Intermediate Microeconomics. Lecture 6

Utility Maximization

Oscar Gálvez-Soriano[1]

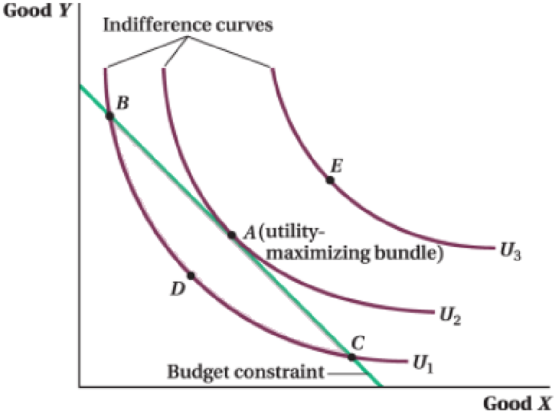
[1]University of Houston
Department of Economics

Summer 2021

Contents

Contents

Consumer's Optimization Problem



Goolsbee et al., *Microeconomics*, 3e, © 2020 Worth Publishers

Figure: Consumer's Optimal Choice

Consumer's Optimization Problem

- Mathematically, the tangency of the indifference curve and budget constraint means that they have the same slope at the optimal consumption bundle
- When the consumer spends all her income and maximizes her utility, her optimal consumption bundle is the one at which the MRS exactly equals the ratio of their prices

$$MRS_{XY} = \frac{P_X}{P_Y}$$

$$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$$

Example: Optimization Problem

Suppose Antonio gets utility from consuming two goods, burgers and fries. His utility function is given by

$$U = \sqrt{BF} = B^{\frac{1}{2}} F^{\frac{1}{2}}$$

Antonio's income is \$20, and the prices of burgers and fries are \$5 and \$2, respectively.

- What are Antonio's utility-maximizing quantities of burgers and fries?

First, write down the budget constraint

$$20 = 5B + 2F$$

Example: Optimization Problem

Second, use the first order conditions (FOC) to find Antonio's marginal utilities

$$MU_B = \frac{1}{2} B^{-\frac{1}{2}} F^{\frac{1}{2}}$$

$$MU_F = \frac{1}{2} B^{\frac{1}{2}} F^{-\frac{1}{2}}$$

and compute the MRS

$$MRS_{BF} = \frac{MU_B}{MU_F} = \frac{\frac{1}{2} B^{-\frac{1}{2}} F^{\frac{1}{2}}}{\frac{1}{2} B^{\frac{1}{2}} F^{-\frac{1}{2}}} = \frac{F^{\frac{1}{2}} F^{\frac{1}{2}}}{B^{\frac{1}{2}} B^{\frac{1}{2}}} = \frac{F}{B}$$

Example: Optimization Problem

Now, we can use the optimal condition

$$MRS_{BF} = \frac{P_B}{P_F}$$

$$\frac{F}{B} = \frac{5}{2} \Rightarrow F = \frac{5}{2}B$$

- This condition tells us that Antonio maximizes his utility when he consumes fries to burgers at a 5 to 2 ratio
- However, we do not yet know exactly what quantities Antonio will choose to consume

Example: Optimization Problem

To figure that out, we can use the budget constraint

$$20 = 5B + 2F \;\Rightarrow\; B = \frac{20}{5} - \frac{2}{5}F$$

Finally, substitute the optimal condition we just found

$$B = 4 - \frac{2}{5}(\frac{5}{2}B) \;\Rightarrow\; 2B = 4 \;\Rightarrow\; \boxed{B^* = 2}$$

$$F = \frac{5}{2}(2) \;\Rightarrow\; \boxed{F^* = 5}$$

Therefore, given his budget constraint, Antonio maximizes his utility by consuming 2 burgers and 5 servings of fries.

The general case of Cobb-Douglas preferences

Consider the following Cobb-Douglas utility function

$$u(x_1, x_2) = x_1^a x_2^b$$

which we would like to maximize subject to the following budget constraint

$$m = p_1 x_1 + p_2 x_2$$

Follow the previous steps to find the optimal choices

$$\boxed{x_1^* = \frac{a}{a+b}\frac{m}{p_1}}$$

$$\boxed{x_2^* = \frac{b}{a+b}\frac{m}{p_2}}$$

Contents

Corner Solutions

- A utility-maximizing bundle located at the "corner" of the budget constraint where the consumer purchases only one of two goods is called a **corner solution**
- A utility-maximizing bundle that contains positive quantities of both goods is called an **interior solution**

