

$$f(K, L) = 4 K^{\frac{3}{4}} L^{\frac{1}{4}}$$

$$MP_L = \frac{\partial f(\cdot)}{\partial L} = \frac{1}{4} 4 K^{\frac{3}{4}} L^{\frac{1}{4}-1} = K^{\frac{3}{4}} L^{-\frac{3}{4}}$$

$$MP_K = \frac{\partial f(\cdot)}{\partial K} = \frac{3}{4} 4 K^{\frac{3}{4}-1} L^{\frac{1}{4}} = 3 K^{-\frac{1}{4}} L^{\frac{1}{4}}$$

$$MRTS_{LK} = \frac{w}{r}$$

$$MRTS_{LK} = \frac{MP_L}{MP_K} = \frac{K^{3/4} L^{-1/4}}{3 K^{1/4} L^{3/4}} = \frac{1}{3} \frac{K^{3/4} L^{-1/4}}{K^{1/4} L^{3/4}}$$

$$MRTS_{LK} = \frac{1}{3} \frac{K}{L} = \frac{2}{10}$$

$$\frac{K}{L} = 3 \frac{2}{10} = \frac{3}{5} \Rightarrow 0.6 : 1$$

$$\boxed{K = \frac{3}{5} L}$$

$$\min w_1 x_1 + w_2 x_2$$

$$\text{st } y = x_1^\alpha x_2^{1-\alpha}$$

USE OPTIMAL
CONDITION

$$MRTS_{1,2} = \frac{w_1}{w_2}$$

$$MRTS_{1,2} = \frac{MP_1}{MP_2}$$

$$MP_1 = \frac{\partial f(\cdot)}{\partial x_1} = \alpha x_1^{\alpha-1} x_2^{1-\alpha}$$

$$MP_2 = \frac{\partial f(\cdot)}{\partial x_2} = (1-\alpha) x_1^\alpha x_2^{1-\alpha-1} = (1-\alpha) x_1^\alpha x_2^{-\alpha}$$

$$MRTS_{12} = \frac{MP_1}{MP_2} = \frac{\alpha X_1^{-1-\alpha} X_2^{1-\alpha}}{(1-\alpha) X_1^{\alpha} X_2^{-\alpha}}$$

$$MRTS_{12} = \frac{\alpha}{1-\alpha} \frac{X_2^{1-\alpha} X_2^{\alpha}}{X_1^{\alpha} X_1^{1-\alpha}} = \frac{\alpha}{1-\alpha} \frac{X_2}{X_1}$$

$$MRTS_{12} = \frac{w_1}{w_2}$$

$$\frac{\alpha}{1-\alpha} \frac{X_2}{X_1} = \frac{w_1}{w_2}$$

 \Rightarrow

$$X_2 = \frac{1-\alpha}{\alpha} \frac{w_1}{w_2} X_1$$

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$$Y = X_1^\alpha X_2^{1-\alpha}$$

$$Y = X_1^\alpha \left[\frac{1-\alpha}{\alpha} \frac{w_1}{w_2} X_1 \right]^{1-\alpha} = X_1^\alpha \left(\frac{1-\alpha}{\alpha} \frac{w_1}{w_2} \right)^{1-\alpha} X_1^{1-\alpha}$$

$$Y = \left(\frac{1-\alpha}{\alpha} \frac{w_1}{w_2} \right)^{1-\alpha} X_1$$

$$X_1^* = \left(\frac{1-\alpha}{\alpha} \frac{w_1}{w_2} \right)^{\alpha-1} Y$$

$$X_1^* = \left(\frac{\alpha}{1-\alpha} \frac{w_2}{w_1} \right)^{1-\alpha} y$$

$$X_2 = \underbrace{\frac{1-\alpha}{\alpha} \frac{w_1}{w_2}}_a \left(\underbrace{\frac{1-\alpha}{\alpha} \frac{w_1}{w_2}}_a \right)^{\alpha-1} y$$

$$X_2^* = \left(\frac{1-\alpha}{\alpha} \frac{w_1}{w_2} \right)^{\alpha} y$$

$$\boxed{MR = MC}$$

OPTIMAL
CONDITION

PERFECT COMPETITION

$$P = MR \Rightarrow \boxed{P = MC}$$

