IN 2400

Indexing and Hashing

Week 11

Outline

- Basic Concepts
- Ordered Indices
- B+ Tree Index Files
- B Tree Index Files
- Static Hashing
- Dynamic Hashing
- Comparison of Ordered Indexing and Hashing
- Index Definition in SQL
- Multiple-Key Access

Basic Concepts

- Indexing is used to speed up access to desired data.
  - E.g. author catalog in library
- A search key is an attribute or set of attributes used to look up records in a file. Unrelated to keys in the db schema.
- An index file consists of records called index entries.
- An index entry for key *k* may consist of
  - An actual data record (with search key value k)
  - A pair (k, rid) where rid is a pointer to the actual data record
- Index entry i as <k(i), P(i)>

Search key	**Pointer**

- Index files are typically much smaller than the original file if the actual data records are in a separate file.
- Two basic kinds of indices:
  - Ordered indices: search keys are stored in sorted order
  - Hash indices: search keys are distributed uniformly across "buckets" using a "hash function".

Index Evaluation Metrics

- Access time for:
  - Equality searches – records with a specified value in an attribute
  - Range searches – records with an attribute value falling within a specified range. (e.g. 10000 < salary < 40000)
- Insertion time
- Deletion time
- Space overhead

Types of Indices

- The records in a file may be unordered or ordered sequentially by some search key.
- A file whose records are unordered is called a heap file.
- In an ordered index, index entries are sorted on the search key value. Other index structures include trees and hash tables.

Ordered Indices

- In an ordered index, index entries are stored sorted on the search key value. E.g., author catalog in library.
- Primary index: in a sequentially ordered file, the index whose search key specifies the sequential order of the file.
  - Also called clustering index
  - The search key of a primary index is usually but not necessarily the primary key.
- Secondary index: an index whose search key specifies an order different from the sequential order of the file. Also called non-clustering index.
- Index-sequential file: ordered sequential file with a primary index.

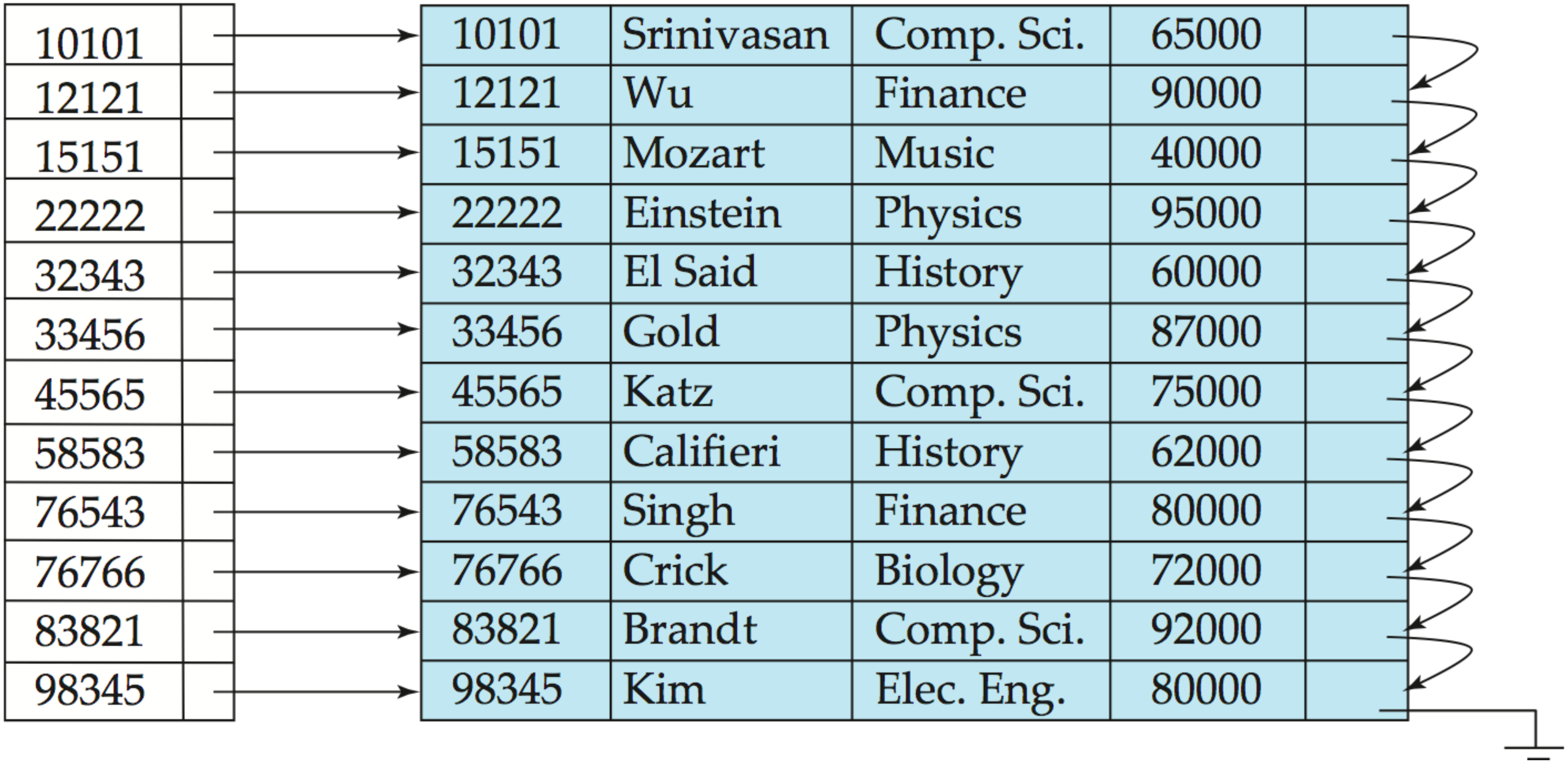
Ordered indexes can be grouped as

- Dense index – an index entry for every search key value (hence every record) in the data file.

- Sparse (nondense) index – has index entry for only some of the search values/

Dense Index Files

- **Dense index** — Index record appears for every search-key value in the file.
- E.g. index on *ID* attribute of *instructor* relation



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15151	
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83821	
98345	

10101	Srinivasan	Comp. Sci.	65000	
12121	Wu	Finance	90000	
15151	Mozart	Music	40000	
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33456	Gold	Physics	87000	
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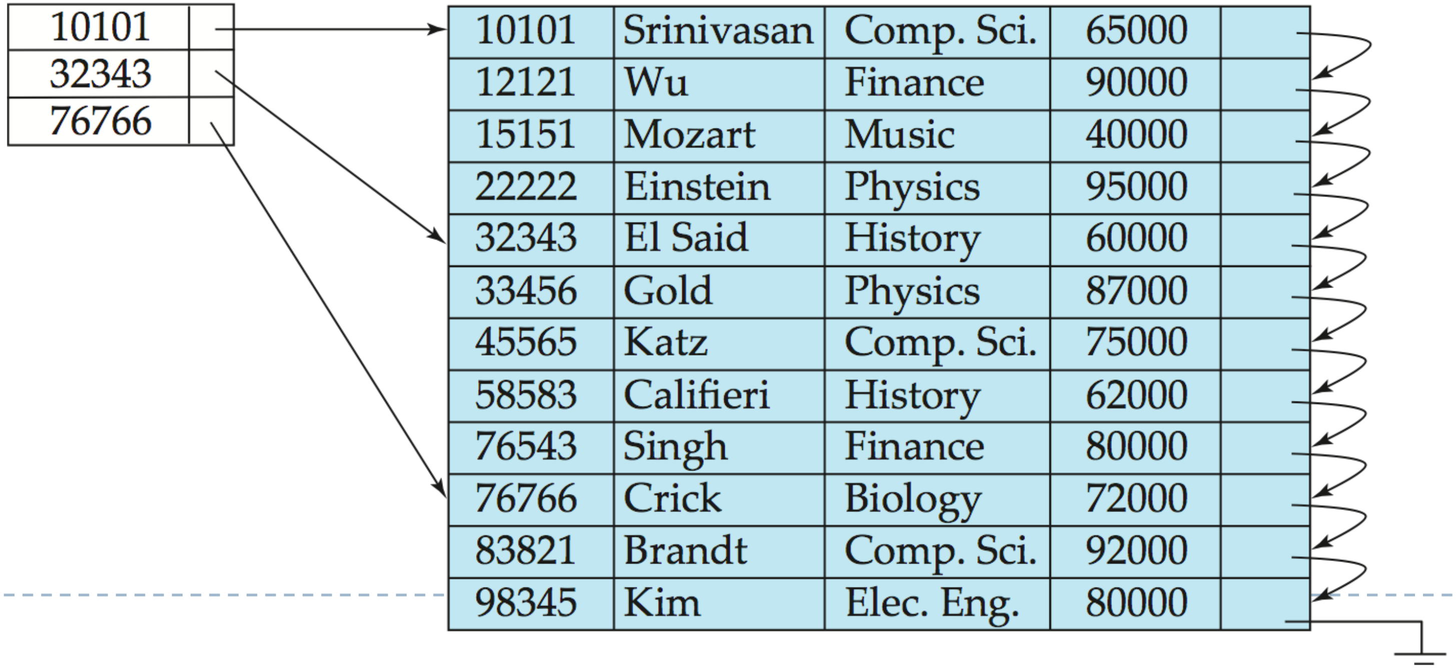
58583	Califieri	History	62000	
76543	Singh	Finance	80000	
76766	Crick	Biology	72000	
83821	Brandt	Comp. Sci.	92000	
98345	Kim	Elec. Eng.	80000	

Primary index – records of a file are physically ordered on key field.

Sparse Index Files

- **Sparse Index**: contains index records for only some search-key values.
  - Applicable when records are sequentially ordered on search-key
- To locate a record with search-key value *K* we:
  - Find index record with largest search-key value < *K*
  - Search file sequentially starting at the record to which the index record points



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10101	Srinivasan	Comp. Sci.	65000	
12121	Wu	Finance	90000	
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76766	Crick	Biology	72000	
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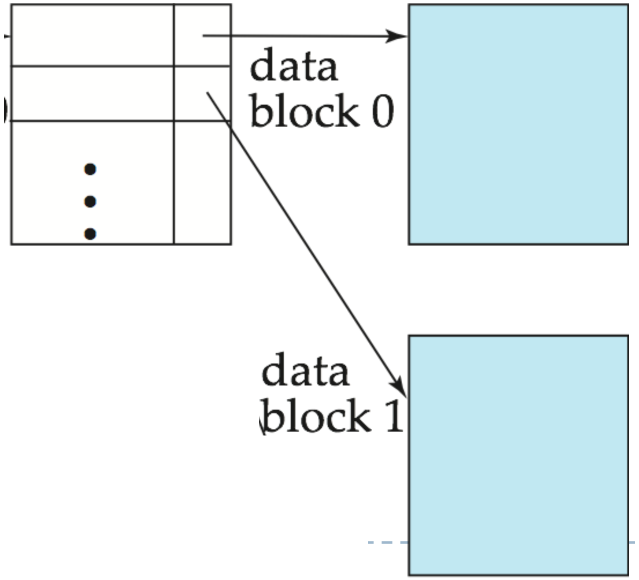
98345	Kim	Elec. Eng.	80000	

Sparse Index Files (Cont.)

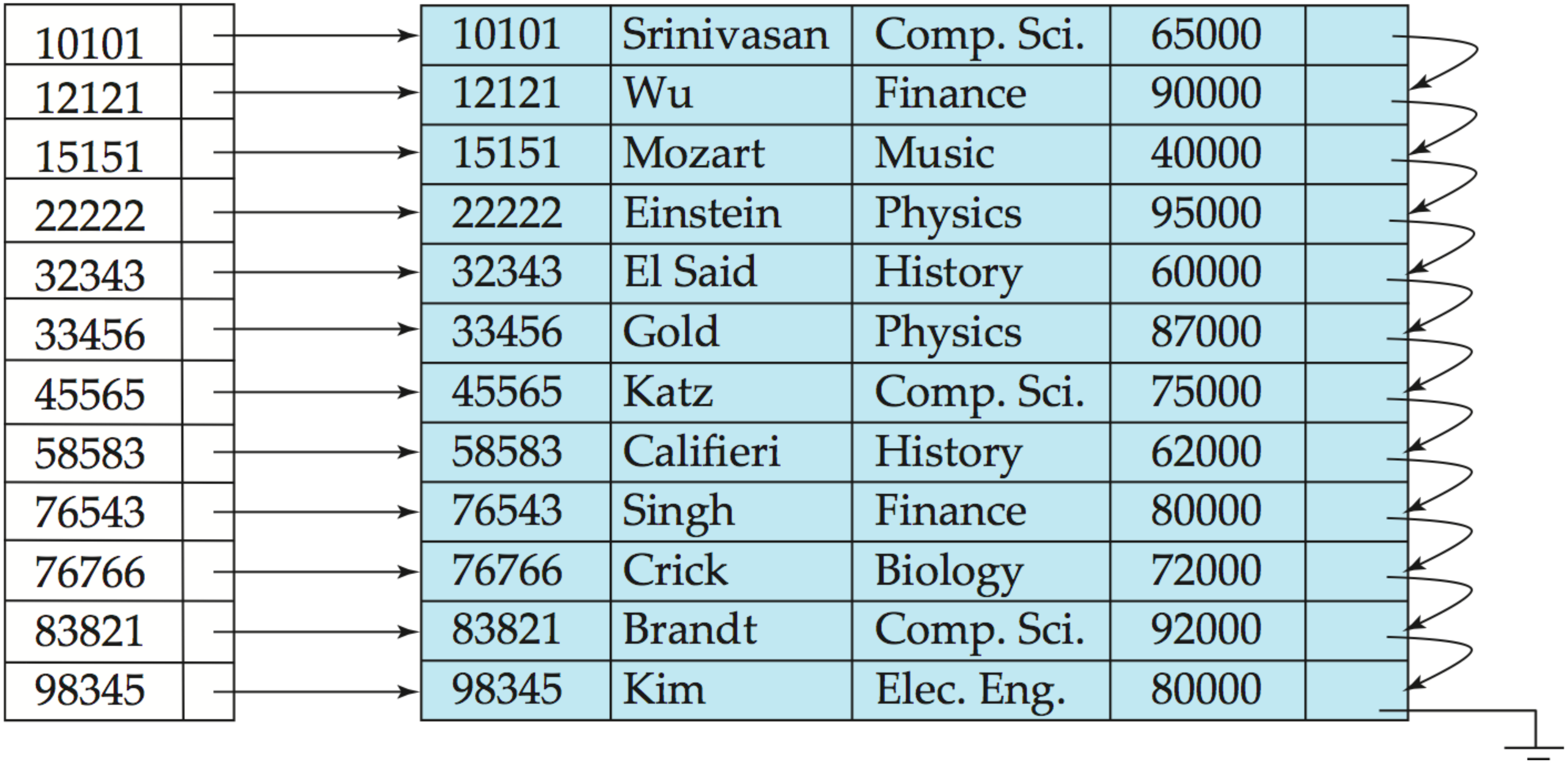
- Compared to dense indices:
  - Less space and less maintenance overhead for insertions and deletions.
  - Generally slower than dense index for locating records.
- **Good tradeoff**: sparse index with an index entry for every block in file, corresponding to least search-key value in the block.



Primary Index Files

Use ordering key field of the file.

