

$$f(K,L) = 10K + 5L \;\Rightarrow\; MRTS_{LK} = \frac{MP_L}{MP_K} = \frac{5}{10} = \boxed{\frac{1}{2}}$$

Cars and drivers might be close to perfect complements in the production of taxi rides. Therefore, the production function would be

$$f(K,L) = min\{L,K\}$$

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Isocost Lines

- As we discussed earlier, the firm wishes to minimize its costs of producing a given quantity of output
- We need to add in the costs of those choices
- The isocost line is a curve that shows all the input combinations that yield the same cost

$$C = rK + \omega L$$

$$K = \frac{C}{r} - \frac{\omega}{r}L$$

Isocost Lines

As an example, suppose $r = 20$ and $\omega = 10$

$$C = 20K + 10L \;\Rightarrow\; K = \frac{C}{20} - \frac{1}{2}L$$

In the next figure you can see the isocost curves, where the slope is $-\frac{\omega}{r} = -\frac{1}{2}$

Isocost Lines

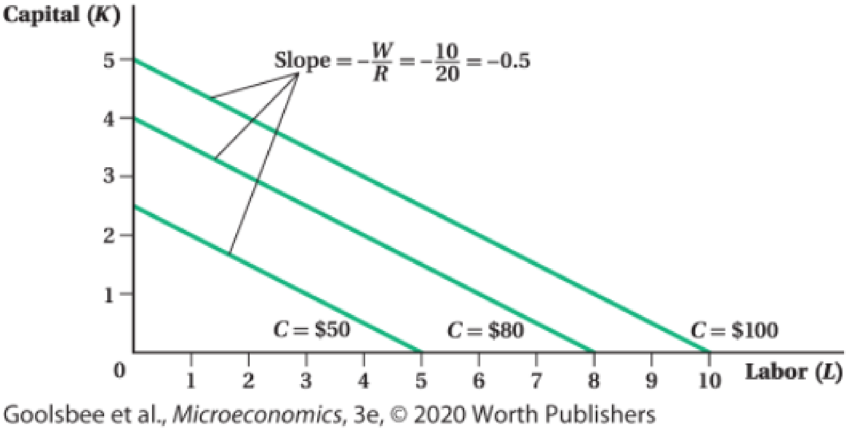

Figure: Isocost Lines

Input Price Changes

Suppose labor's price (ω) rises from \$10 to \$20. This will change the slope from $-\frac{1}{2}$ to -1, as follows

$$C = rK + \omega L$$

$$C = 20K + 20L \Rightarrow K = \frac{C}{20} - L$$

Input Price Changes

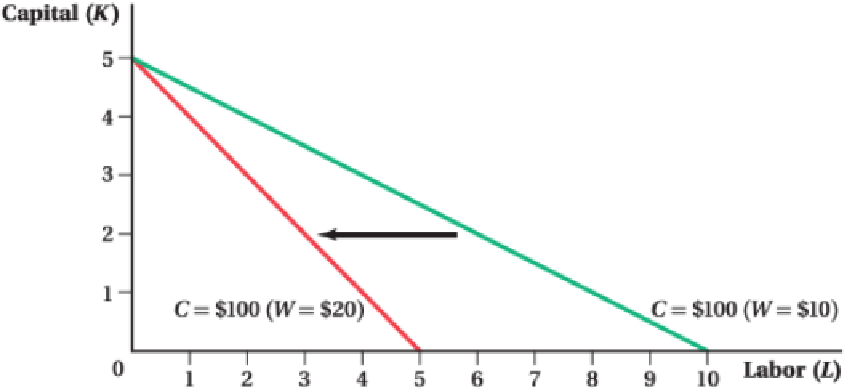

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Figure: When Labor Becomes More Expensive, the Isocost Line Becomes Steeper

Input Price Changes

Now, suppose that the price of capital (r) increases from \$20 to \$40. This will change the slope from $-\frac{1}{2}$ to $-\frac{1}{4}$, as follows

