Dive into Deep Learning

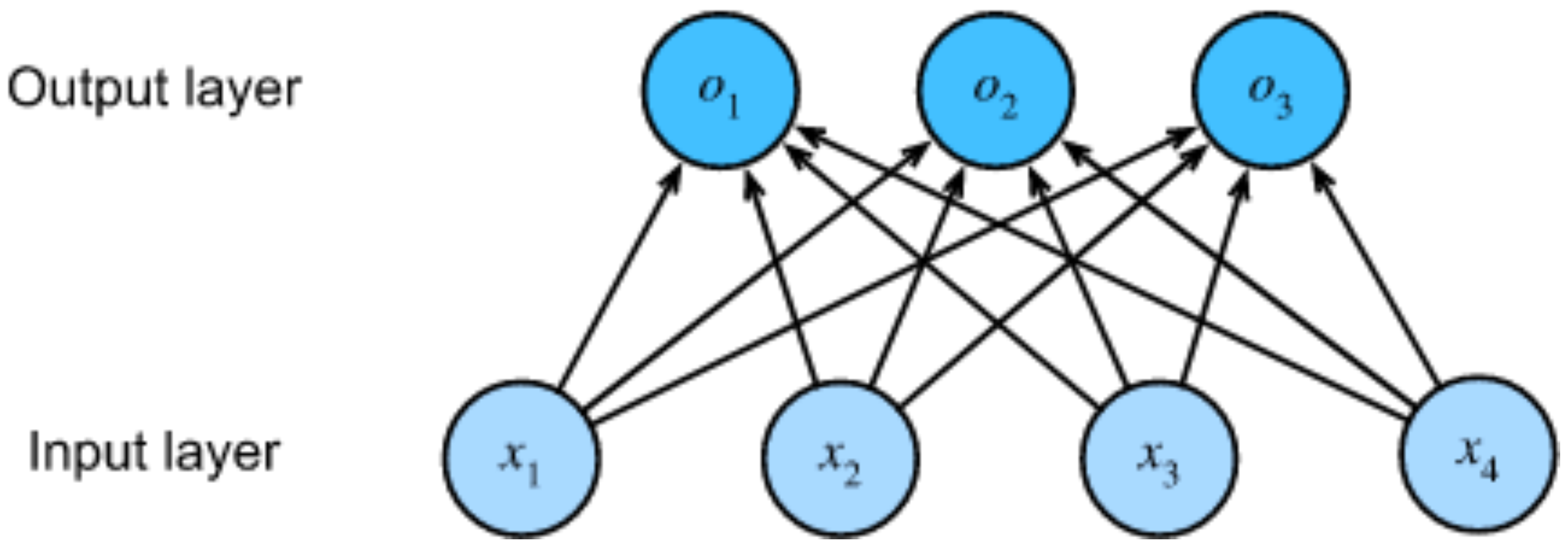
Chapter 4. Multilayer Perceptrons

Wayne L, Oct 18, 2020

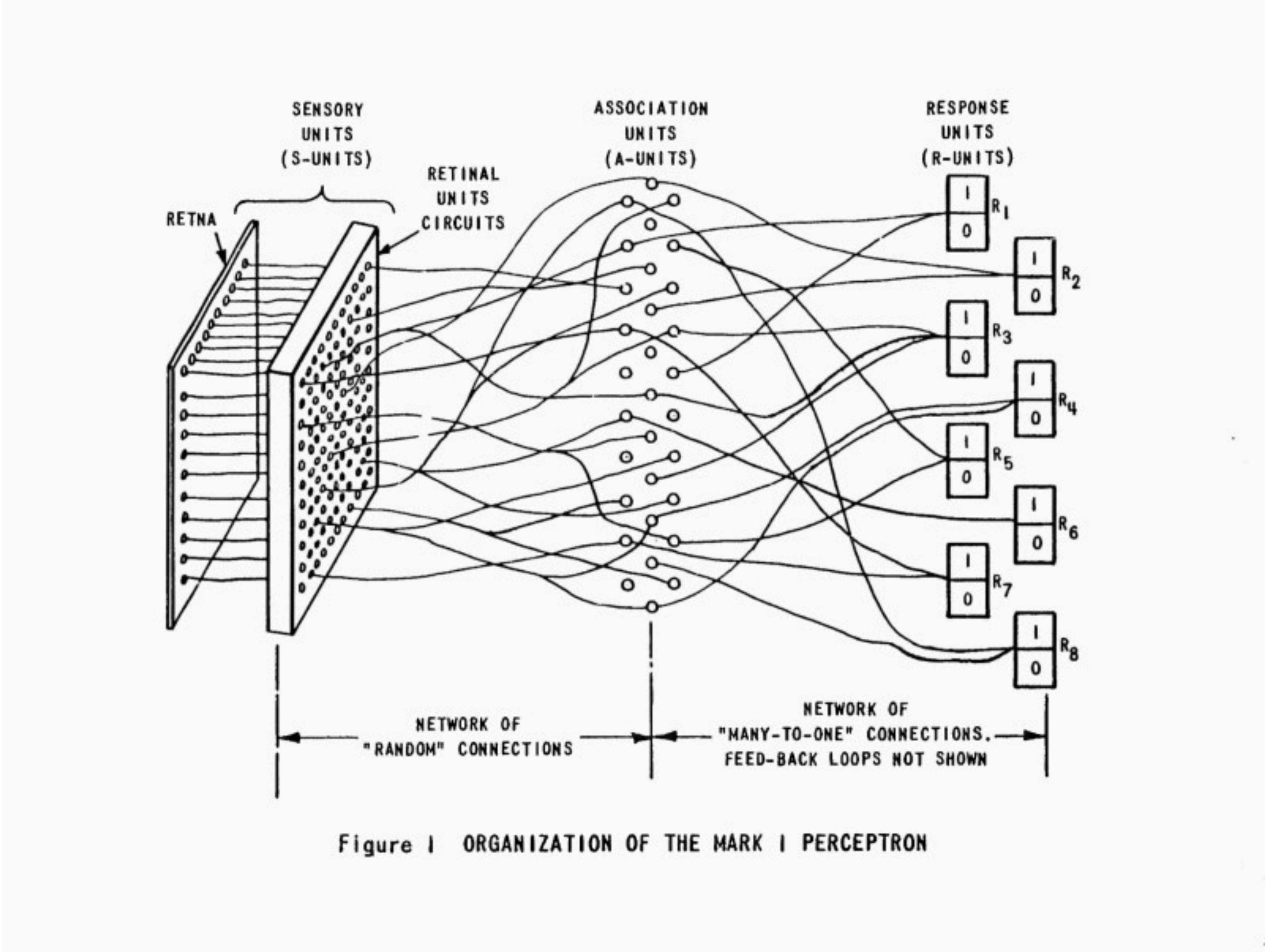
Previously

Perceptron (Linear = Single = Simple = Feedforward)

- This model mapped our inputs directly to our outputs via single affine transformation, followed by a softmax operation.