- E.g. index on *ID* attribute of *instructor* relation



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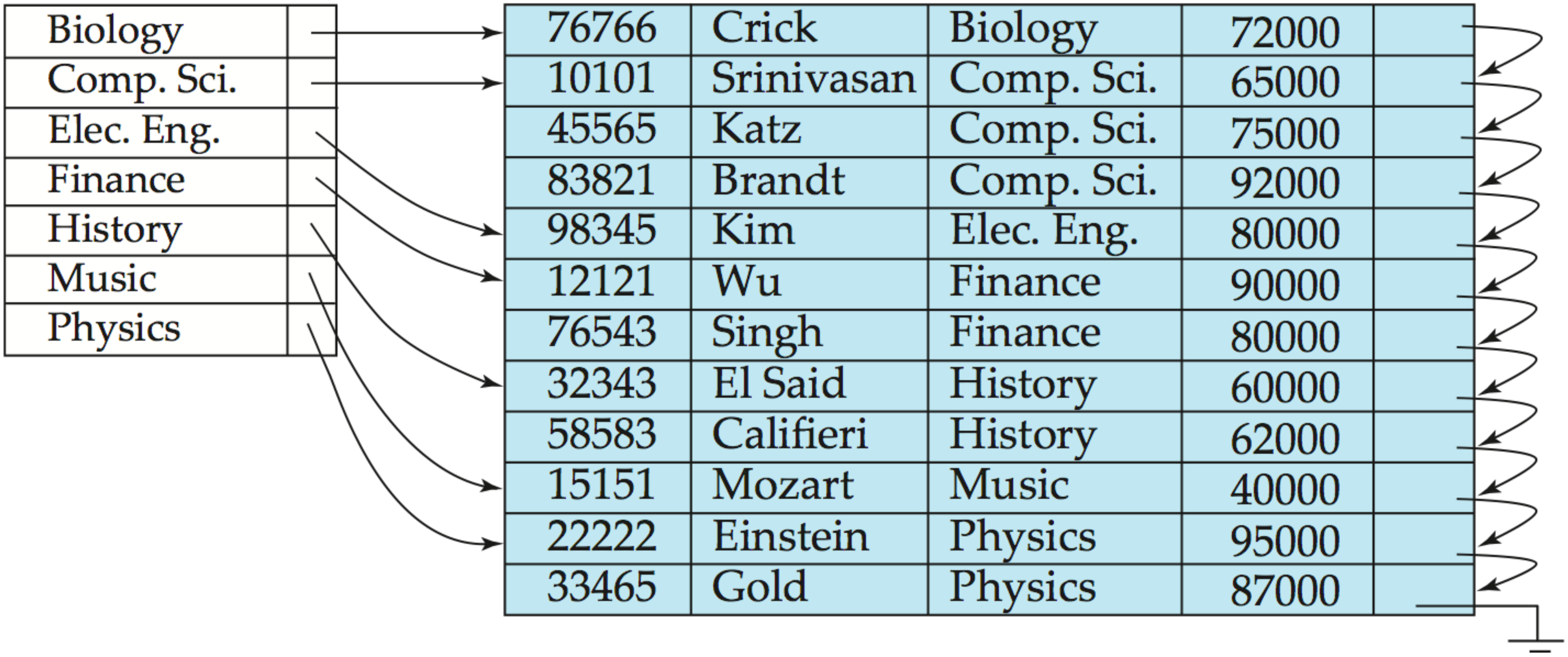
98345	Kim	Elec. Eng.	80000	

Primary index – records of a file are physically ordered on key field.

Clustering Index

Records are physically ordered on a non-key field (does not have a distinct value for each record).

Index on *dept_name*, with *instructor* file sorted on *dept_name*



Biology	
Comp. Sci.	
Elec. Eng.	
Finance	
History	
Music	
Physics	

76766	Crick	Biology	72000	
10101	Srinivasan	Comp. Sci.	65000	
45565	Katz	Comp. Sci.	75000	
83821	Brandt	Comp. Sci.	92000	
98345	Kim	Elec. Eng.	80000	
12121	Wu	Finance	90000	
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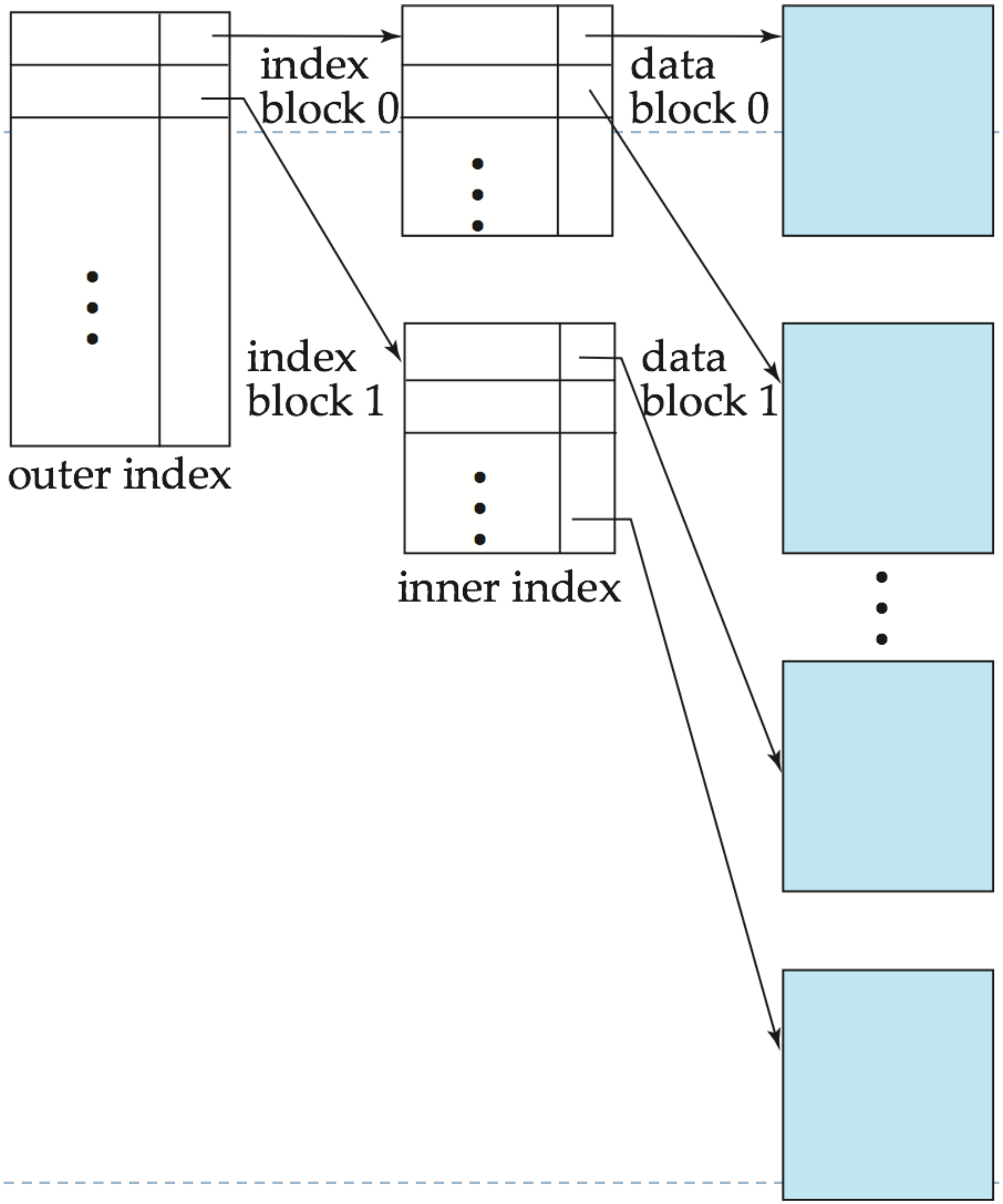
32343	El Said	History	60000	
58583	Califieri	History	62000	
15151	Mozart	Music	40000	
22222	Einstein	Physics	95000	
33465	Gold	Physics	87000	

Clustering index – records of a file are physically ordered on a non key field.

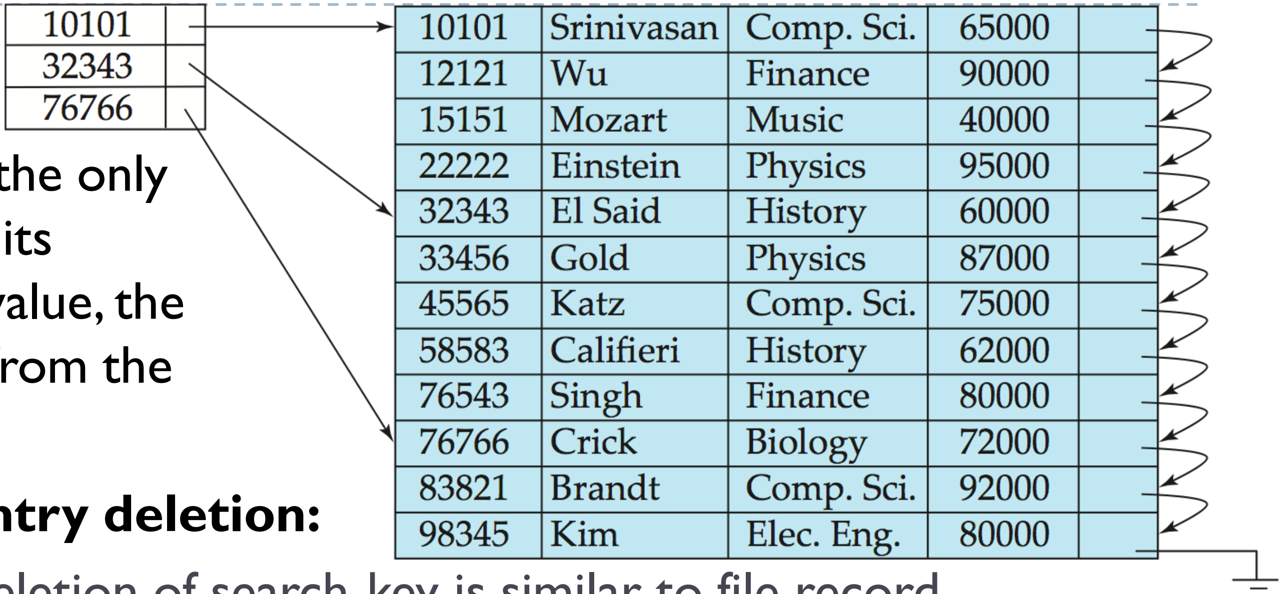
Multilevel Index

- If primary index does not fit in memory, access becomes expensive.
- Solution: treat primary index kept on disk as a sequential file and construct a sparse index on it.
  - outer index – a sparse index of primary index
  - inner index – the primary index file
- If even outer index is too large to fit in main memory, yet another level of index can be created, and so on.
- Indices at all levels must be updated on insertion or deletion from the file.

Multilevel Index (Cont.)



Index Update: Record Deletion



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32343
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- If deleted record was the only record in the file with its particular search-key value, the search-key is deleted from the index also.

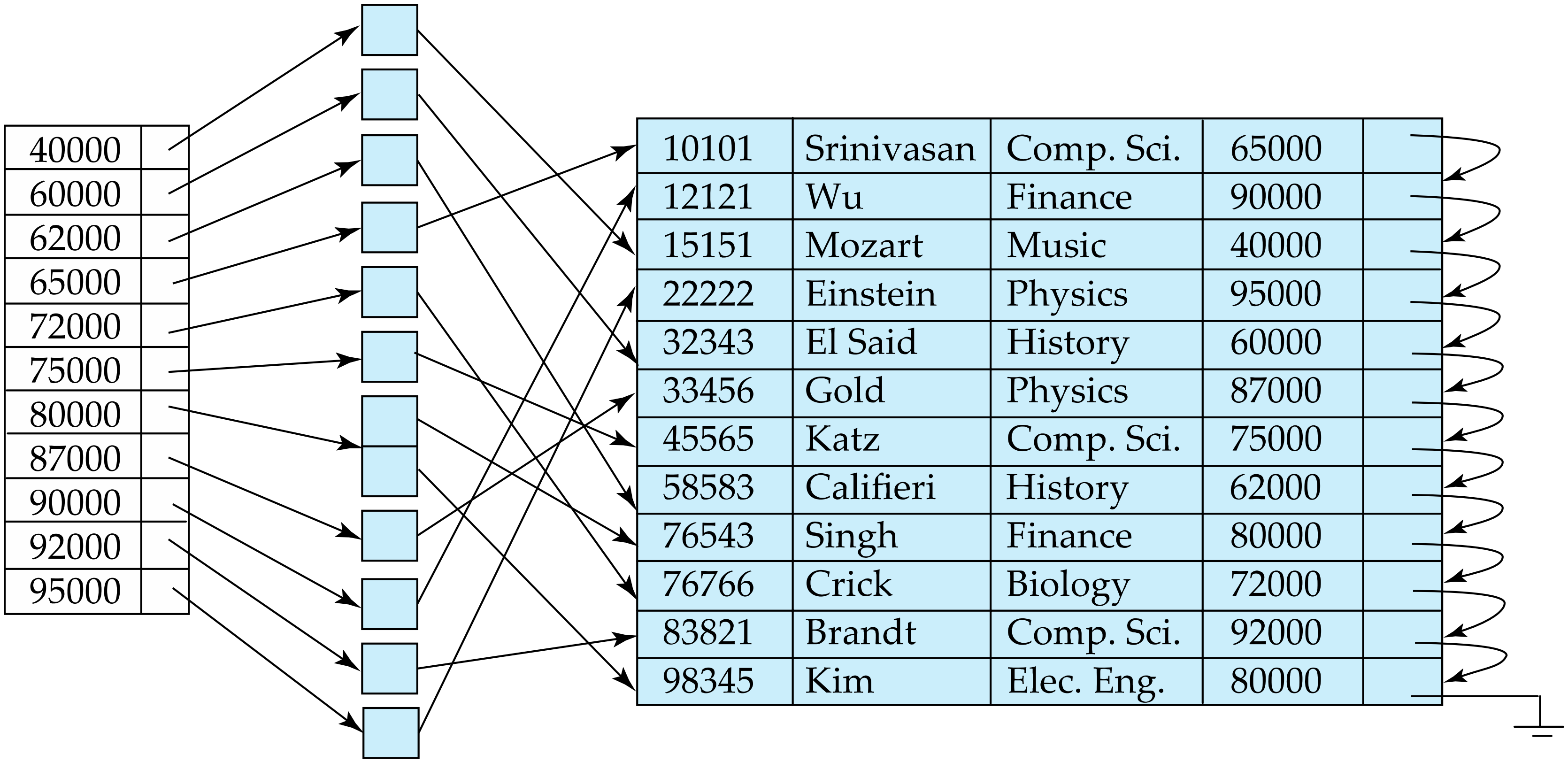
- **Single-level index entry deletion:**
  - **Dense indices** – deletion of search-key is similar to file record deletion.
  - **Sparse indices** –
    - if an entry for the search key exists in the index, it is deleted by replacing the entry in the index with the next search-key value in the file (in search-key order).
    - If the next search-key value already has an index entry, the entry is deleted instead of being replaced.

Index Update: Record Insertion

- **Single-level index insertion:**
  - Perform a lookup using the search-key value appearing in the record to be inserted.
  - **Dense indices** – if the search-key value does not appear in the index, insert it.
  - **Sparse indices** – if index stores an entry for each block of the file, no change needs to be made to the index unless a new block is created.
    - If a new block is created, the first search-key value appearing in the new block is inserted into the index.

- **Multilevel insertion and deletion:** algorithms are simple extensions of the single-level algorithms

Secondary Indices Example



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10101	Srinivasan	Comp. Sci.	65000
12121	Wu	Finance	90000
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Secondary index on *salary* field of *instructor*

- Index record points to a bucket that contains pointers to all the actual records with that particular search-key value.
- Secondary indices have to be dense

Primary and Secondary Indices

- Indices offer substantial benefits when searching for records.
- BUT: Updating indices imposes overhead on database modification --when a file is modified, every index on the file must be updated,
- Sequential scan using primary index is efficient, but a sequential scan using a secondary index is expensive
  - Each record access may fetch a new block from disk
  - Block fetch requires about 5 to 10 milliseconds, versus about 100 nanoseconds for memory access

Secondary Indices

- Frequently, one wants to find all the records whose values in a certain field (which is not the search-key of the primary index) satisfy some condition.
  - Example 1: In the *instructor* relation stored sequentially by ID, we may want to find all instructors in a particular department
  - Example 2: as above, but where we want to find all instructors with a specified salary or with salary in a specified range of values
- We can have a secondary index with an index record for each search-key value

B^+-Tree Index Files

B^+-tree indices are an alternative to indexed-sequential files.

- Disadvantage of indexed-sequential files
  - performance degrades as file grows, since many overflow blocks get created.
  - Periodic reorganization of entire file is required.
- Advantage of B^+-tree index files:
  - automatically reorganizes itself with small, local, changes, in the face of insertions and deletions.
  - Reorganization of entire file is not required to maintain performance.
- (Minor) disadvantage of B^+-trees:
  - extra insertion and deletion overhead, space overhead.
- Advantages of B^+-trees outweigh disadvantages
  - B^+-trees are used extensively

Account file

Brighton	A-217	750
Downtown	A-101	500
Downtown	A-110	600
Miami	A-215	700
Perryridge	A-102	400
Perryridge	A-201	900
Perryridge	A-218	700
Redwood	A-222	700
Round Hill	A-305	350

B^+-Tree Index Files (Cont.)

