Intermediate Microeconomics: Optional Problem Set 2

Vasundhara Tanwar

Professor Oscar Galvez-Soriano
University of Houston

Summer 2021

Problem:

$$\min_{x_1,x_2} \omega_1 x_1 + \omega_2 x_2 \quad s.t. \quad y = x_1^{\alpha} x_2^{1-\alpha}$$

Solution: Use the optimal condition $MRTS = \frac{\omega_1}{\omega_2}$

$$MRTS = \frac{MP_1}{MP_2} = \frac{\alpha x_1^{\alpha-1} x_2^{1-\alpha}}{(1-\alpha) x_1^{\alpha} x_2^{-\alpha}} = \frac{\alpha x_2}{(1-\alpha) x_1}$$

$$\frac{\alpha x_2}{(1-\alpha)x_1} = \frac{\omega_1}{\omega_2} \implies x_2 = \frac{\omega_1}{\omega_2}\frac{1-\alpha}{\alpha}x_1 \quad OR \quad x_1 = \frac{\omega_2}{\omega_1}\frac{\alpha}{1-\alpha}x_2 \tag{1}$$

Plug in equation (1) in the production function to get conditional factor demands,

$$x_1^{\alpha}(\frac{\omega_1}{\omega_2}\frac{1-\alpha}{\alpha}x_1)^{1-\alpha} = y$$
$$x_1^* = (\frac{\omega_2}{\omega_1}\frac{\alpha}{1-\alpha})^{1-\alpha} y$$

Similarly,

$$(\frac{\omega_2}{\omega_1}\frac{\alpha}{1-\alpha}x_2)^{\alpha} x_2^{1-\alpha} = y$$
$$x_2^* = (\frac{\omega_1}{\omega_2}\frac{1-\alpha}{\alpha})^{\alpha} y$$