$$y = \mathbf{wx} + b$$

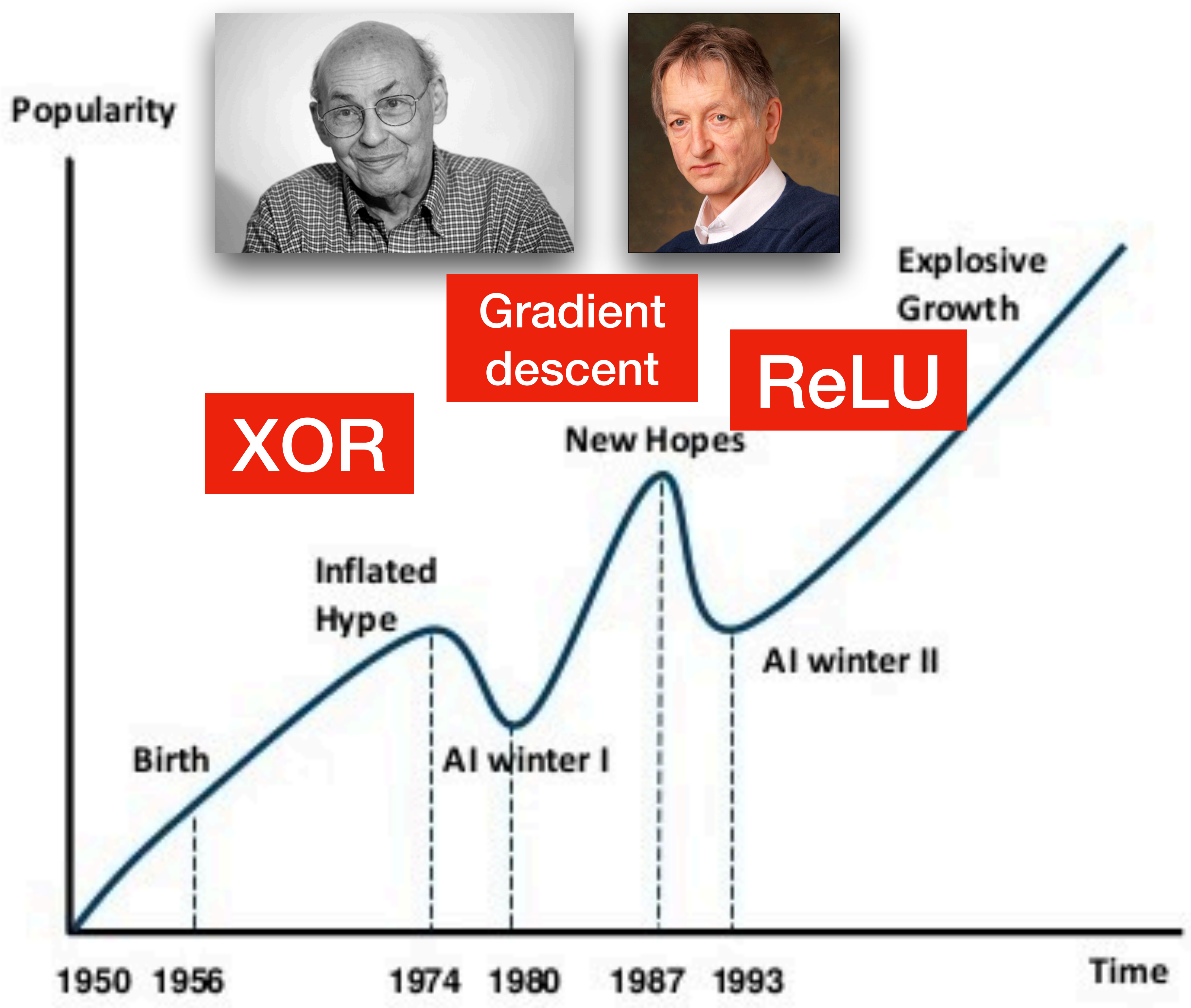


Mark 1 Perceptron (1959), Frank Rosenblatt

Previously

AI History

AI HAS A LONG HISTORY OF BEING "THE NEXT BIG THING"...