A B^+-tree is a rooted tree satisfying the following properties:

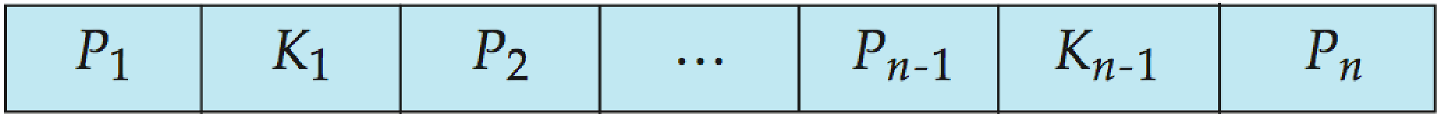
In a B+ tree of order (or branching factor) n:

- All paths from root to leaf are of the same length
- Each node that is not a root or a leaf has between $\lceil n/2 \rceil$ and n children.
- A leaf node has between $\lceil (n-1)/2 \rceil$ and $n-1$ values
- Special cases:
  - If the root is not a leaf, it has at least 2 children.
  - If the root is a leaf (that is, there are no other nodes in the tree), it can have between 0 and $(n-1)$ values.

ceiling(x) = $\lceil x \rceil$ is the smallest integer greater than or equal to x

B+-Tree Node Structure

- Typical node



P_1	K_1	P_2	…	P_{n-1}	K_{n-1}	P_n

  - K_i are the search-key values
  - P_i are pointers to children (for non-leaf nodes) or pointers to records or buckets of records (for leaf nodes).
- The search-keys in a node are ordered

$$K_1 < K_2 < K_3 < \ldots < K_{n-1}$$

(Initially assume no duplicate keys)

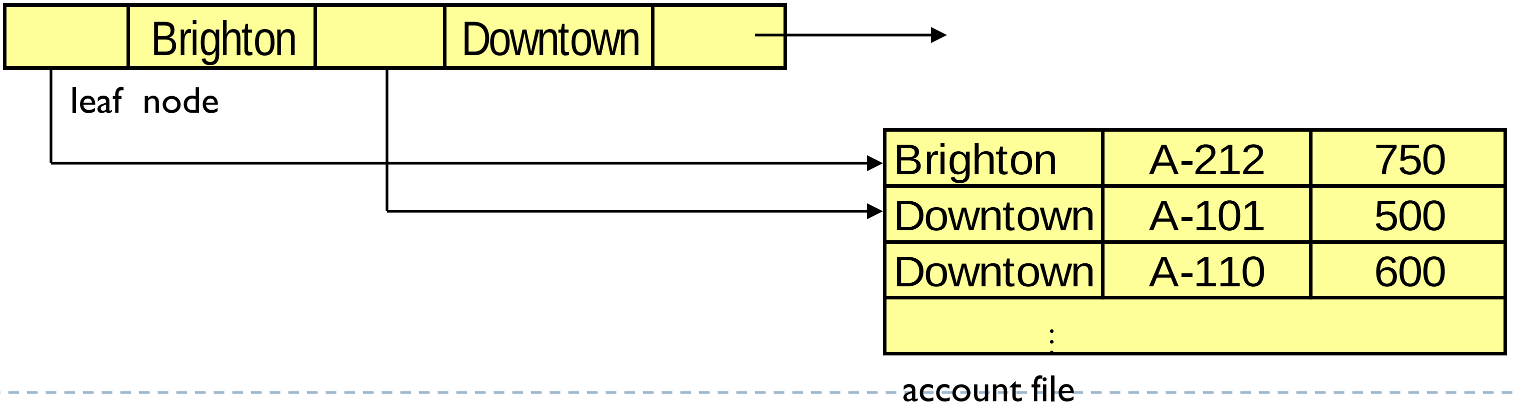
Leaf Nodes in B^+-Trees

Properties of a leaf node:

- For i = 1,2,…, n-1, pointer P_i either points to a file record with search-key value K_i, or to a bucket of pointers to file records, each record having search-key value K_i. Only need bucket structure if search-key does not form a primary key.

- If L_i, L_j are leaf nodes and i < j, L_i's search-key values are less than L_j's search-key values

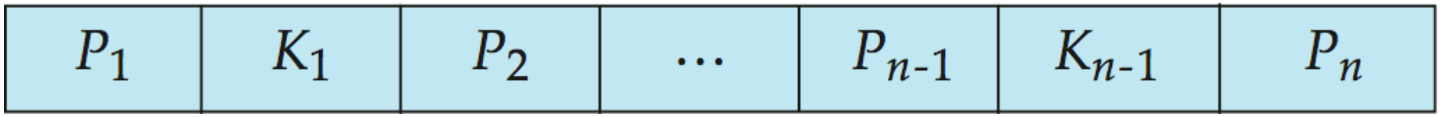
- P_n points to next leaf node in search-key order



Brighton	A-212	750
Downtown	A-101	500
Downtown	A-110	600
:		

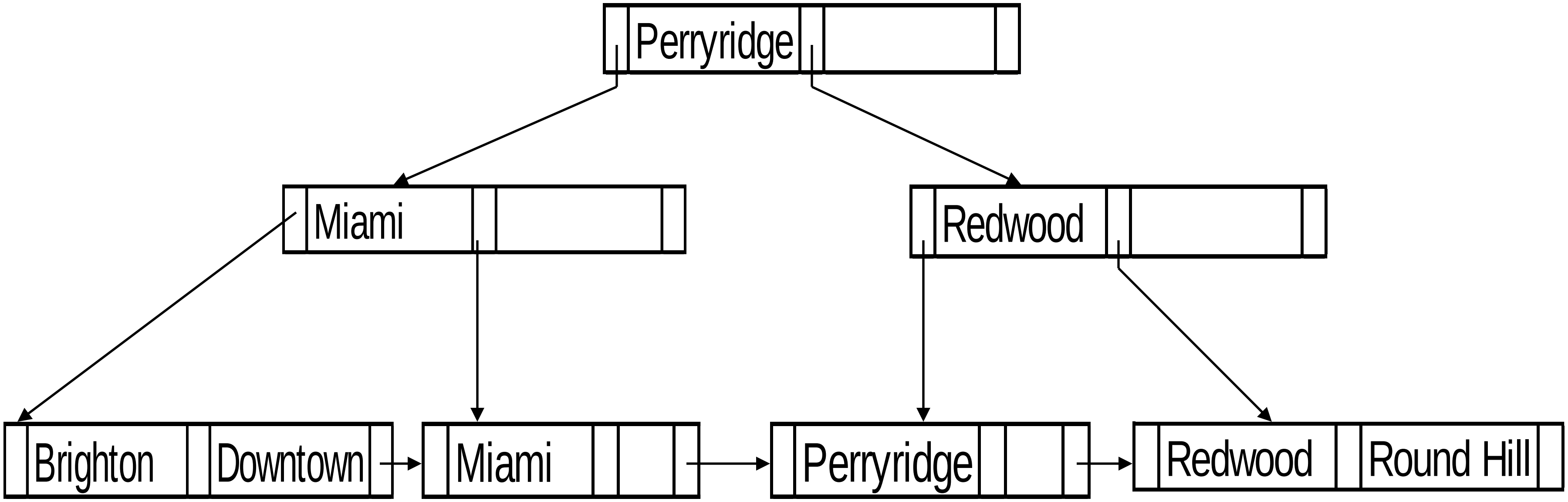
Non-Leaf Nodes in B+-Trees

- Non leaf nodes form a multi-level sparse index on the leaf nodes. For a non-leaf node with *m* pointers:
  - All the search-keys in the subtree to which P_1 points are less than K_1
  - For $2 \le i \le n - 1$, all the search-keys in the subtree to which P_i points have values greater than or equal to K_{i-1} and less than K_i
  - All the search-keys in the subtree to which P_n points have values greater than or equal to K_{n-1}



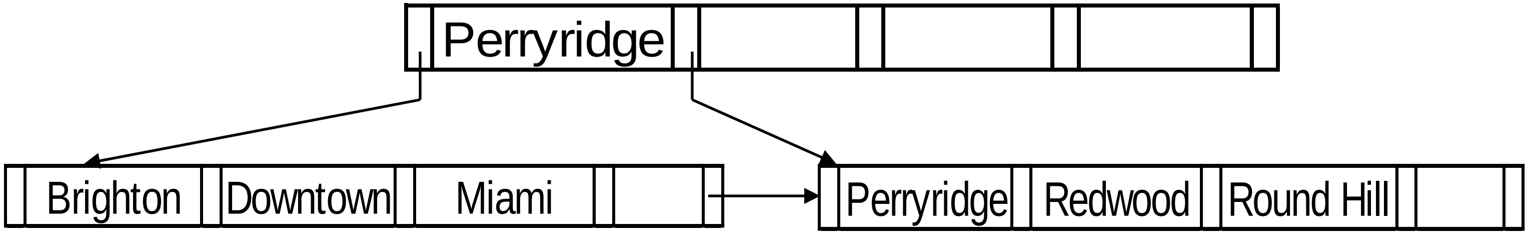
P_1	K_1	P_2	$\ldots$	P_{n-1}	K_{n-1}	P_n

Examples of a B$^+$-tree



B$^+$-tree for account file (n=3)

Example of a B$^+$-tree



B$^+$-tree for account file (n=5)

- Leaf nodes must have between 2 and 4 values ($\lceil (n-1)/2 \rceil$ and n – 1, with n=5).
- Non-leaf nodes other than root must have between 3 and 5 children ($\lceil n/2 \rceil$ and n with n = 5).
- Root must have at least 2 children

Observations about B$^+$-trees

- Since the inter-node connections are done by pointers, "logically" close blocks need not be "physically" close.
- The non-leaf levels of the B$^+$-tree form a hierarchy of sparse indices.
- The B$^+$-tree contains a relatively small number of levels
    - Level below root has at least $2*\lceil n/2 \rceil$ values
    - Next level has at least $2*\lceil n/2 \rceil * \lceil n/2 \rceil$ values
    - .. etc.
  - If there are *K* search-key values in the file, the tree height is no more than $\lceil \log_{\lceil n/2 \rceil}(K) \rceil$
  - thus searches can be conducted efficiently.
- Insertions and deletions to the main file can be handled efficiently, as the index can be restructured in logarithmic time (as we shall see).

Queries on B^+-Trees

- Find all records with a search-key value of k.
  - Start with the root node
  - Examine the node for the smallest search-key value > k.
  - If such a value exists, assume it is K_i. Then follow P_i to the child node
  - Otherwise $k \geq K_{n-1}$, where there are n pointers in the node, Then follow P_n to the child node.
  - If the node reached by following the pointer above is not a leaf node, repeat the above procedure on the node, and follow the corresponding pointer.
  - Eventually reach a leaf node. If key K_i = k, follow pointer P_i to the desired record or bucket. Else no record with search-key value k exists.

Queries on B^+-Trees

Find all records with a search-key value of k.

- 1. C=root
- 2. Repeat
  - 1. Examine C for the smallest search-key value > k.
  - 2. If such a value exists, assume it is K_i. Then set C = P_i
  - 3. Otherwise k ≥ K_{n-1}. Set C = P_n

    Until C is a leaf node
- 3. If for some i, key K_i = k follow pointer P_i to the desired record or bucket.
- 4. Else no record with search-key value k exists.

Queries on B^{+}-Trees (Cont.)

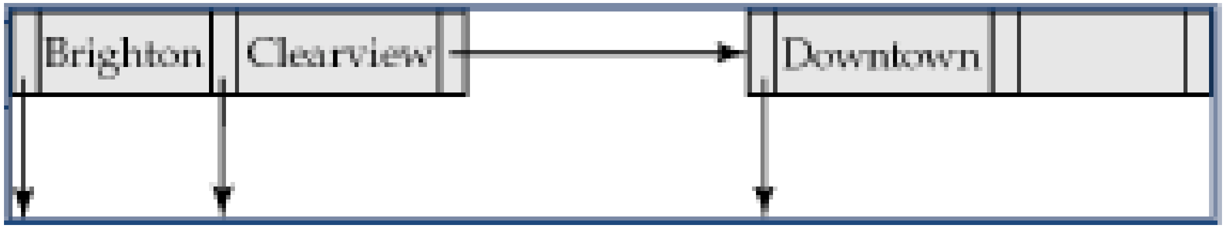
- If there are K search-key values in the file, the height of the tree is no more than $\lceil \log_{\lceil n/2 \rceil}(K) \rceil$.
- A node is generally the same size as a disk block, typically 4 kilobytes
  - and n is typically around 100 (40 bytes per index entry).
- With 1 million search key values and n = 100
  - at most log_{50}(1,000,000) = 4 nodes are accessed in a lookup.
- Contrast this with a balanced binary tree with 1 million search key values — around 20 nodes are accessed in a lookup
  - above difference is significant since every node access may need a disk I/O, costing around 20 milliseconds

Updates on B^+-Trees: Insertion

1. Find the leaf node in which the search-key value would appear
2. If the search-key value is already present in the leaf node
    1. Add record to the file
3. If the search-key value is not present, then
    1. Add the record to the main file (and create a bucket if necessary)
    2. If there is room in the leaf node, insert (key-value, pointer) pair in the leaf node
    3. Otherwise, split the node (along with the new (key-value, pointer) entry) as discussed in the next slide.

Updates on B^+-Trees: Insertion (Cont.)

- Splitting a leaf node:
  - take the *n* (search-key value, pointer) pairs (including the one being inserted) in sorted order. Place the first $\lceil n/2 \rceil$ in the original node, and the rest in a new node.
  - let the new node be *p*, and let *k* be the least key value in *p*. Insert (*k*,*p*) in the parent of the node being split.
  - If the parent is full, split it and **propagate** the split further up.
- Splitting of nodes proceeds upwards till a node that is not full is found.
  - In the worst case the root node may be split increasing the height of the tree by 1.

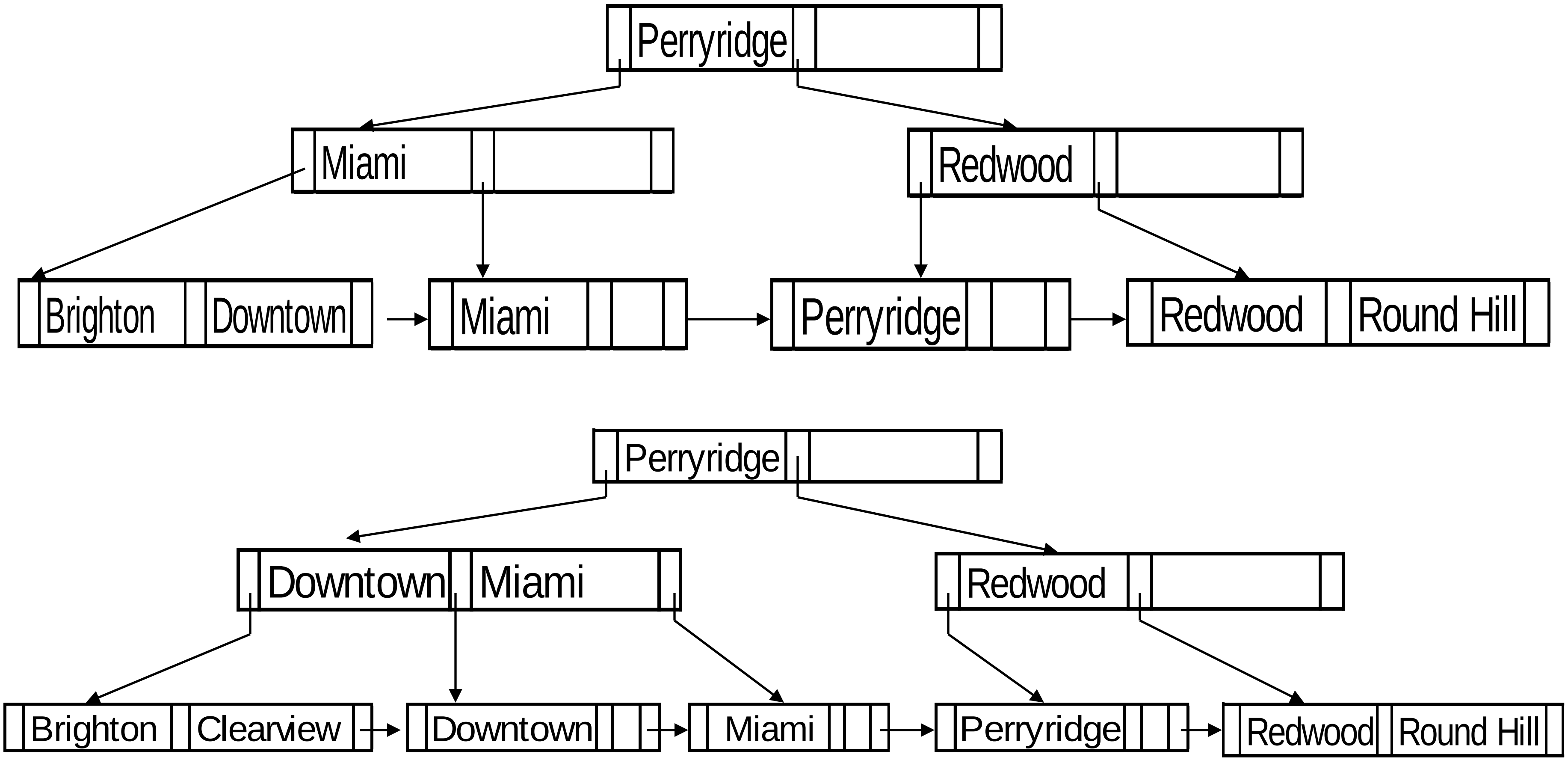


Result of splitting node containing Brighton and Downtown on inserting Clearview.

