Intermediate Microeconomics. Lecture 21

Review on Producer and Consumer Theory (Final Exam)

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Consumer Theory (MRS)

In several questions you will be asked to compute the MRS or implicitly you will need to compute it to solve the consumer's maximization problem

$$MRS_{12} = \frac{MU_{x_1}}{MU_{x_2}}$$

where

$$MU_1 = \frac{\partial u(x_1, x_2)}{\partial x_1}$$

$$MU_2 = \frac{\partial u(x_1, x_2)}{\partial x_2}$$

Consumer Theory (Budget Constraint)

In the test, I will give you information of prices and income. Substitute that information in the following expression

$$m = p_1 x_1 + p_2 x_2$$

which you need to re-express in order to graph it. Just solve for x_2

$$p_2 x_2 = m - p_1 x_1$$

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2} x_1$$

Consumer Theory (Optimization Problem)

Consider the following Cobb-Douglas utility function

$$u(x_1, x_2) = x_1^a x_2^b$$

which we would like to maximize subject to the following budget constraint

$$m = p_1 x_1 + p_2 x_2$$

Derive the Marshallian demands

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Producer Theory (MRTS)

In several questions you will be asked to compute the MRTS or implicitly you will need to compute it to solve the producer's profit maximization problem

$$MRTS_{12} = \frac{MP_L}{MP_K}$$

where

$$MP_L = \frac{\partial f(L,K)}{\partial L}$$

$$MU_K = \frac{\partial f(L,K)}{\partial K}$$

Producer Theory (Isocost function)

In the test, I will give you information of input prices. Substitute that information in the following expression to obtain the isocost function

$$C = \omega L + rK$$

which you need to re-express in order to graph it. Just solve for K

$$rK = C - \omega L$$

$$K = \frac{C}{r} - \frac{\omega}{r} L$$

Producer Theory (Optimization Problem)

Consider the following production function

$$f(K, L) = 4K^{0.75}L^{0.25}$$

Find the marginal product of both inputs

$$MP_K = 3K^{-0.25}L^{0.25}$$

$$MP_L = K^{0.75}L^{-0.75}$$

The isocost function is

$$C = 10K + 2L$$

Producer Theory (Optimization Problem)

- What ratio of capital to labor minimizes total costs?

$$MRTS = \frac{\omega}{r}$$

$$\frac{1}{3}\frac{K}{L} = \frac{1}{5}$$

$$\frac{K}{L} = \frac{3}{5} \Rightarrow 0.6 : 1$$

Producer Theory (Optimization Problem)

- How much capital and labor to rent and hire in order to produce 1,000 units

$$f(K,L) = 4K^{0.75}L^{0.25}$$

$$1,000 = 4(\frac{3}{5}L)^{0.75}L^{0.25} \Rightarrow 1,000 = 2.7269L \Rightarrow \boxed{L = 367}$$

$$K = \frac{3}{5}L \Rightarrow K = \frac{3}{5}(367) \Rightarrow \boxed{K = 220}$$

Producer Theory (Cost Minimization)

Consider the following minimization problem

$$min \;\; \omega_1 x_1 + \omega_2 x_2$$

st

$$y = x_1^{\alpha} x_2^{1-\alpha}$$

Show that the optimal factor demands are

$$\boxed{x_1^* = \left[\frac{1-\alpha}{\alpha}\frac{\omega_2}{\omega_1}\right]^{\alpha} y}$$

$$\boxed{x_2^* = \left[\frac{\alpha}{1-\alpha}\frac{\omega_1}{\omega_2}\right]^{1-\alpha} y}$$

Monopolist (Optimization Problem)

Goolsbee et al., *Microeconomics*, 3e, © 2020 Worth Publishers

Figure: Pop Rocks market demand

Monopolist (Optimization Problem)

Consider the market for Pop Rocks depicted in the previous figure

- If the Pop Rock industry were competitive, what would the competitive price and quantity be?

The optimal condition for a competitive market is $P = MC$

$$\boxed{P = 0.10}$$

To find quantity we substitute this price in the inverse demand function

$$\frac{1}{10} = 1 - \frac{1}{100}Q \Rightarrow \frac{1}{100}Q = \frac{9}{10} \Rightarrow \boxed{Q = 90}$$

Monopolist (Optimization Problem)

- If the Pop Rock industry were competitive, what would the consumer and producer surpluses be, respectively?

$$CS = (\frac{1}{2}) * (90,000) * (\frac{9}{10}) \Rightarrow \boxed{CS = 40,500}$$

$$PS = (\frac{1}{2}) * (90,000) * (0) \Rightarrow \boxed{PS = 0}$$

Monopolist (Optimization Problem)

- Suppose that gangland figure Tommy Vercetti monopolizes the Pop Rock market. What quantity and price would he choose to maximize profit?