$$C = rK + \omega L$$

$$C = 40K + 10L \ \Rightarrow \ K = \frac{C}{40} - \frac{1}{4}L$$

Input Price Changes

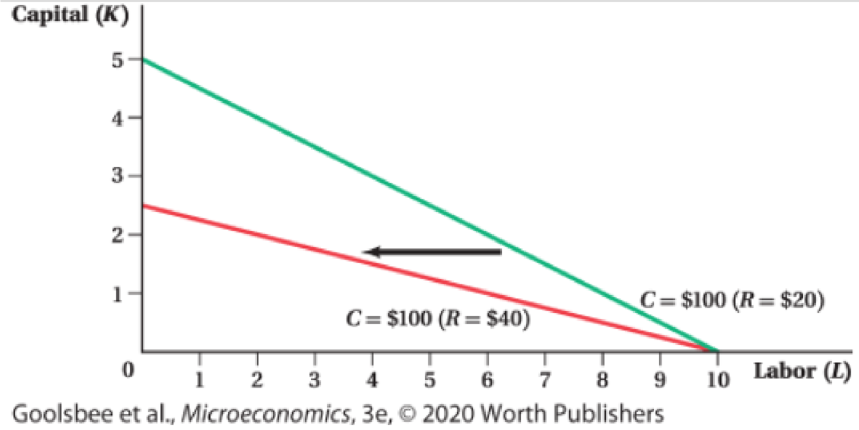

Figure: When Capital Becomes More Expensive, the Isocost Line Becomes Flatter

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Cost Minimization

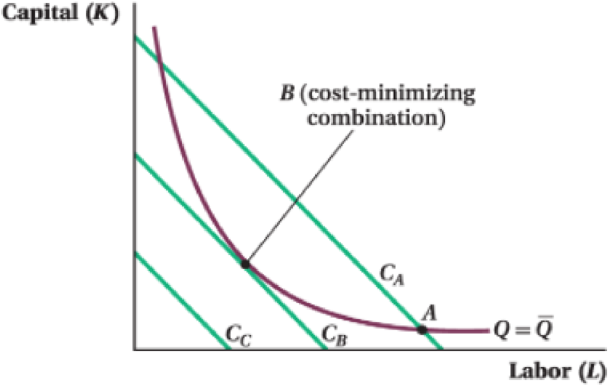

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Figure: Cost Minimization

Cost Minimization

The minimized total cost of producing a given quantity is where the isocost line is tangent to the isoquant

$$MRTS_{LK} = \frac{\omega}{r}$$

$$\frac{MP_L}{MP_K} = \frac{\omega}{r}$$

This condition has an important economic interpretation. Rearrange the terms in the equation

$$MP_K \omega = MP_L r$$

$$\frac{MP_K}{r} = \frac{MP_L}{\omega}$$

Cost Minimization

$$\frac{MP_K}{r} = \frac{MP_L}{\omega}$$

- One way to interpret these ratios is that they measure the marginal product per dollar spent on each input
- Alternatively, we can think of each of these ratios as the firm's marginal benefit-to-cost ratio of hiring an input
- For example, if the firm produced an input bundle where this wasn't true

$$\frac{MP_K}{r} > \frac{MP_L}{\omega}$$

- then the firm's benefit-to-cost ratio for capital is higher than for labor

Effects of an Input Price Change

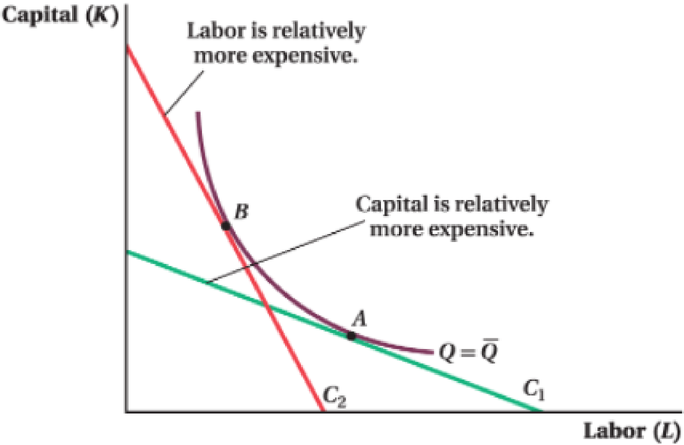

Figure: A Change in the Relative Price of Labor Leads to a New Cost-Minimizing Input Choice

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Returns to Scale

- Economists use the term returns to scale to describe a change in the amount of output in response to a proportional increase in all the inputs
- A production function, $f(L, K)$, has **constant returns to scale** if changing the amount of capital and labor by some multiple changes the quantity of output by exactly the same multiple

$$f(\lambda L, \lambda K) = \lambda f(L, K)$$

Returns to Scale

- A production function has **increasing returns to scale** if changing all inputs by the same proportion changes output more than proportionately

$$f(\lambda L, \lambda K) > \lambda f(L, K)$$

- **Decreasing returns to scale** exist if adjusting all inputs by the same multiple changes output by less than that multiple

$$f(\lambda L, \lambda K) < \lambda f(L, K)$$

Returns to Scale: examples

For each of the following production functions, determine if they exhibit constant, decreasing, or increasing returns to scale

$$f(L, K) = 2K + 15K$$

$$f(L, K) = min\{3K, 4L\}$$

$$f(L, K) = 15K^{0.5}L^{0.4}$$