- Next step: insert entry with (Downtown,pointer-to-new-node) into parent

Updates on B+-Trees: Insertion(Cont.)

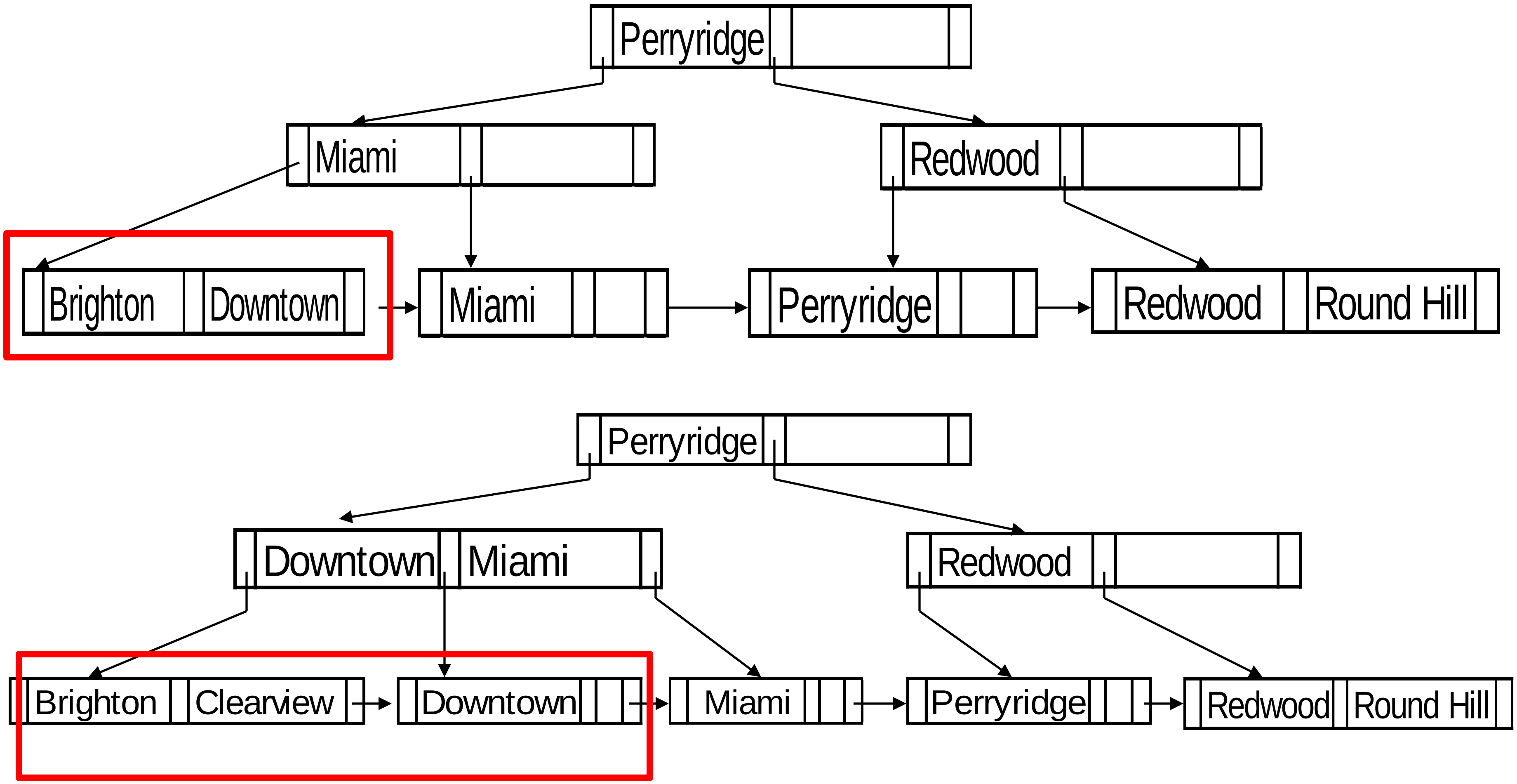
B+-Tree before and after insertion of "Clearview"



Updates on B+-Trees: Insertion(Cont.)

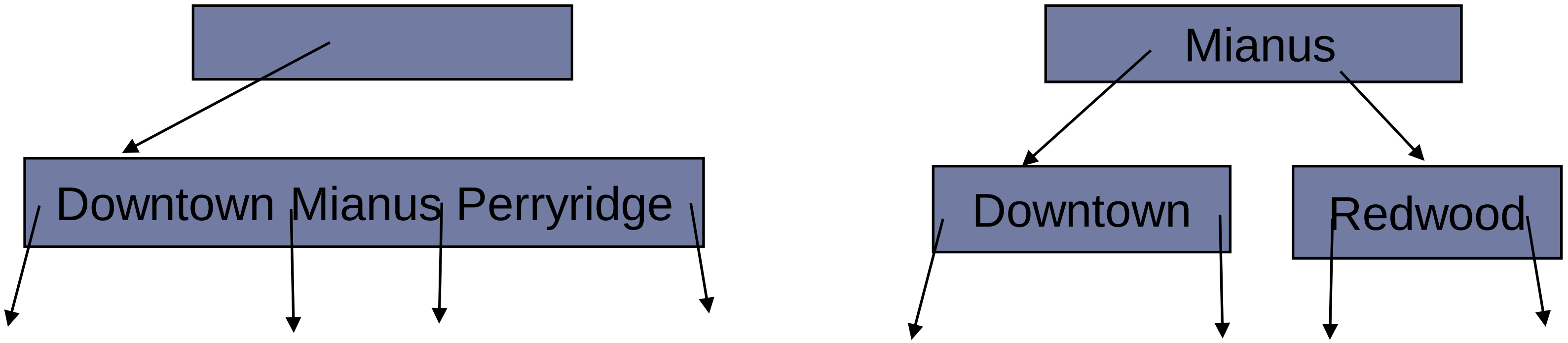
B+-Tree before and after insertion of "Clearview"

B+ Tree of order 3



Insertion in B$^+$-Trees (Cont.)

- Splitting a non-leaf node: when inserting (k,p) into an already full internal node N
  - Copy N to an in-memory area M with space for n+1 pointers and n keys
  - Insert (k,p) into M
  - Copy $P_1,K_1, \ldots, K_{\lceil n/2 \rceil -1},P_{\lceil n/2 \rceil}$ from M back into node N
  - Copy $P_{\lceil n/2 \rceil +1},K_{\lceil n/2 \rceil +1},\ldots,K_n,P_{n+1}$ from M into newly allocated node N’
  - Insert $(K_{\lceil n/2 \rceil},N')$ into parent N



Downtown Mianus Perryridge

Mianus

Downtown

Redwood

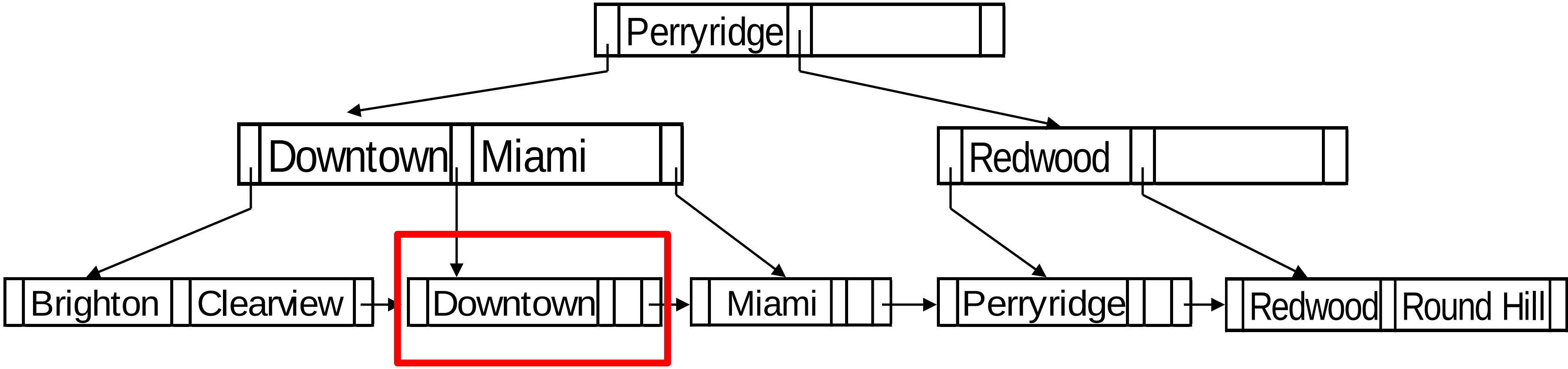
Updates on B^+-Trees: Deletion

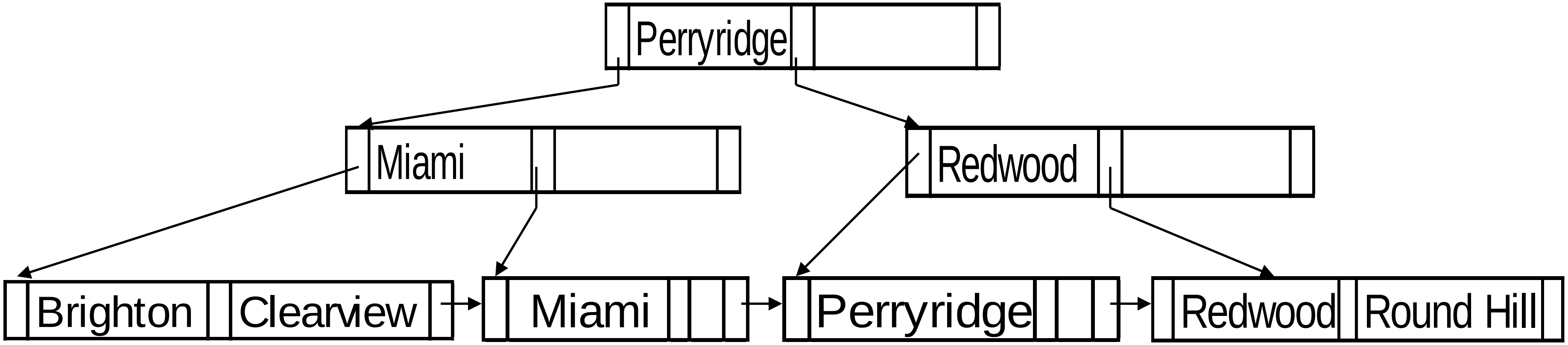
- Find the record to be deleted, and remove it from the main file and from the bucket (if present)
- Remove (search-key value, pointer) from the leaf node if there is no bucket or if the bucket has become empty
- If the node has too few entries due to the removal, and the entries in the node and a sibling fit into a single node, then ***merge siblings*:**
  - Insert all the search-key values in the two nodes into a single node (the one on the left), and delete the other node.
  - Delete the pair (K_{i-1}, P_i), where P_i is the pointer to the deleted node, from its parent, recursively using the above procedure.

Updates on B^+-Trees: Deletion

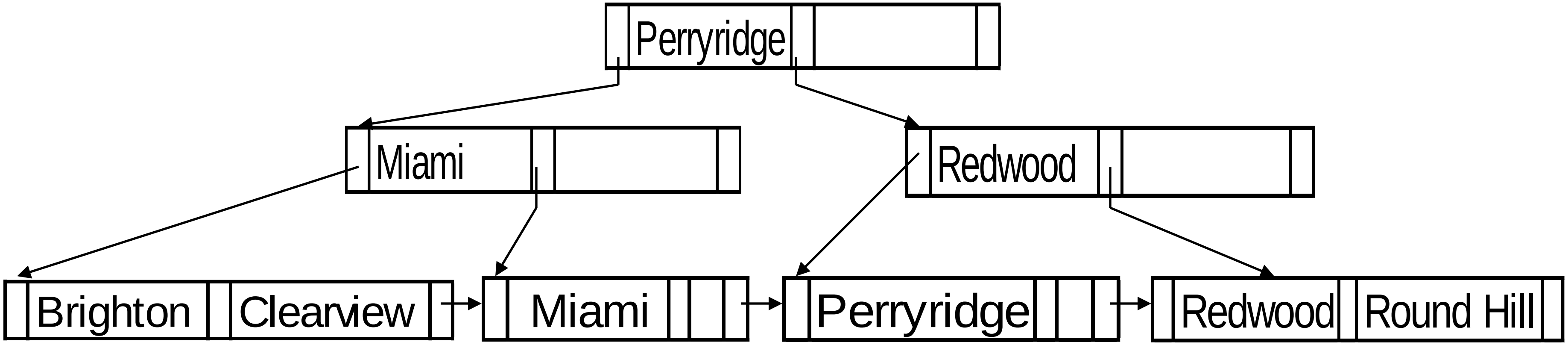
- Otherwise, if the node has too few entries due to the removal, but the entries in the node and a sibling do not fit into a single node, then **redistribute pointers**:
  - Redistribute the pointers between the node and a sibling such that both have more than the minimum number of entries.
  - Update the corresponding search-key value in the parent of the node.
- The node deletions may cascade upwards till a node which has $\lceil n/2 \rceil$ or more pointers is found.
- If the root node has only one pointer after deletion, it is deleted and the sole child becomes the root.

Examples of B$^+$-Tree Deletion (1)





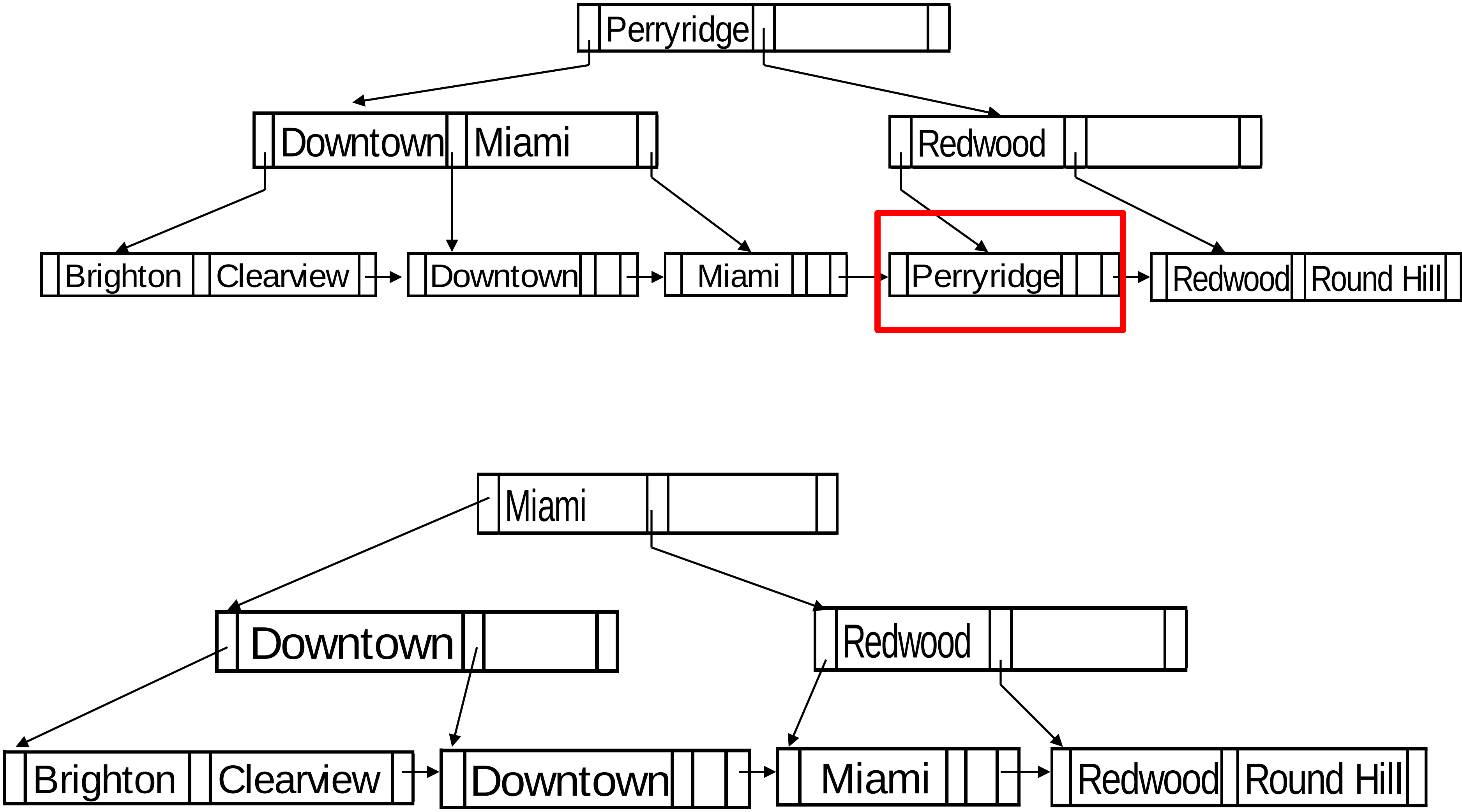
Examples of B$^+$-Tree Deletion



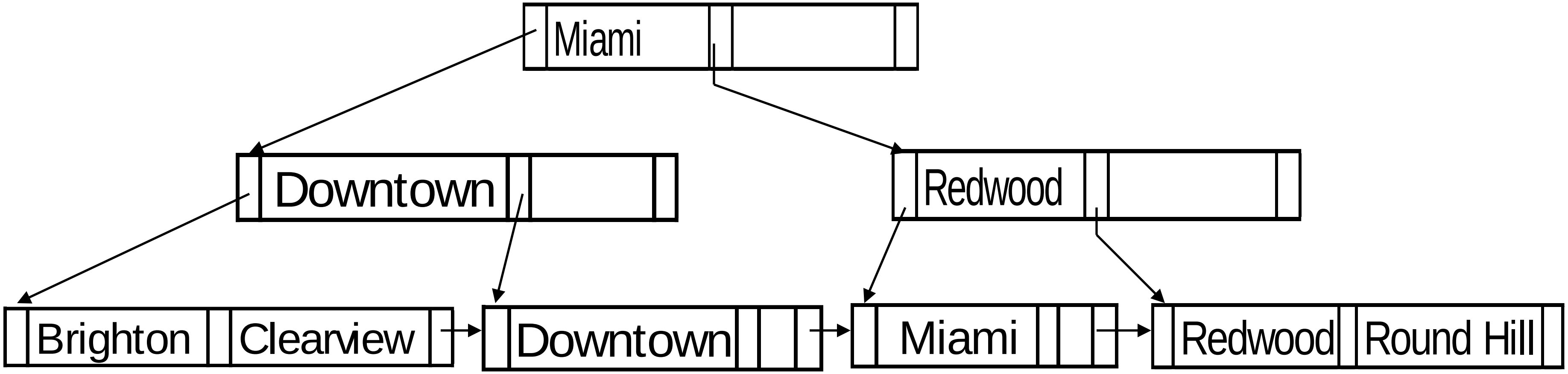
Result after deleting “Downtown” from account

- The removal of the leaf node containing “Downtown” did not result in its parent having too little pointers. So the cascaded deletions stopped with the deleted leaf node’s parent.