Figure: Monopolist demand

Monopolist (Optimization Problem)

First find the MR deriving TR with respect to Q

$$TR = P(Q) \cdot Q = (1 - \frac{1}{100}Q) * Q = Q - \frac{1}{100}Q^2$$

$$MR = \frac{dTR(Q)}{dQ} = \frac{d(Q - \frac{1}{100}Q^2)}{dQ} = 1 - \frac{1}{50}Q$$

Then use the optimal condition $MR = MC$

$$1 - \frac{1}{50}Q = \frac{1}{10} \Rightarrow \frac{1}{50}Q = \frac{9}{10} \Rightarrow \boxed{Q = 45}$$

Finally, find the monopoly price substituting the optimal quantity in the demand equation

$$P = 1 - \frac{1}{100}(45) \Rightarrow \boxed{P = 0.55}$$

Monopolist (Optimization Problem)

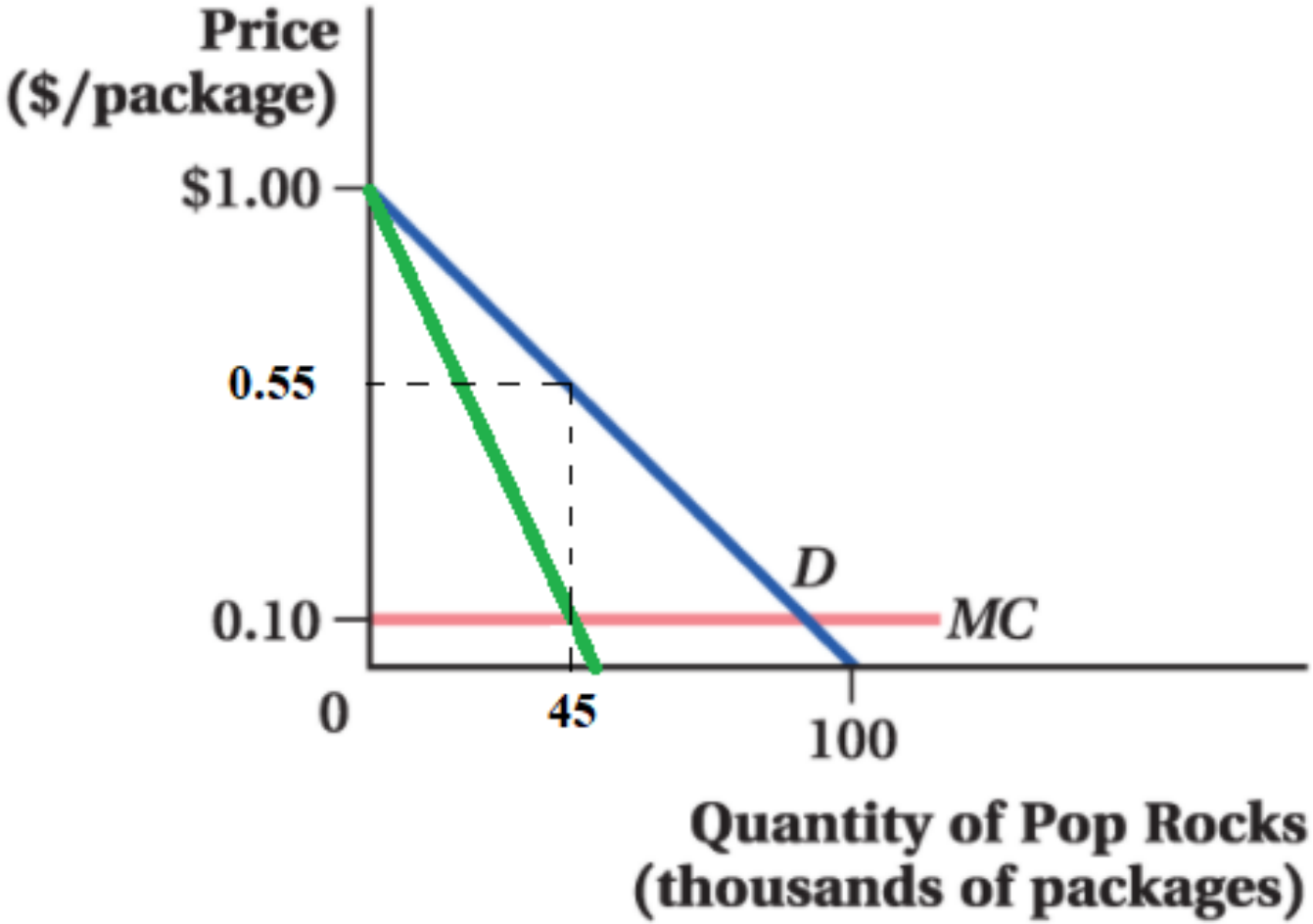

Figure: Monopolist price and quantity

Monopolist (Optimization Problem)

- Calculate the consumer and producer surpluses of Vercetti's Pop Rock monopoly

$$CS = (\frac{1}{2}) * (45,000) * (0.45) \Rightarrow \boxed{CS = 10,125}$$

$$PS = (45,000) * (0.45) \Rightarrow \boxed{PS = 20,250}$$

- How big is the deadweight loss (DWL) caused by the monopoly?

$$DWL = (\frac{1}{2}) * (45,000) * (0.45) \Rightarrow \boxed{DWL = 10,125}$$