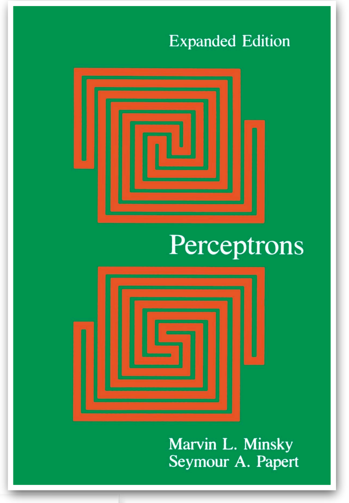


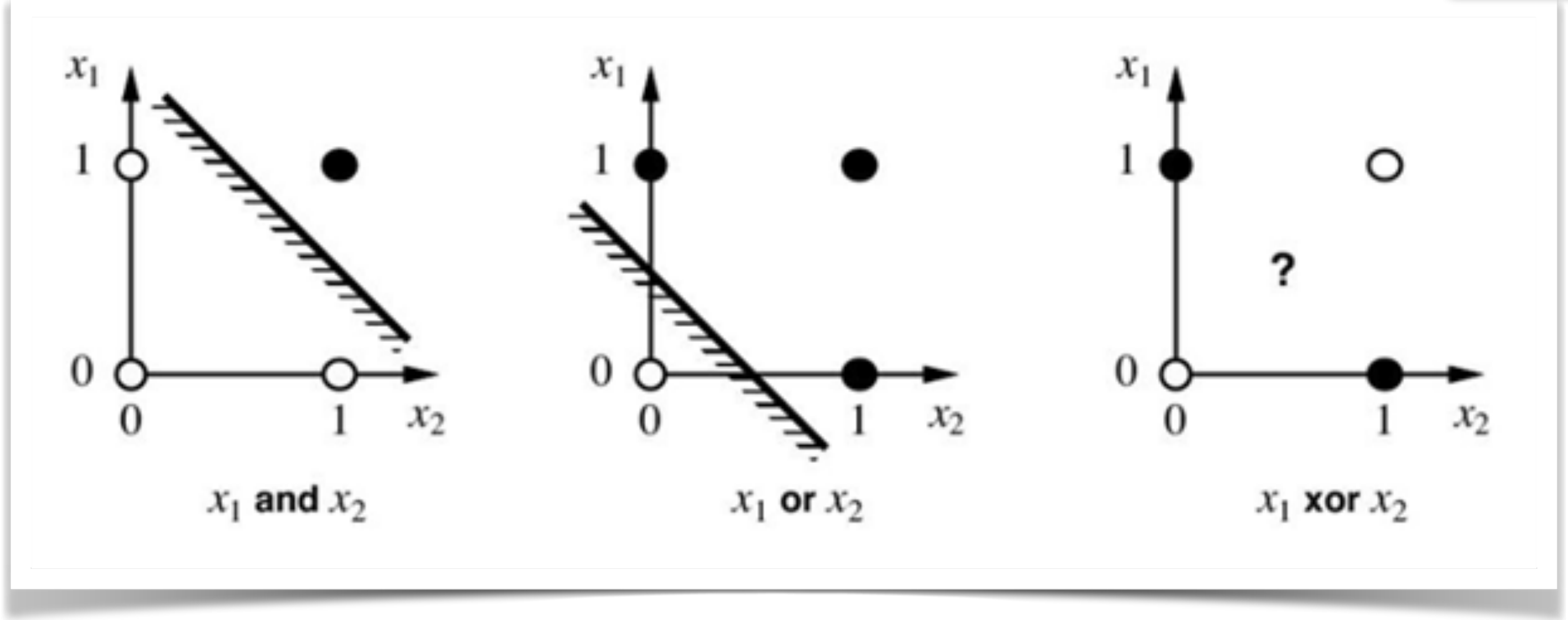
Timeline of AI Development

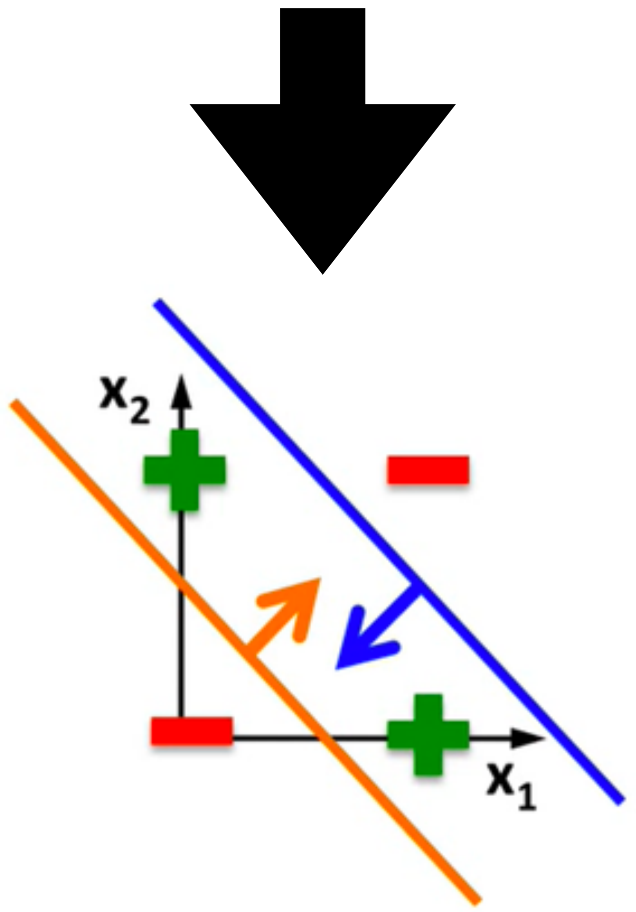
- **1950s-1960s**: First AI boom - the age of reasoning, prototype AI developed
- **1970s**: AI winter I
- **1980s-1990s**: Second AI boom: the age of Knowledge representation (appearance of expert systems capable of reproducing human decision-making)
- **1990s**: AI winter II
- **1997**: Deep Blue beats Gary Kasparov
- **2006**: University of Toronto develops Deep Learning
- **2011**: IBM's Watson won Jeopardy
- **2016**: Go software based on Deep Learning beats world's champions