Examples of B$^+$-Tree Deletion (2)



Examples of B+-Tree Deletion (Cont.)



Deletion of “Perryridge” instead of “Downtown”

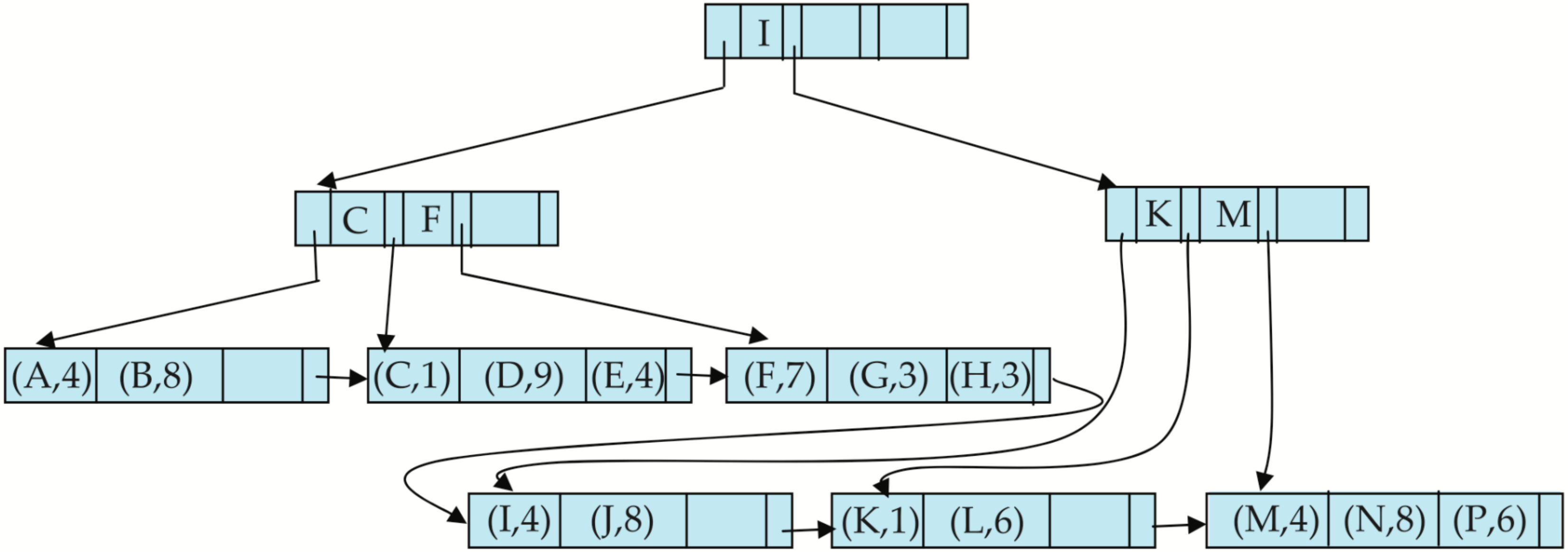
- The deleted “Perryridge” node’s parent became too small, but its sibling did not have space to accept one more pointer, so redistribution is performed. Observe that the root node’s search-key value changes as a result.

- Leaf with “Perryridge” becomes underfull (actually empty, in this special case) and merged with its sibling.
- As a result “Perryridge” node’s parent became underfull, and was merged with its sibling
    - Value separating two nodes (at parent) moves into merged node
    - Entry deleted from parent
- Root node then has only one child, and is deleted

B^+-Tree File Organization

- Index file degradation problem is solved by using B^+-Tree indices.
- Data file degradation problem is solved by using B^+-Tree File Organization.
- The leaf nodes in a B^+-tree file organization store records, instead of pointers.
- Leaf nodes are still required to be half full
  - Since records are larger than pointers, the maximum number of records that can be stored in a leaf node is less than the number of pointers in a nonleaf node.
- Insertion and deletion are handled in the same way as insertion and deletion of entries in a B^+-tree index.

B$^+$-Tree File Organization (Cont.)



Example of B$^+$-tree File Organization

- Good space utilization important since records use more space than pointers.
- To improve space utilization, involve more sibling nodes in redistribution during splits and merges
  - Involving 2 siblings in redistribution (to avoid split / merge where possible) results in each node having at least $\lfloor 2n/3 \rfloor$ entries

Other Issues in Indexing

- **Record relocation and secondary indices**
  - If a record moves, all secondary indices that store record pointers have to be updated
  - Node splits in B$^+$-tree file organizations become very expensive
  - *Solution*: use primary-index search key instead of record pointer in secondary index
    - Extra traversal of primary index to locate record
      - □ Higher cost for queries, but node splits are cheap
    - Add record-id if primary-index search key is non-unique

Multiple-Key Access

- Use multiple indices for certain types of queries.
- Example:

  select *ID*

  from *instructor*

  where *dept_name* = "Finance" **and** *salary* = 80000
- Possible strategies for processing query using indices on single attributes:
  1. Use index on *dept_name* to find instructors with department name Finance; test *salary = 80000*
  2. Use index on *salary* to find instructors with a salary of $80000; test *dept_name* = "Finance".
  3. Use *dept_name* index to find pointers to all records pertaining to the "Finance" department. Similarly use index on *salary*. Take intersection of both sets of pointers obtained.

Indices on Multiple Keys

- **Composite search keys** are search keys containing more than one attribute
  - E.g. (*dept_name, salary*)
- Lexicographic ordering: $(a_1, a_2) < (b_1, b_2)$ if either
  - $a_1 < b_1$, or
  - $a_1=b_1$ and $a_2 < b_2$

Indices on Multiple Attributes

Suppose we have an index on combined search-key (*dept_name*, *salary*).

- With the **where** clause

  where *dept_name* = "Finance" **and** *salary* = 80000

  the index on (*dept_name*, *salary*) can be used to fetch only records that satisfy both conditions.
  - Using separate indices in less efficient — we may fetch many records (or pointers) that satisfy only one of the conditions.
- Can also efficiently handle

  where *dept_name* = "Finance" **and** *salary* < 80000
- But cannot efficiently handle

  where *dept_name* < "Finance" **and** *balance* = 80000
  - May fetch many records that satisfy the first but not the second condition

Hashing

Static Hashing

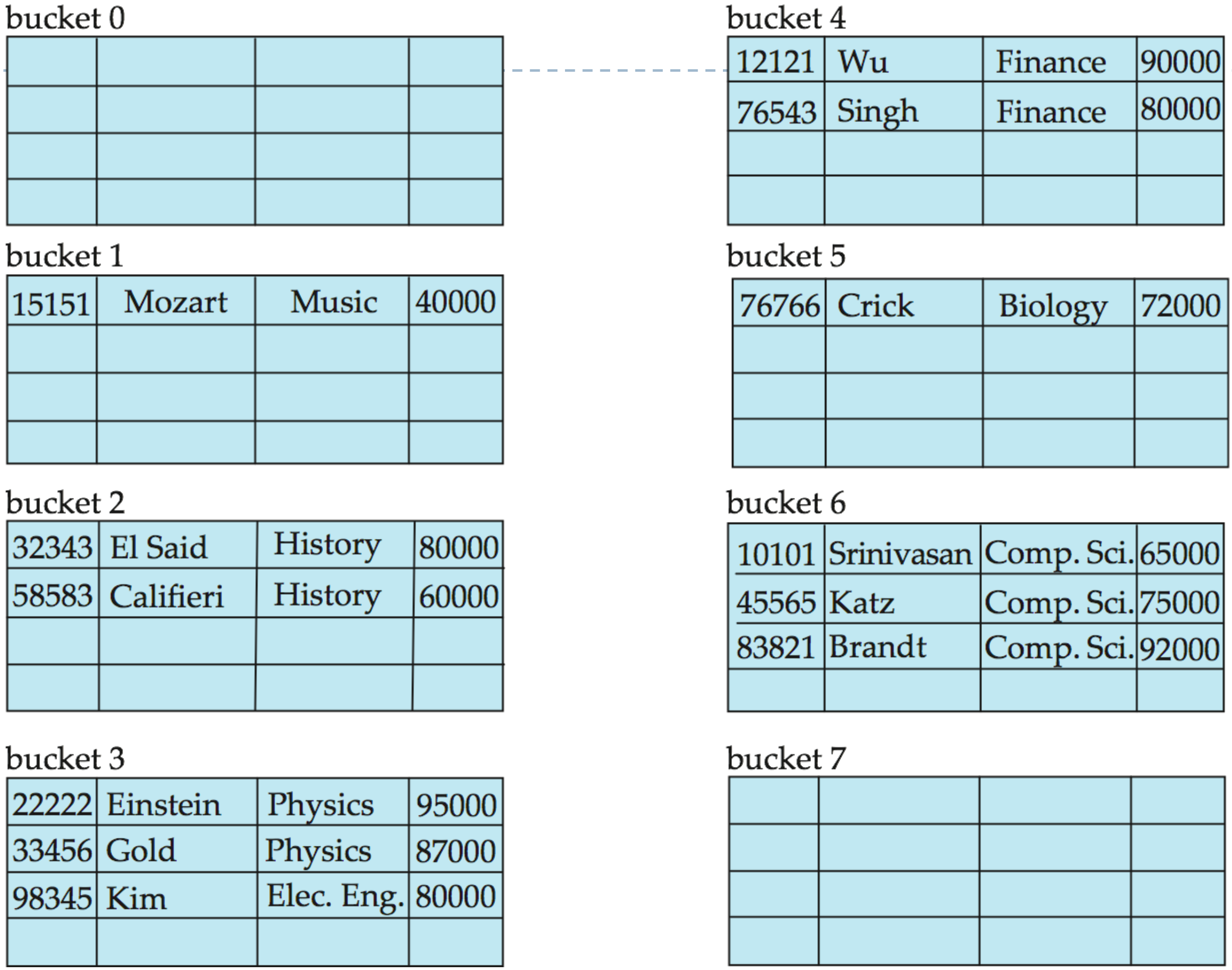
- A **bucket** is a unit of storage containing one or more records (a bucket is typically a disk block).
- In a **hash file organization** we obtain the bucket of a record directly from its search-key value using a **hash function.**
- Hash function *h* is a function from the set of all search-key values *K* to the set of all bucket addresses *B*.
- Hash function is used to locate records for access, insertion as well as deletion.
- Records with different search-key values may be mapped to the same bucket; thus entire bucket has to be searched sequentially to locate a record.

Example of Hash File Organization

Hash file organization of *instructor* file, using *dept_name* as key (See figure in next slide.)

- There are 10 buckets,
- The binary representation of the *i*th character is assumed to be the integer *i.*
- The hash function returns the sum of the binary representations of the characters modulo 10
  - E.g. h(Music) = 1 h(History) = 2
    h(Physics) = 3 h(Elec. Eng.) = 3

Example of Hash File Organization



bucket 0

bucket 1

15151	Mozart	Music	40000

bucket 2

32343	El Said	History	80000
58583	Califieri	History	60000

bucket 3

22222	Einstein	Physics	95000
33456	Gold	Physics	87000
98345	Kim	Elec. Eng.	80000

bucket 4

12121	Wu	Finance	90000
76543	Singh	Finance	80000

bucket 5

76766	Crick	Biology	72000

bucket 6

10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000
83821	Brandt	Comp. Sci.	92000

bucket 7

Hash file organization of *instructor* file, using *dept_name* as key (see previous slide for details).

Hash Functions

- Worst hash function maps all search-key values to the same bucket; this makes access time proportional to the number of search-key values in the file.
- An ideal hash function is **uniform**, i.e., each bucket is assigned the same number of search-key values from the set of *all* possible values.
- Ideal hash function is **random**, so each bucket will have the same number of records assigned to it irrespective of the *actual distribution* of search-key values in the file.
- Typical hash functions perform computation on the internal binary representation of the search-key.
  - For example, for a string search-key, the binary representations of all the characters in the string could be added and the sum modulo the number of buckets could be returned. .

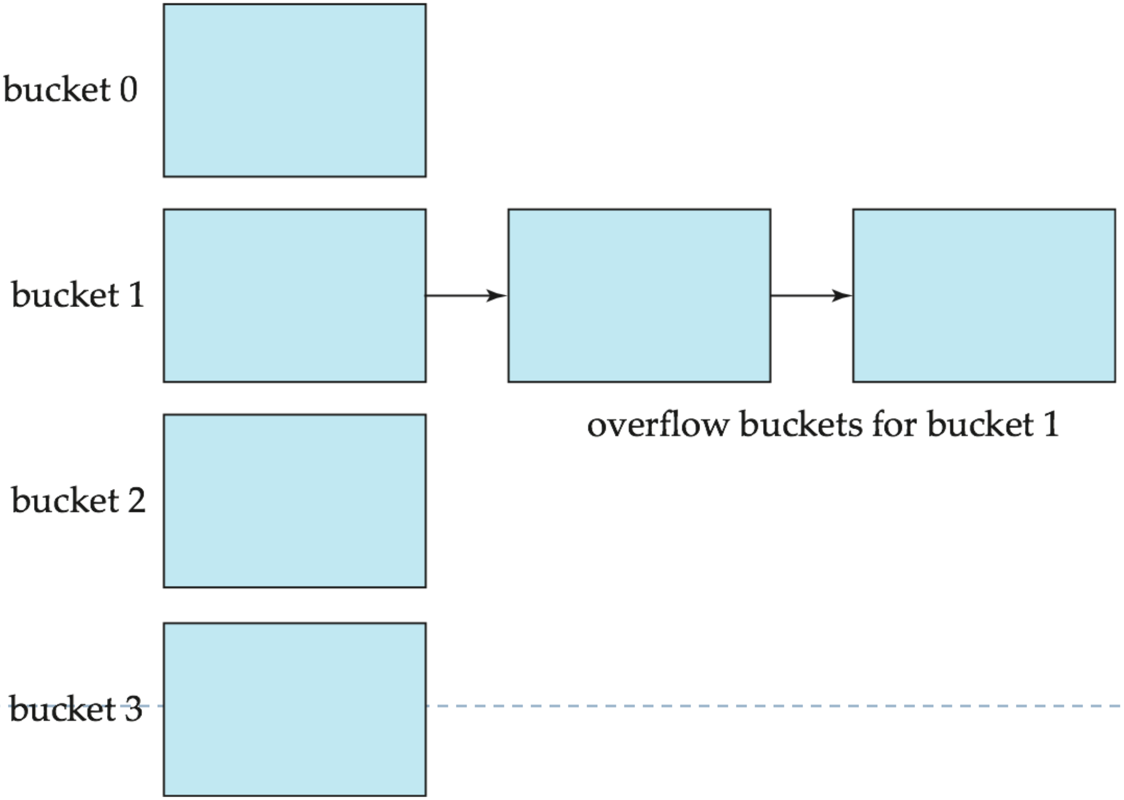
Handling of Bucket Overflows

- Bucket overflow can occur because of
  - Insufficient buckets
  - Skew in distribution of records. This can occur due to two reasons:
    - multiple records have same search-key value
    - chosen hash function produces non-uniform distribution of key values

- Although the probability of bucket overflow can be reduced, it cannot be eliminated; it is handled by using ***overflow buckets***.