Corner Solutions

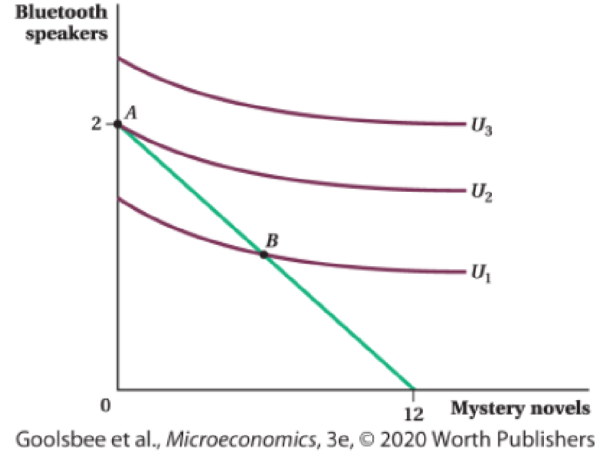


Figure: Corner Solution

Perfect Substitutes

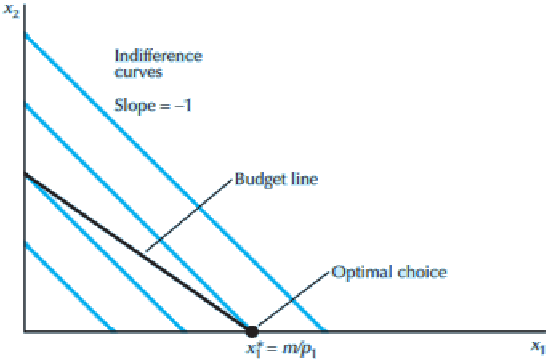


Figure: Optimal choice with perfect substitutes (Source: Varian, Intermediate Microeconomics 8e, 2010)

Perfect Substitutes

Our maximization problem is

$$Max \;\; u(x_1, x_2) = ax_1 + bx_2$$

s.t.

$$m = p_1x_1 + p_2x_2$$

From the FOC we know that

$$MRS_{12} = \frac{a}{b}$$

Perfect Substitutes

BUT, there is **no tangency** point. So, instead of using the optimal condition, we think of all possible solutions.

The demand function for good 1 will be

$$x_1^* = \begin{cases} \frac{m}{p_1} & \frac{p_1}{p_2} < \frac{a}{b} \\ \in [0, \frac{m}{p_1}] & \frac{p_1}{p_2} = \frac{a}{b} \\ 0 & \frac{p_1}{p_2} > \frac{a}{b} \end{cases}$$

Similarly for good 2

Perfect Complements

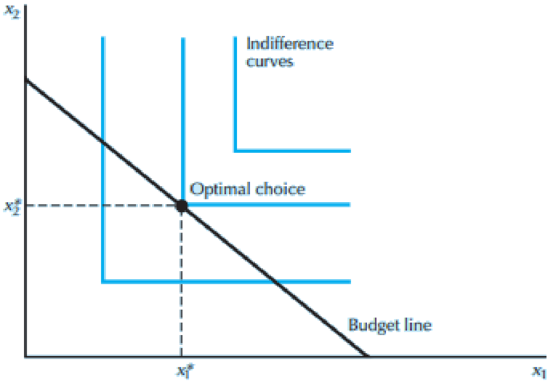


Figure: Optimal choice with perfect complements (Source: Varian, Intermediate Microeconomics 8e, 2010)

Perfect Complements

Our maximization problem is

$$Max \;\; u(x_1, x_2) = min\{ax_1, bx_2\}$$

s.t.

$$m = p_1x_1 + p_2x_2$$

- Since there is **NO tangency** point (why?), let us solve for the optimal choice algebraically

Perfect Complements

- We know that this consumer is purchasing the same proportional amount of good 1 and good 2, no matter what the prices are

$$ax_1 = bx_2 \;\Rightarrow\; x_2 = \frac{a}{b}x_1$$

- Then we have to satisfy the budget constraint

$$m = p_1 x_1 + p_2(\frac{a}{b}x_1)$$

Perfect Complements

Solving for x_1

$$m = p_1 x_1 + p_2(\frac{a}{b}x_1) \;\Rightarrow\; m = (p_1 + \frac{a}{b}p_2)x_1$$

$$\boxed{x_1^* = \frac{m}{p_1 + \frac{a}{b}p_2}}$$

And going back to the optimal proportion to find x_2

$$x_2 = \frac{a}{b}x_1 \;\Rightarrow\; \boxed{x_2^* = \frac{a}{b}(\frac{m}{p_1 + \frac{a}{b}p_2})}$$