XOR

Perceptrons (1969), Marvin minsky







No one on earth had found a viable way to train

Multilayer Perceptrons

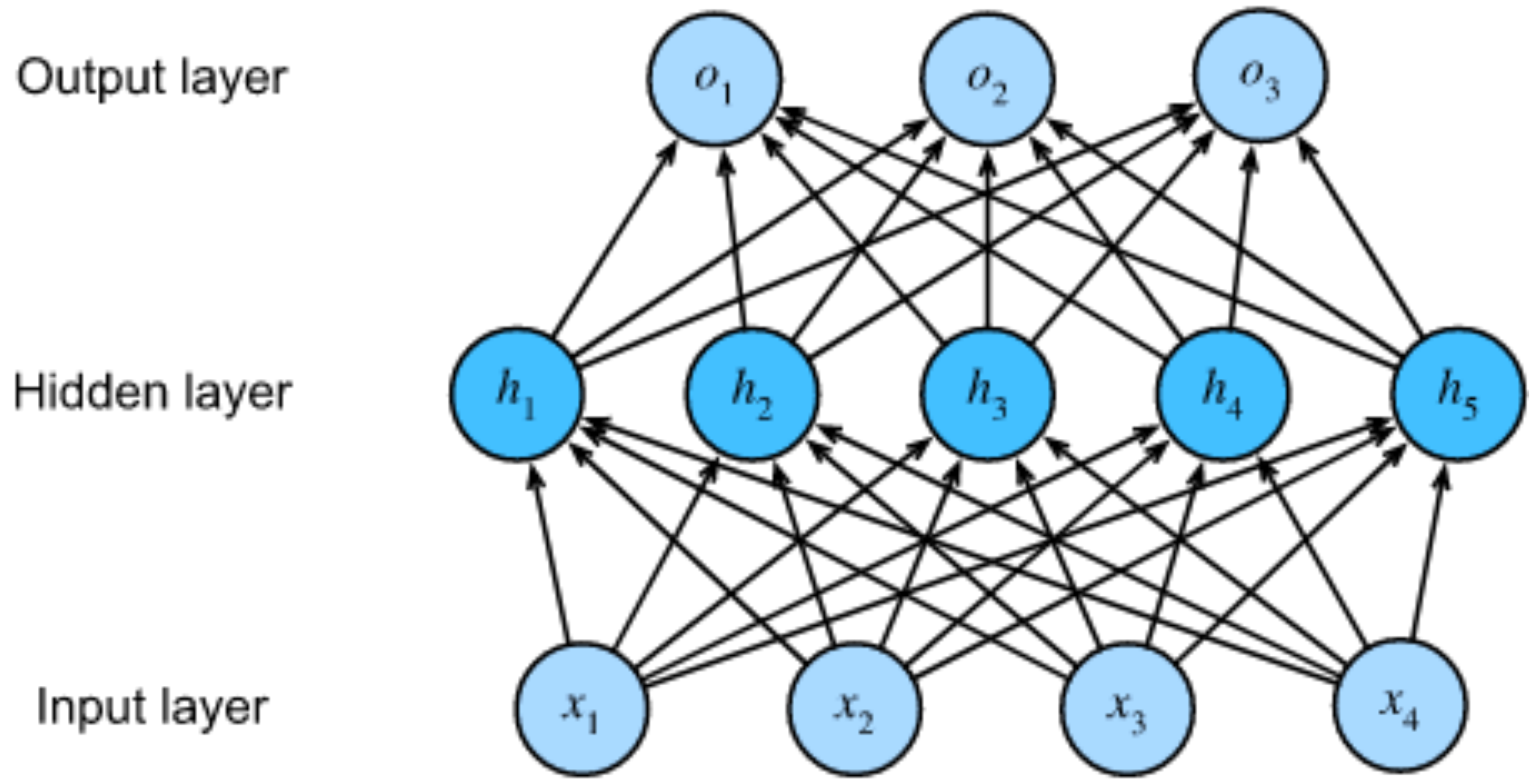
Perceptrons (Multi = Deep feedforward)

- The goal of a feedforward network is to approximate some function f^*.