Handling of Bucket Overflows (Cont.)

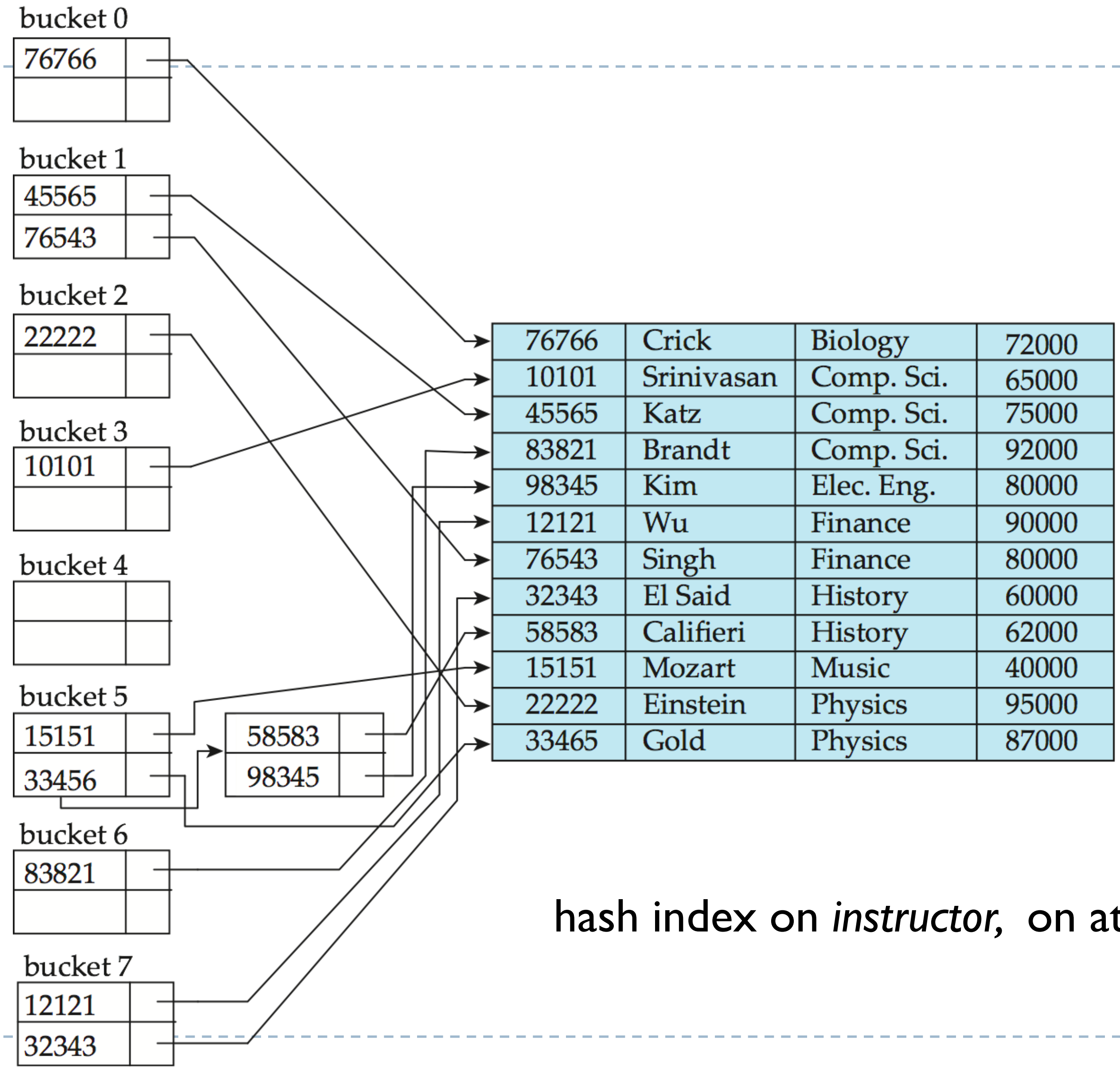
- **Overflow chaining** – the overflow buckets of a given bucket are chained together in a linked list.
- Above scheme is called **closed hashing**.
  - An alternative, called **open hashing**, which does not use overflow buckets, is not suitable for database applications.



Hash Indices

- Hashing can be used not only for file organization, but also for index-structure creation.
- A **hash index** organizes the search keys, with their associated record pointers, into a hash file structure.
- Strictly speaking, hash indices are always secondary indices
  - if the file itself is organized using hashing, a separate primary hash index on it using the same search-key is unnecessary.
  - However, we use the term hash index to refer to both secondary index structures and hash organized files.

Example of Hash Index



76766	Crick	Biology	72000
10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000
12121	Wu	Finance	90000
76543	Singh	Finance	80000
32343	El Said	History	60000
58583	Califieri	History	62000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
33465	Gold	Physics	87000

hash index on *instructor*, on attribute *ID*

Deficiencies of Static Hashing

- In static hashing, function *h* maps search-key values to a fixed set of *B* of bucket addresses. Databases grow or shrink with time.
  - If initial number of buckets is too small, and file grows, performance will degrade due to too much overflows.
  - If space is allocated for anticipated growth, a significant amount of space will be wasted initially (and buckets will be underfull).
  - If database shrinks, again space will be wasted.

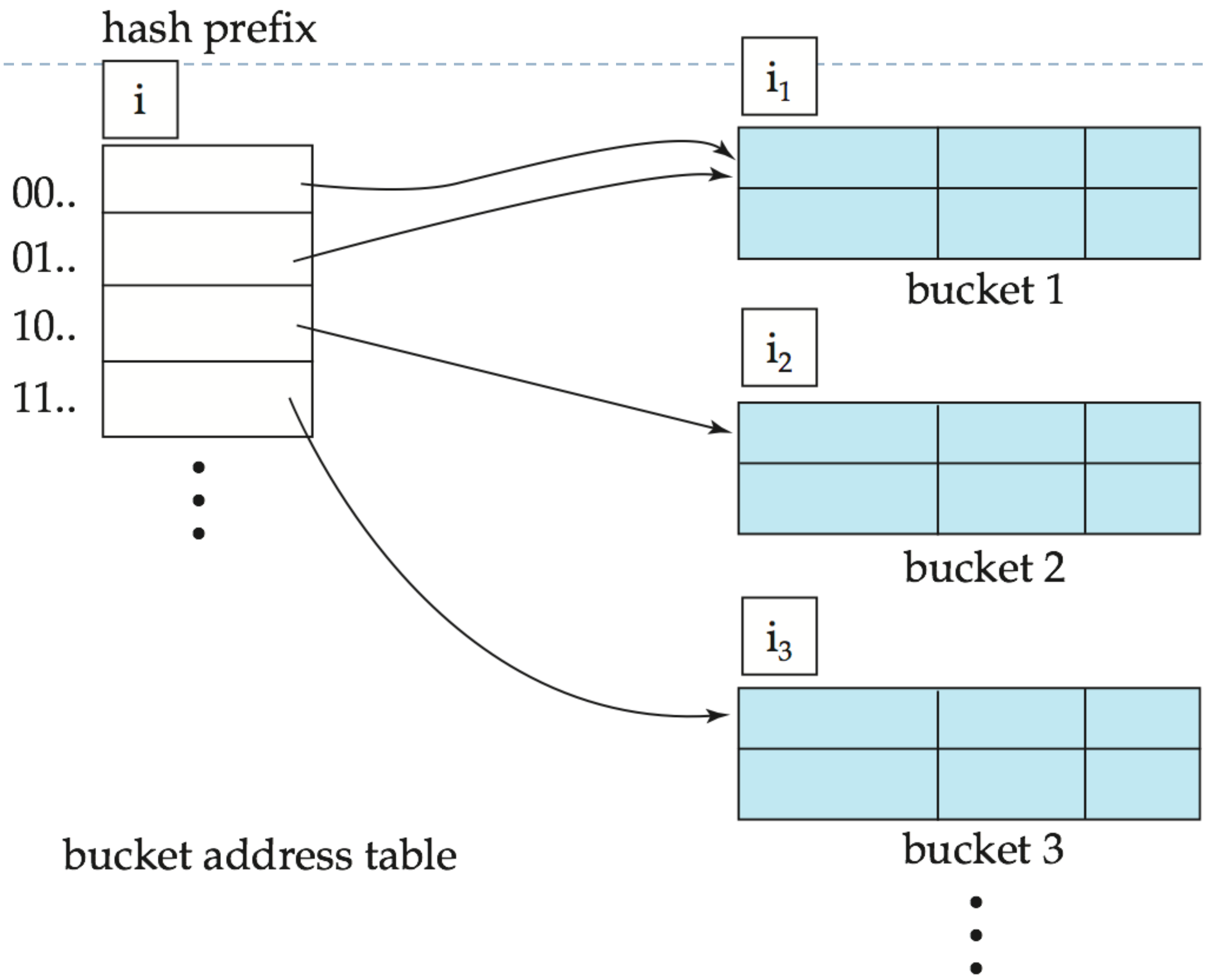
- One solution: periodic re-organization of the file with a new hash function
  - Expensive, disrupts normal operations

- Better solution: allow the number of buckets to be modified dynamically.

Dynamic Hashing

- Good for database that grows and shrinks in size
- Allows the hash function to be modified dynamically
- **Extendable hashing** – one form of dynamic hashing
  - Hash function generates values over a large range — typically *b*-bit integers, with *b* = 32.
  - At any time use only a prefix of the hash function to index into a table of bucket addresses.
  - Let the length of the prefix be *i* bits, $0 \leq i \leq 32$.
    - Bucket address table size = $2^{i.}$ Initially *i* = 0
    - Value of *i* grows and shrinks as the size of the database grows and shrinks.
  - Multiple entries in the bucket address table may point to a bucket (why?)
  - Thus, actual number of buckets is < 2^i
    - The number of buckets also changes dynamically due to coalescing and splitting of buckets.

General Extendable Hash Structure



In this structure, $i_2 = i_3 = i$, whereas $i_1 = i - 1$ (see next slide for details)

Use of Extendable Hash Structure

- Each bucket *j* stores a value i_j
  - *All* the entries that point to the same bucket have the same values on the first i_j bits.

- To locate the bucket containing search-key K_j:

  1. Compute $h(K_j) = X$
  2. Use the first *i* high order bits of *X* as a displacement into bucket address table, and follow the pointer to appropriate bucket

- To insert a record with search-key value K_j
  - follow same procedure as look-up and locate the bucket, say *j*.
  - If there is room in the bucket *j* insert record in the bucket.
  - Else the bucket must be split and insertion re-attempted (next slide.)
    - Overflow buckets used instead in some cases (will see shortly)

Insertion in Extendable Hash Structure (Cont)

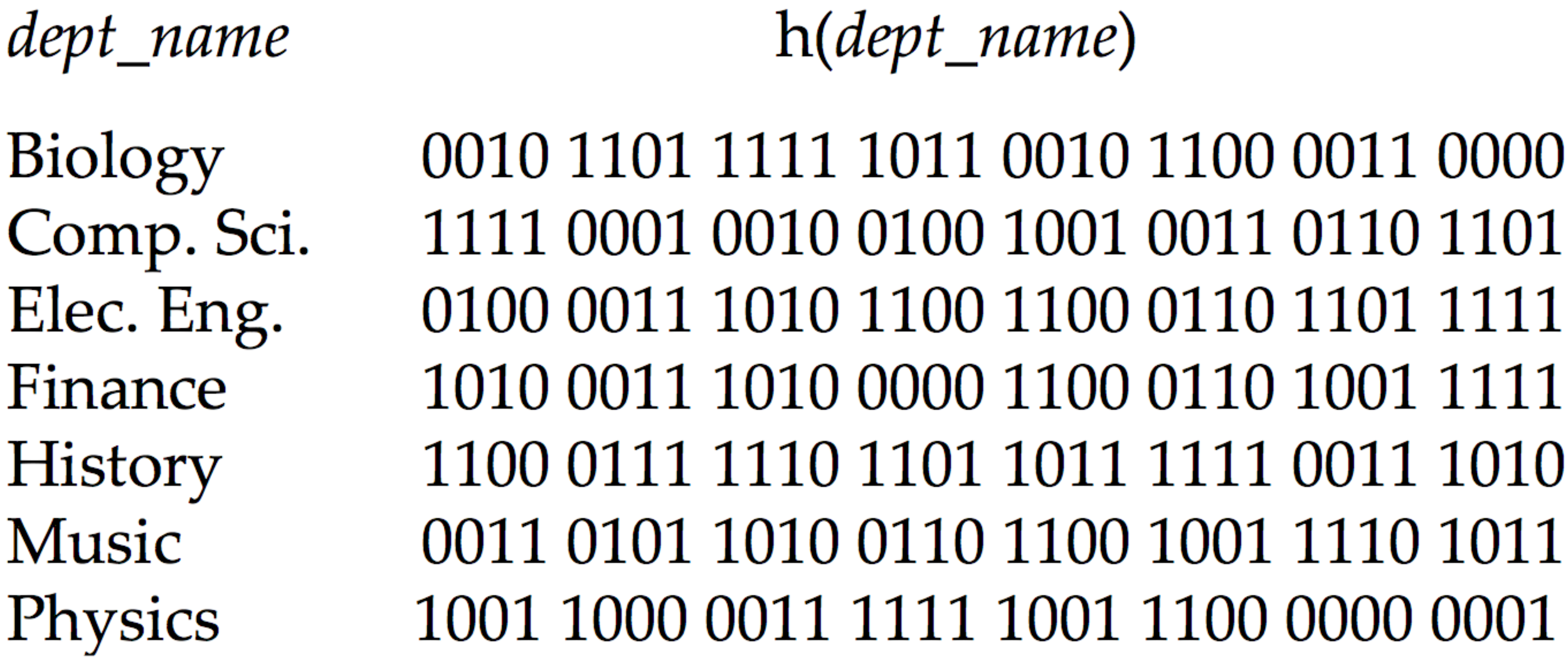
To split a bucket *j* when inserting record with search-key value K_j:

- If $i > i_j$ (more than one pointer to bucket *j*)
  - allocate a new bucket *z*, and set $i_j = i_z = (i_j + 1)$
  - Update the second half of the bucket address table entries originally pointing to *j*, to point to *z*
  - remove each record in bucket *j* and reinsert (in *j* or *z*)
  - recompute new bucket for K_j and insert record in the bucket (further splitting is required if the bucket is still full)
- If $i = i_j$ (only one pointer to bucket *j*)
  - If *i* reaches some limit *b*, or too many splits have happened in this insertion, create an overflow bucket
  - Else
    - increment *i* and double the size of the bucket address table.
    - replace each entry in the table by two entries that point to the same bucket.
    - recompute new bucket address table entry for K_j
      Now $i > i_j$ so use the first case above.

