Intermediate Microeconomics: Optional Problem Set

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Problem:

$$\max_{x_1,x_2} x_1^a x_2^b \quad s.t. \quad m = p_1x_1 + p_2x_2$$

Solution method 1: Use the optimal condition $MRS_{12} = \frac{p_1}{p_2}$

$$MRS_{12} = \frac{MU_1}{MU_2} = \frac{ax_1^{a-1}x_2^b}{bx_1^a x_2^{b-1}} = \frac{ax_1}{bx_2}$$

$$\frac{ax_1}{bx_2} = \frac{p_1}{p_2} \implies x_2 = \frac{p_1}{p_2}\frac{b}{a}x_1 \tag{1}$$

Plug Equation 1 in budget constraint

$$m = p_1x_1 + p_2(\frac{p_1}{p_2}\frac{b}{a}x_1)$$

$$m = p_1x_1 + (p_1\frac{b}{a}x_1) = (1+\frac{b}{a})p_1x_1 = (\frac{a+b}{a})p_1x_1$$

$$\boxed{x_1^* = \frac{a}{a+b}\frac{m}{p_1}} \tag{2}$$

Finally, plug Equation 2 in Equation 1

$$x_2 = \frac{p_1}{p_2}\frac{b}{a}(\frac{a}{a+b}\frac{m}{p_1}) = \frac{1}{p_2}b(\frac{1}{a+b}m)$$

$$\boxed{x_2^* = \frac{b}{a+b}\frac{m}{p_2}}$$

Solution method 2: Use the Lagrangian

$$\mathcal{L}(x_1, x_2, m) \equiv x_1^a x_2^b + \lambda(m - p_1x_1 - p_2x_2)$$

Take First Order Conditions:

$$x_1: \quad ax_1^{a-1}x_2^b - \lambda p_1 = 0 \implies ax_1^{a-1}x_2^b = \lambda p_1 \tag{3}$$
$$x_2: \quad bx_1^a x_2^{b-1} - \lambda p_2 = 0 \implies bx_1^a x_2^{b-1} = \lambda p_2 \tag{4}$$

$$\lambda: \quad m = p_1x_1 + p_2x_2 \tag{5}$$

To solve the above system, divide equation (3) and (4)

$$\frac{(5)}{(6)} \implies \frac{ax_1^{a-1}x_2^b}{bx_1^a x_2^{b-1}} = \frac{\lambda p_1}{\lambda p_2} = \frac{p_1}{p_2}$$

Simplifying this gives us the same relationship as before:

$$x_2 = \frac{p_1}{p_2}\frac{b}{a}x_1 \tag{6}$$

Using the relationship from equation (6) in the budget constraint to get x_1^* and x_2^*

$$p_1x_1 + p_2\frac{p_1}{p_2}\frac{b}{a}x_1 = m$$

$$p_1x_1(1 + \frac{b}{a}) = m$$

$$\boxed{x_1^* = \frac{m}{p_1}\frac{a}{(a+b)}}$$

Plug the above back in (6) to get

$$x_2 = \frac{p_1}{p_2}\frac{b}{a}\frac{m}{p_1}\frac{a}{(a+b)}$$

$$\boxed{x_2^* = \frac{m}{p_2}\frac{b}{(a+b)}}$$