- For example, for a classifier, $y = f^*(x)$ maps an input x to a category y.
  A feedforward network defines a mapping $\mathbf{y} = f(x; \theta)$ and learns the value of the parameters θ that result in the best function approximation.

- ***Universal approximation theorem*** means that regardless of what function we are trying to learn, we know that a large MLP will be able to represent this function.

$$f(x) = f^*(x) \approx y$$



$$\mathbf{O} = \mathbf{H}\mathbf{W}^{(2)} + \mathbf{b}^{(2)}$$

$$\mathbf{H} = \mathbf{X}\mathbf{W}^{(1)} + \mathbf{b}^{(1)}$$

$$\mathbf{W} = \mathbf{W}^{(1)}\mathbf{W}^{(2)}$$
$$\mathbf{b} = \mathbf{b}^{(1)}\mathbf{W}^{(2)} + \mathbf{b}^{(2)}$$

$$\mathbf{O} = (\mathbf{X}\mathbf{W}^{(1)} + \mathbf{b}^{(1)})\mathbf{W}^{(2)} + \mathbf{b}^{(2)} = \mathbf{X}\mathbf{W}^{(1)}\mathbf{W}^{(2)} + \mathbf{b}^{(1)}\mathbf{W}^{(2)} + \mathbf{b}^{(2)} = \mathbf{X}\mathbf{W} + \mathbf{b}$$

Even though MLP is going deeper, it can be equivalent with single-layer model !

Multilayer Perceptrons

From Linear to Nonlinear

- In order to realize the potential of multilayer architectures, we need one more key ingredient: a nonlinear *activation function* σ to be more ***expressive***.
- MLPs are ***universal approximators***, however, it doe not mean that we can solve all of problems with MLPs. In fact, we can approximate many functions much more compactly by using deeper (or wider) networks.
- Each neuron acts as a ***linear SVM***, however, …
  - its output is not interpreted immediately,
  - but it becomes a new feature,
  - to be forwarded to the next layer for further analysis **#SVMcascade**

$$\mathbf{H}^{(1)} = \sigma_1(\mathbf{X}\mathbf{W}^{(1)} + \mathbf{b}^{(1)})$$

$$\mathbf{H}^{(2)} = \sigma_2(\mathbf{H}^{(1)}\mathbf{W}^{(2)} + \mathbf{b}^{(2)})$$

Multilayer Perceptrons

Activation function

- ***Activation function*** decide whether a neuron should be activated or not by calculating the weighted sum and further adding bias with it.
- They are ***differentiable*** operators to transform input signal to outputs, while most of them add non-linearity.

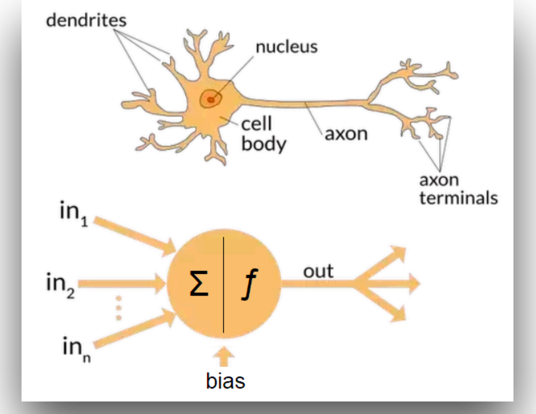
`nn.ELU`	Applies the element-wise function:
`nn.Hardshrink`	Applies the hard shrinkage function element-wise:
`nn.Hardsigmoid`	Applies the element-wise function:
`nn.Hardtanh`	Applies the HardTanh function element-wise
`nn.Hardswish`	Applies the hardswish function, element-wise, as described in the paper:
`nn.LeakyReLU`	Applies the element-wise function:
`nn.LogSigmoid`	Applies the element-wise function:
`nn.MultiheadAttention`	Allows the model to jointly attend to information from different representation subspaces.
`nn.PReLU`	Applies the element-wise function:
`nn.ReLU`	Applies the rectified linear unit function element-wise:
`nn.ReLU6`	Applies the element-wise function:
`nn.RReLU`	Applies the randomized leaky rectified liner unit function, element-wise, as described in the paper:
`nn.SELU`	Applied element-wise, as:
`nn.CELU`	Applies the element-wise function:
`nn.GELU`	Applies the Gaussian Error Linear Units function:
`nn.Sigmoid`	Applies the element-wise function:
`nn.Softplus`	Applies the element-wise function:
`nn.Softshrink`	Applies the soft shrinkage function elementwise:
`nn.Softsign`	Applies the element-wise function:
`nn.Tanh`	Applies the element-wise function:
`nn.Tanhshrink`	Applies the element-wise function:
`nn.Threshold`	Thresholds each element of the input Tensor.