Deletion in Extendable Hash Structure

- To delete a key value,
  - locate it in its bucket and remove it.
  - The bucket itself can be removed if it becomes empty (with appropriate updates to the bucket address table).
  - Coalescing of buckets can be done (can coalesce only with a "*buddy*" bucket having same value of i_j and same $i_j - 1$ prefix, if it is present)
  - Decreasing bucket address table size is also possible
    - Note: decreasing bucket address table size is an expensive operation and should be done only if number of buckets becomes much smaller than the size of the table

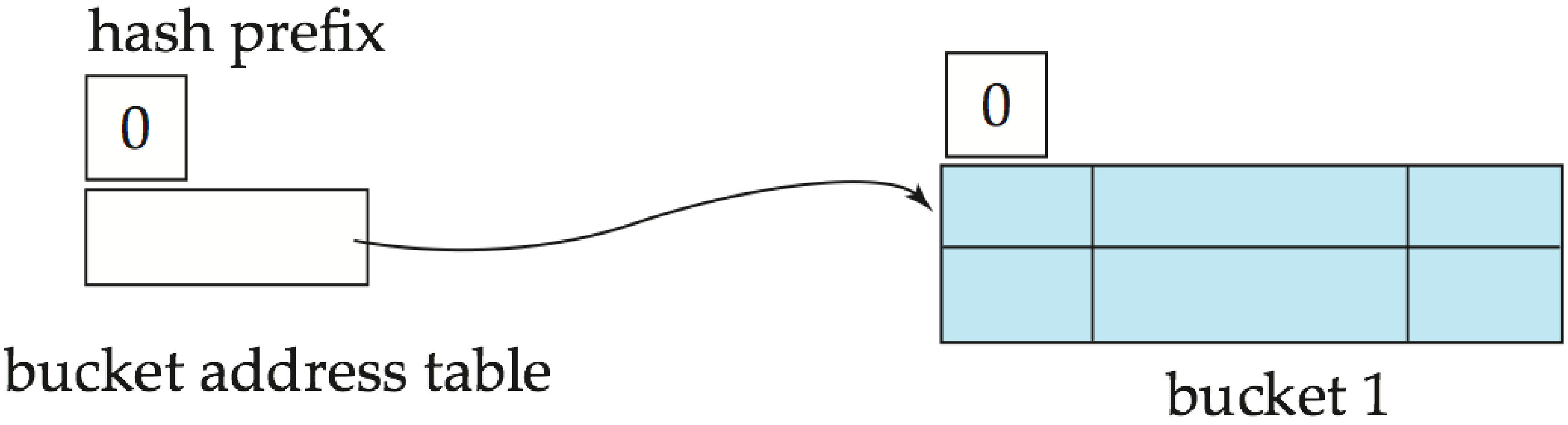
Use of Extendable Hash Structure: Example



dept_name	h(*dept_name*)
Biology	0010 1101 1111 1011 0010 1100 0011 0000
Comp. Sci.	1111 0001 0010 0100 1001 0011 0110 1101
Elec. Eng.	0100 0011 1010 1100 1100 0110 1101 1111
Finance	1010 0011 1010 0000 1100 0110 1001 1111
History	1100 0111 1110 1101 1011 1111 0011 1010
Music	0011 0101 1010 0110 1100 1001 1110 1011
Physics	1001 1000 0011 1111 1001 1100 0000 0001

Example (Cont.)

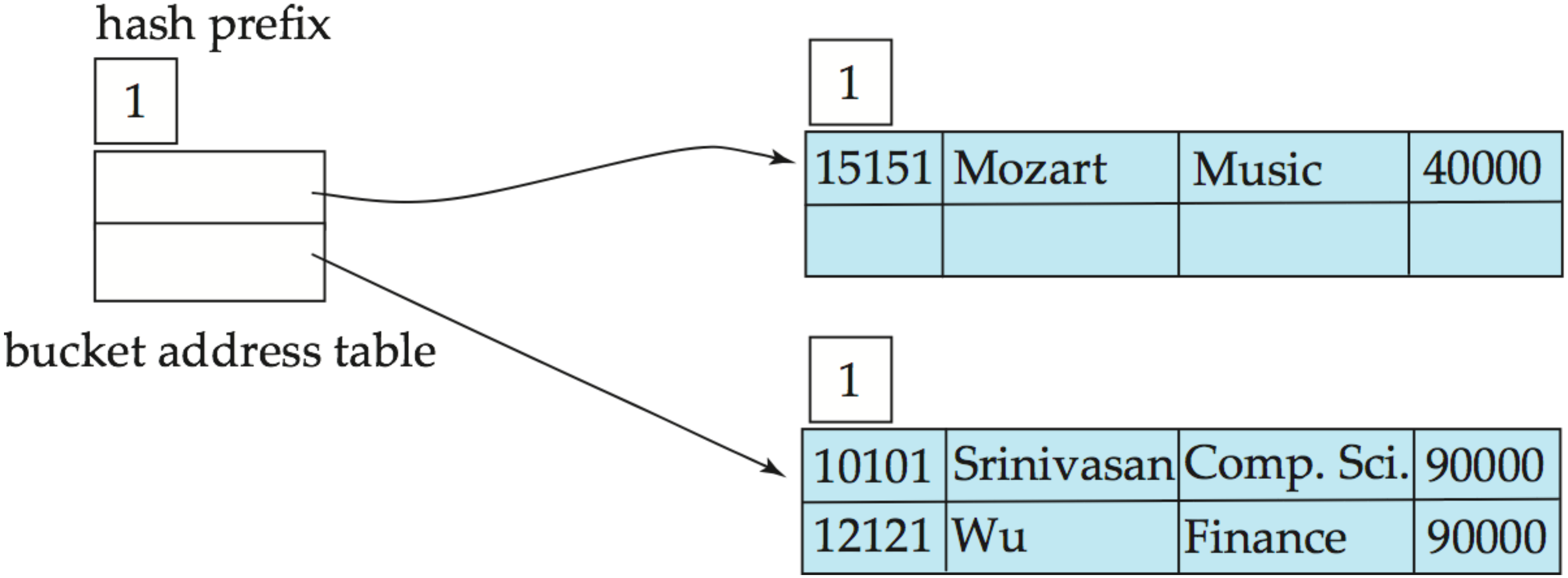
- Initial Hash structure; bucket size = 2



Example (Cont.)

dept_name	
Biology	0010
Comp. Sci.	1111
Elec. Eng.	0100
Finance	1010
History	1100
Music	0011
Physics	1001

- Hash structure after insertion of "Mozart", "Srinivasan", and "Wu" records



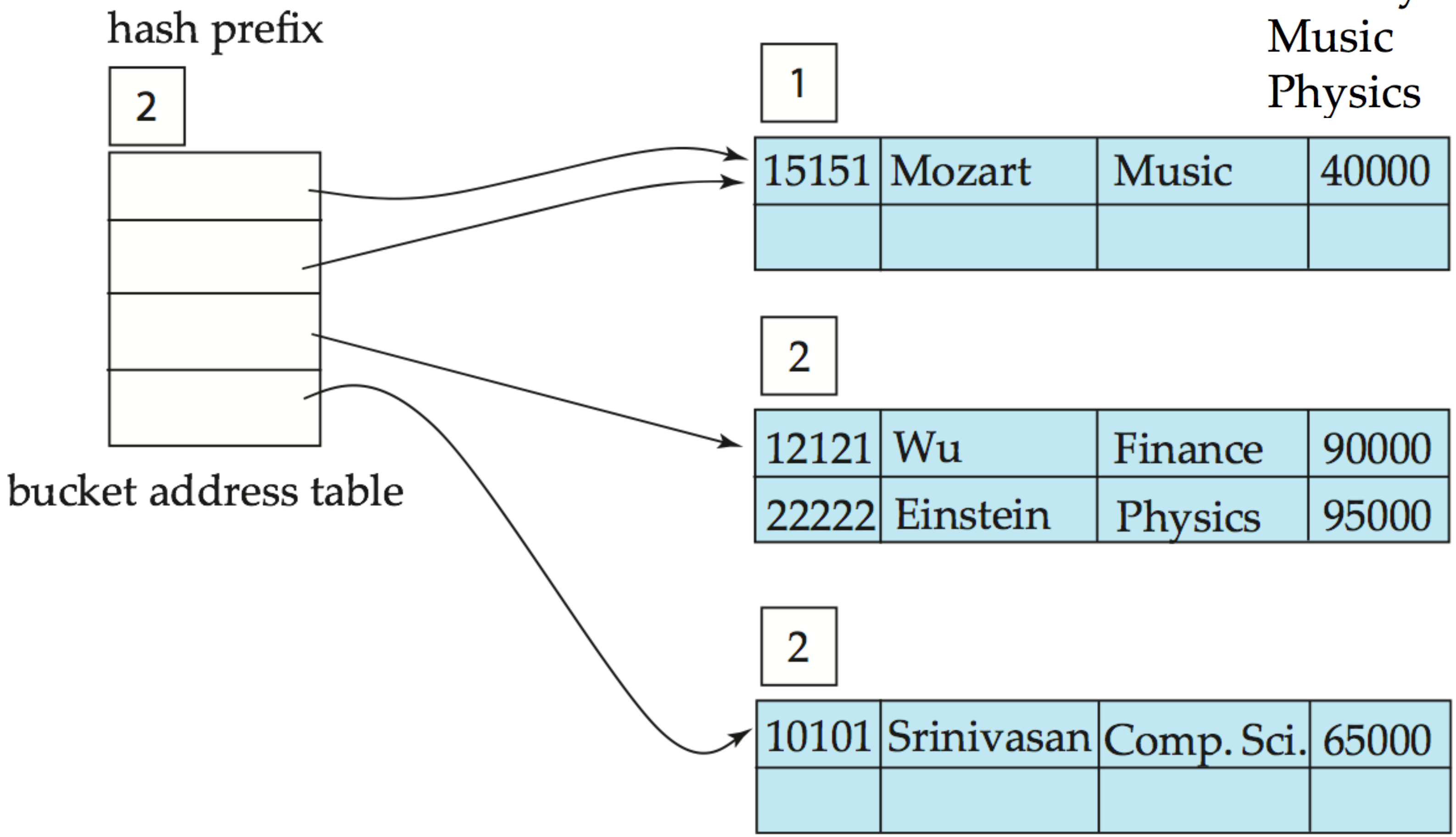
15151	Mozart	Music	40000

10101	Srinivasan	Comp. Sci.	90000
12121	Wu	Finance	90000

Example (Cont.)

- Hash structure after insertion of Einstein record

dept_name	
Biology	0010
Comp. Sci.	1111
Elec. Eng.	0100
Finance	1010
History	1100
Music	0011
Physics	1001



15151	Mozart	Music	40000

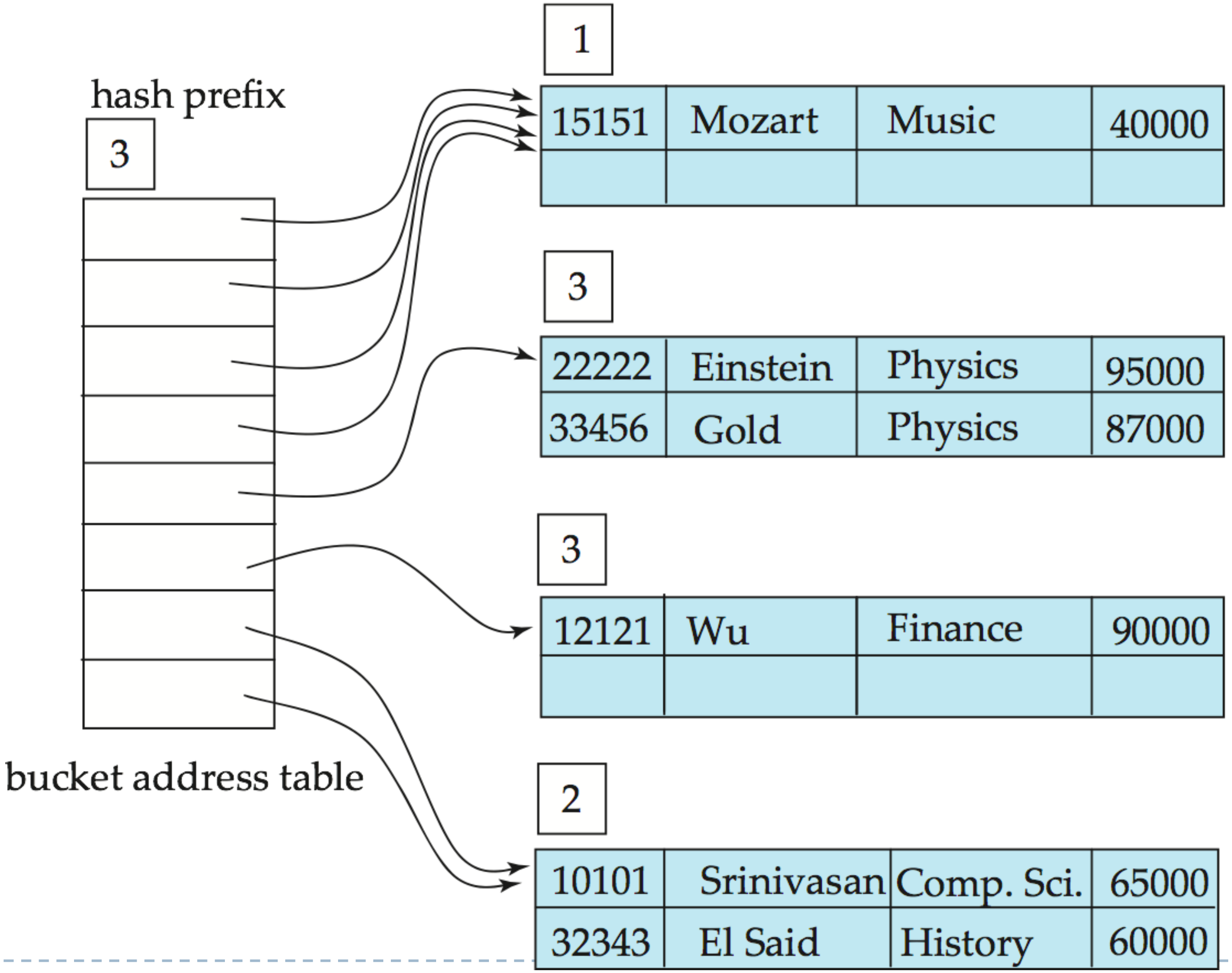
12121	Wu	Finance	90000
22222	Einstein	Physics	95000

10101	Srinivasan	Comp. Sci.	65000

Example (Cont.)

- Hash structure after insertion of Gold and El Said records

dept_name	
Biology	0010
Comp. Sci.	1111
Elec. Eng.	0100
Finance	1010
History	1100
Music	0011
Physics	1001



hash prefix

3

bucket address table

1

15151	Mozart	Music	40000

3

22222	Einstein	Physics	95000
33456	Gold	Physics	87000

3

12121	Wu	Finance	90000

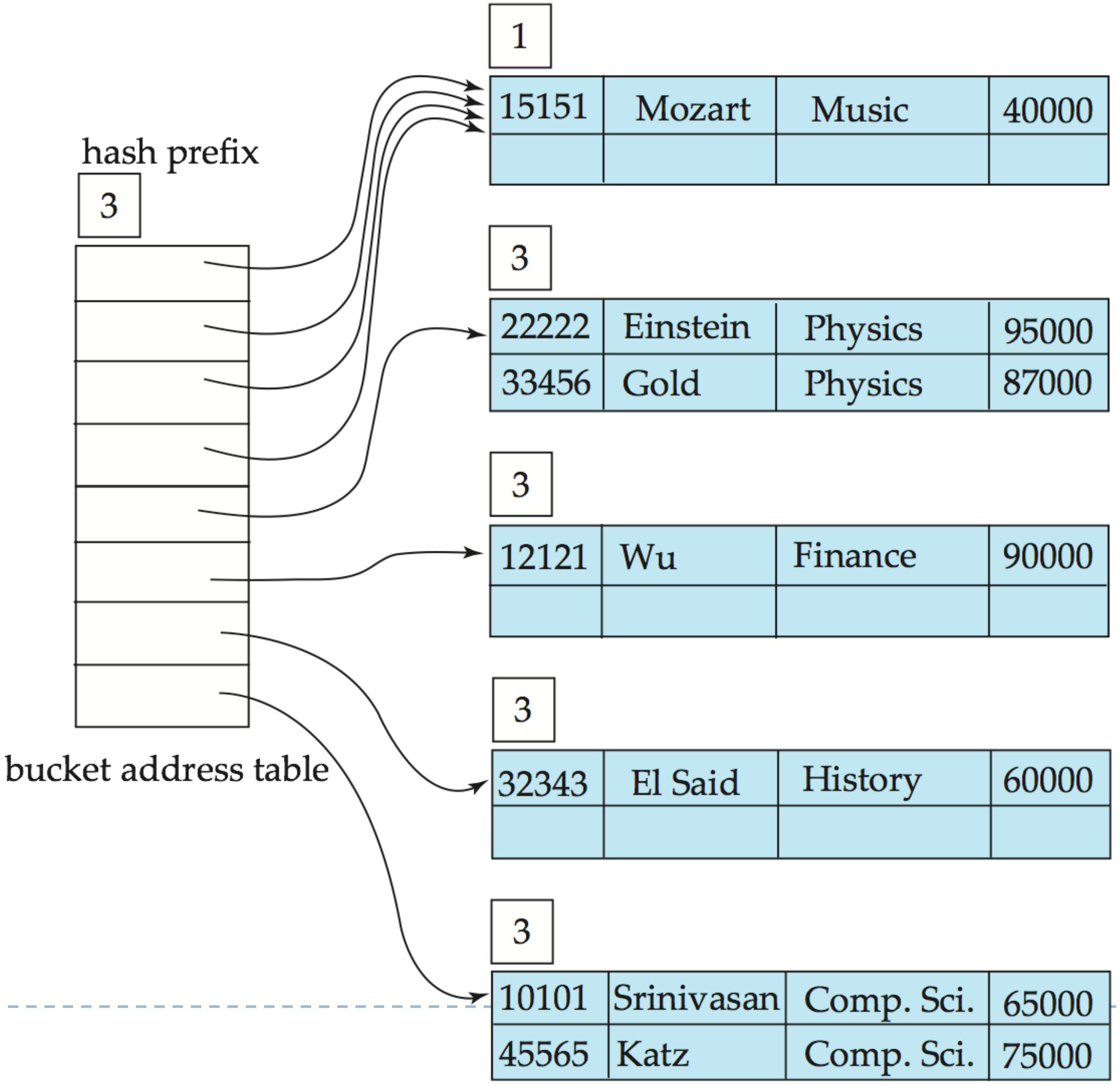
2

10101	Srinivasan	Comp. Sci.	65000
32343	El Said	History	60000

Example (Cont.)

dept_name	
Biology	0010
Comp. Sci.	1111
Elec. Eng.	0100
Finance	1010
History	1100
Music	0011
Physics	1001

- Hash structure after insertion of Katz record



15151	Mozart	Music	40000

22222	Einstein	Physics	95000
33456	Gold	Physics	87000

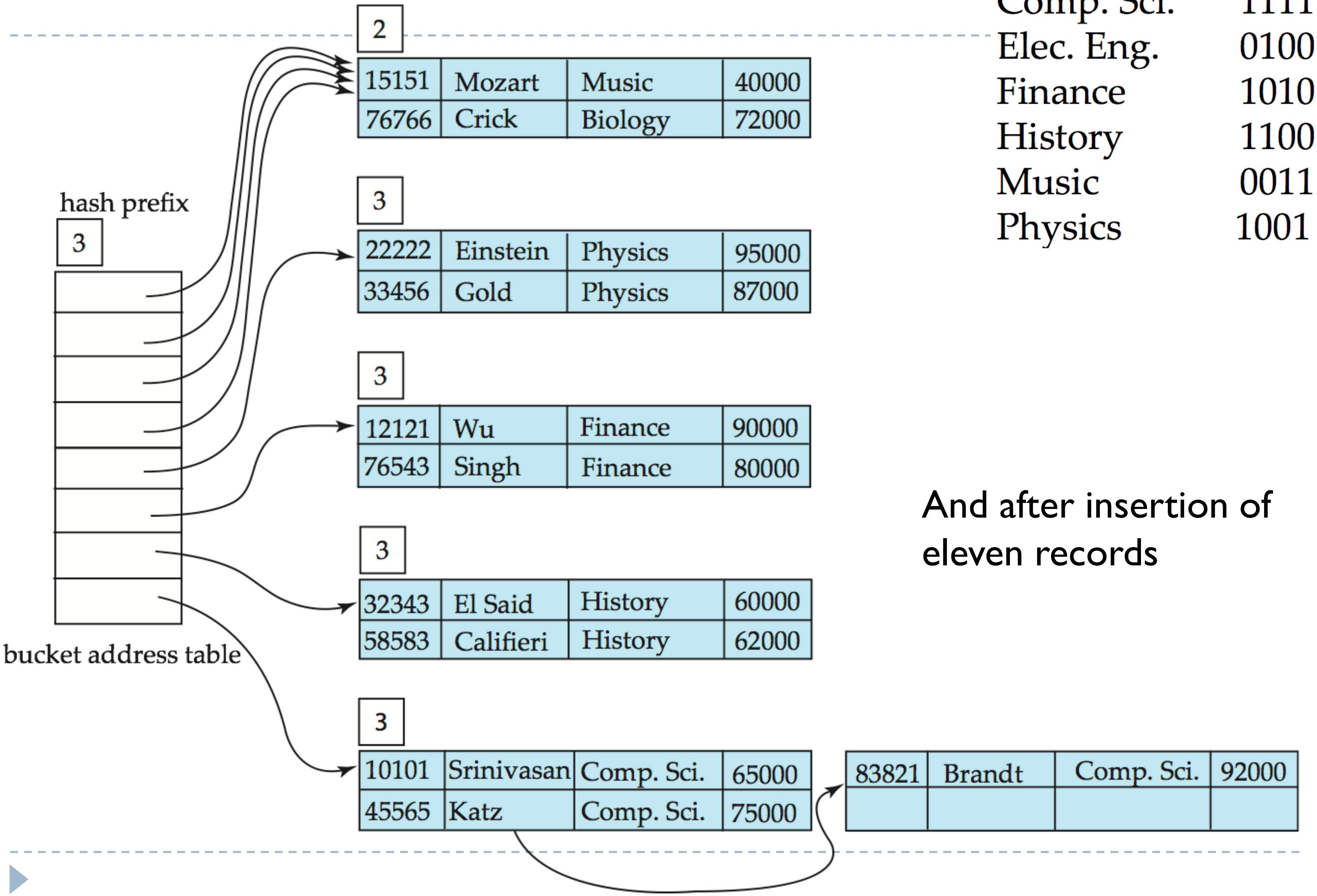
12121	Wu	Finance	90000

32343	El Said	History	60000

10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000

Example (Cont.)

dept_name	
Biology	0010
Comp. Sci.	1111
Elec. Eng.	0100
Finance	1010
History	1100
Music	0011
Physics	1001



15151	Mozart	Music	40000
76766	Crick	Biology	72000

22222	Einstein	Physics	95000
33456	Gold	Physics	87000

12121	Wu	Finance	90000
76543	Singh	Finance	80000

32343	El Said	History	60000
58583	Califieri	History	62000

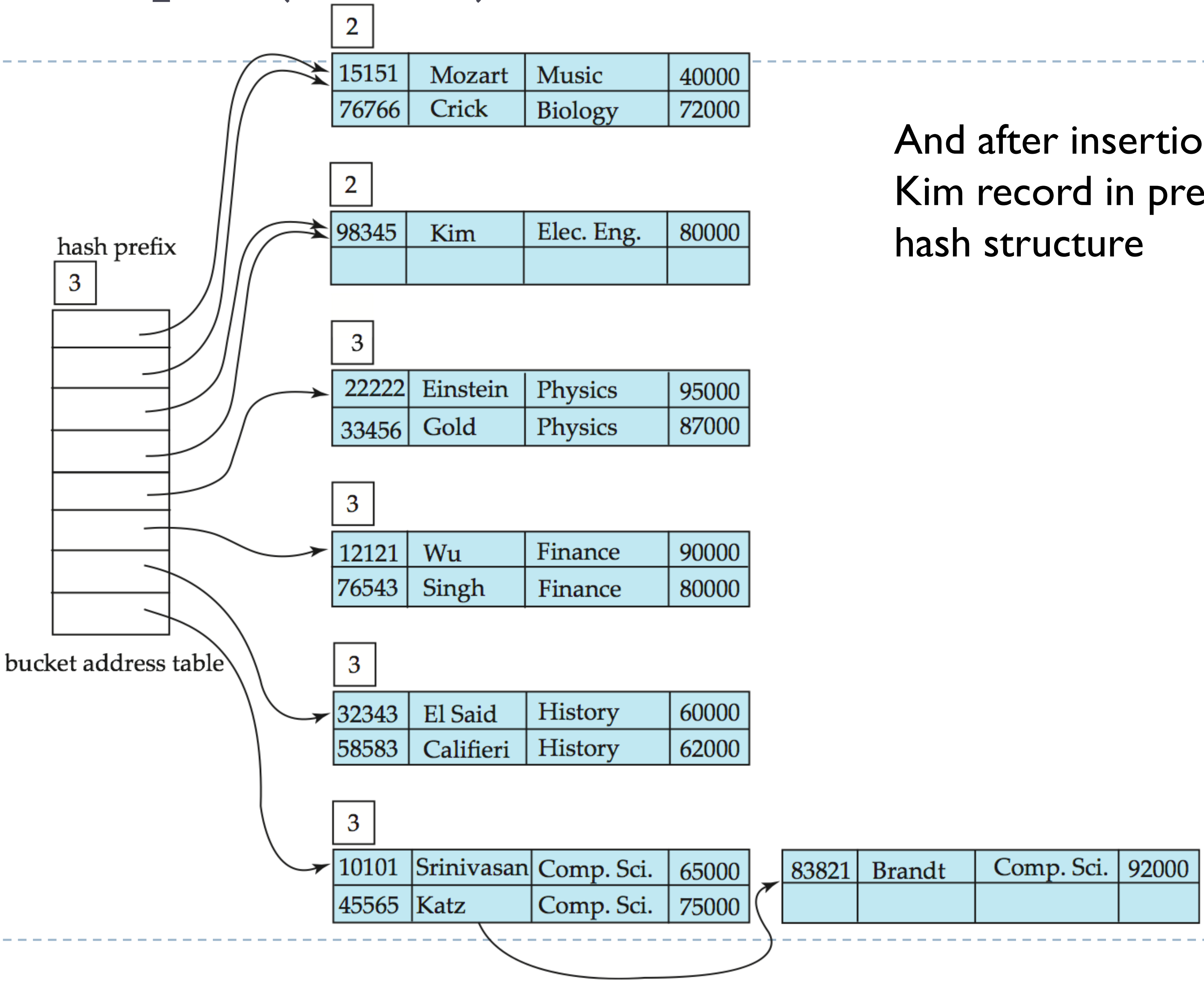
10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000

83821	Brandt	Comp. Sci.	92000

And after insertion of eleven records

Example (Cont.)

And after insertion of Kim record in previous hash structure



15151	Mozart	Music	40000
76766	Crick	Biology	72000

98345	Kim	Elec. Eng.	80000

22222	Einstein	Physics	95000
33456	Gold	Physics	87000

12121	Wu	Finance	90000
76543	Singh	Finance	80000

32343	El Said	History	60000
58583	Califieri	History	62000

10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000

83821	Brandt	Comp. Sci.	92000

Extendable Hashing vs. Other Schemes

- Benefits of extendable hashing:
  - Hash performance does not degrade with growth of file
  - Minimal space overhead
- Disadvantages of extendable hashing
  - Extra level of indirection to find desired record
  - Bucket address table may itself become very big (larger than memory)
    - Cannot allocate very large contiguous areas on disk either
    - Solution: B$^+$-tree structure to locate desired record in bucket address table
  - Changing size of bucket address table is an expensive operation
- Linear hashing is an alternative mechanism
  - Allows incremental growth of its directory (equivalent to bucket address table)
  - At the cost of more bucket overflows

Comparison of Ordered Indexing and Hashing

- Cost of periodic re-organization
- Relative frequency of insertions and deletions
- Is it desirable to optimize average access time at the expense of worst-case access time?
- Expected type of queries:
  - Hashing is generally better at retrieving records having a specified value of the key.
  - If range queries are common, ordered indices are to be preferred
- In practice:
  - PostgreSQL supports hash indices, but discourages use due to poor performance
  - Oracle supports static hash organization, but not hash indices
  - SQLServer supports only B^+-trees

Index Definition in SQL

- Create an index

  ```
  create index <index-name> on <relation-name>
    (<attribute-list>)
  ```

  E.g.: **create index** *b-index* **on** *branch(branch_name)*

- Use **create unique index** to indirectly specify and enforce the condition that the search key is a candidate key.
  - Not really required if SQL **unique** integrity constraint is supported
- To drop an index

  ```
              drop index <index-name>
  ```

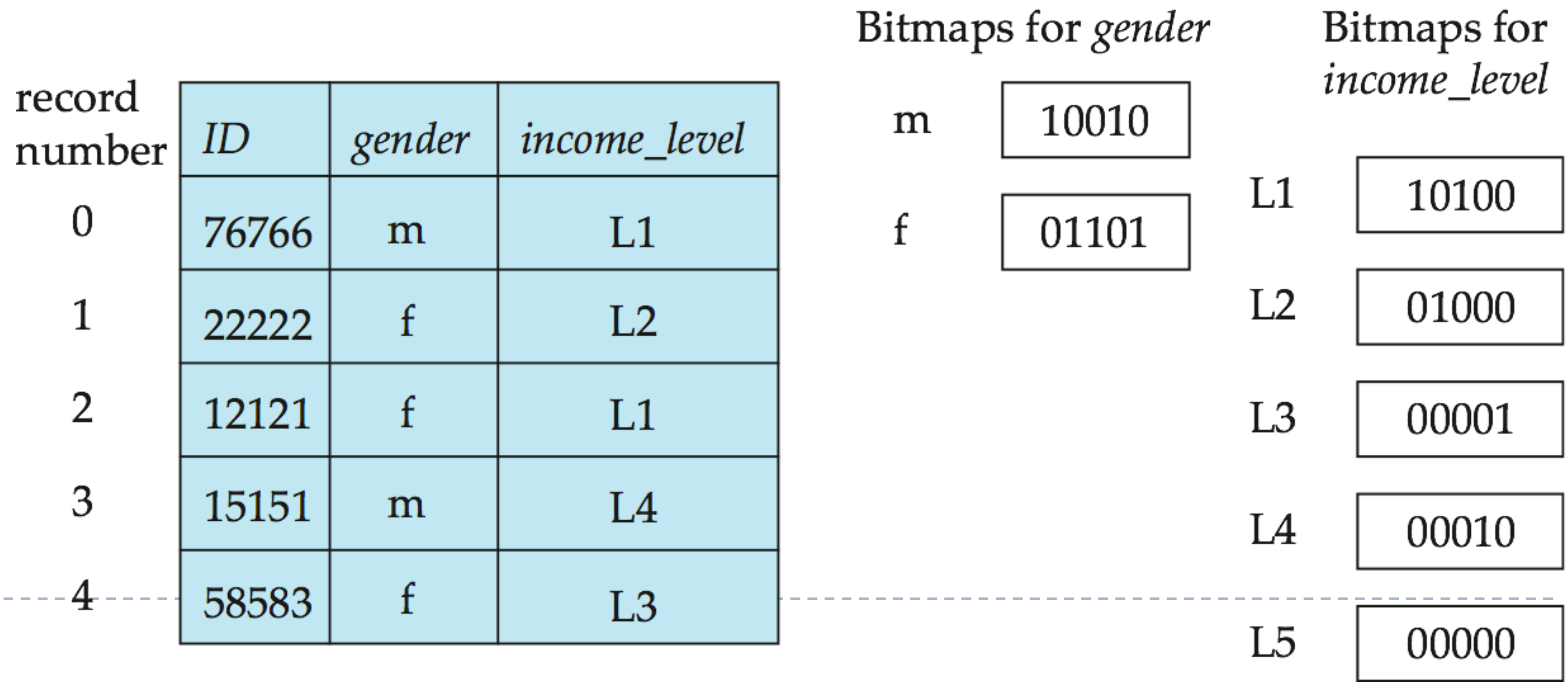
- Most database systems allow specification of type of index, and clustering.

Bitmap Indices

- Bitmap indices are a special type of index designed for efficient querying on multiple keys
- Records in a relation are assumed to be numbered sequentially from, say, 0
  - Given a number *n* it must be easy to retrieve record *n*
    - Particularly easy if records are of fixed size
- Applicable on attributes that take on a relatively small number of distinct values
  - E.g. gender, country, state, …
  - E.g. income-level (income broken up into a small number of levels such as 0-9999, 10000-19999, 20000-50000, 50000- infinity)
- A bitmap is simply an array of bits

Bitmap Indices (Cont.)

- In its simplest form a bitmap index on an attribute has a bitmap for each value of the attribute
  - Bitmap has as many bits as records
  - In a bitmap for value v, the bit for a record is 1 if the record has the value v for the attribute, and is 0 otherwise



record number	*ID*	*gender*	*income_level*
0	76766	m	L1
1	22222	f	L2
2	12121	f	L1
3	15151	m	L4
4	58583	f	L3

Bitmaps for *gender*

m	10010
f	01101

Bitmaps for *income_level*

L1	10100
L2	01000
L3	00001
L4	00010
L5	00000

Bitmap Indices (Cont.)

- Bitmap indices are useful for queries on multiple attributes
  - not particularly useful for single attribute queries
- Queries are answered using bitmap operations
  - Intersection (and)
  - Union (or)
  - Complementation (not)

Bitmap Indices (Cont.)

- Each operation takes two bitmaps of the same size and applies the operation on corresponding bits to get the result bitmap
  - E.g. 100110 AND 110011 = 100010

    100110 OR 110011 = 110111

    NOT 100110 = 011001
  - Males with income level L1: 10010 AND 10100 = 10000
    - Can then retrieve required tuples.
    - Counting number of matching tuples is even faster

Bitmap Indices (Cont.)

- Bitmap indices generally very small compared with relation size
  - E.g. if record is 100 bytes, space for a single bitmap is 1/800 of space used by relation.
    - If number of distinct attribute values is 8, bitmap is only 1% of relation size
- Deletion needs to be handled properly
  - **Existence bitmap** to note if there is a valid record at a record location
  - Needed for complementation
    - not(*A*=*v*): *(NOT bitmap-A-v) AND ExistenceBitmap*
- Should keep bitmaps for all values, even null value
  - To correctly handle SQL null semantics for NOT(*A*=*v*):
    - intersect above result with (NOT *bitmap-A-Null*)