https://pytorch.org/docs/stable/nn.html#non-linear-activations-weighted-sum-nonlinearity

Multilayer Perceptrons

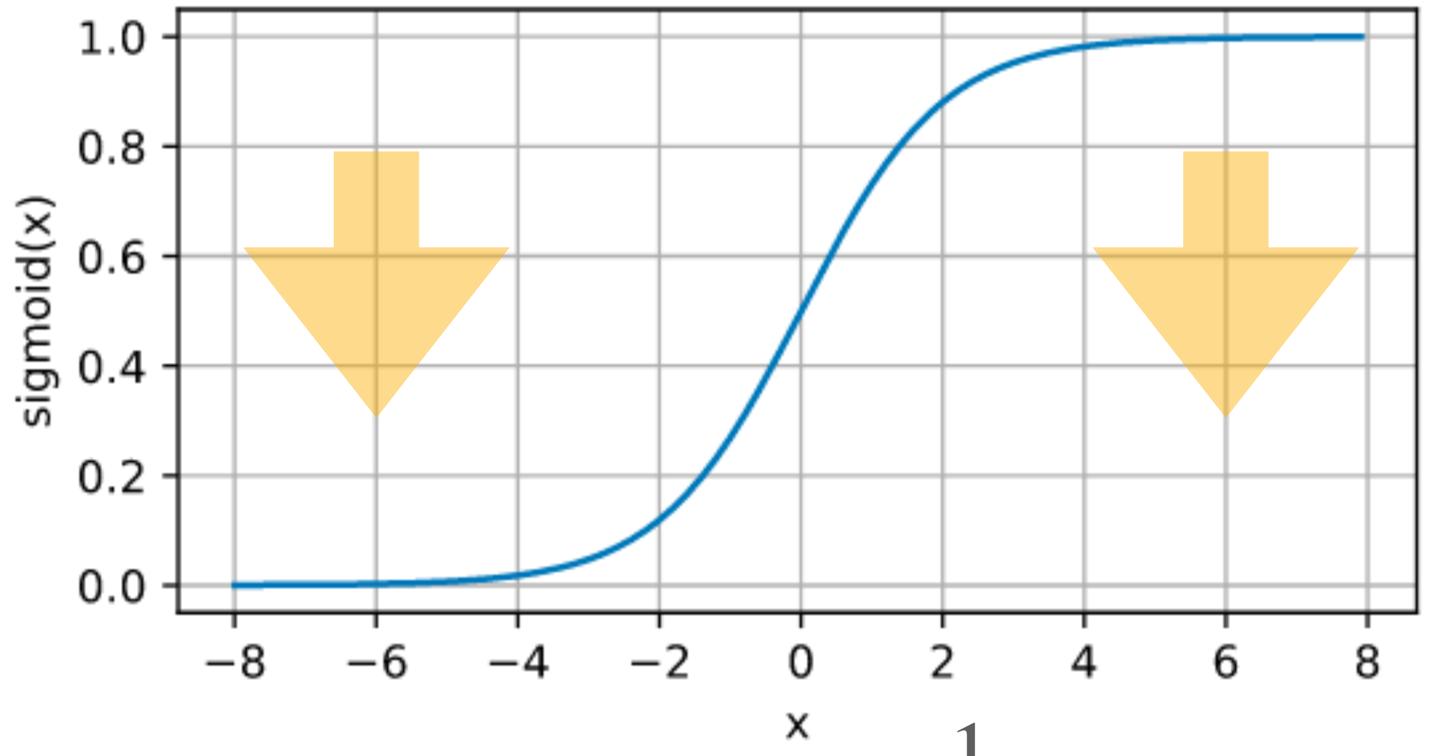
Sigmoid

- In the earliest neural networks, scientists were interested in modeling biological neurons which either fire or do not fire.

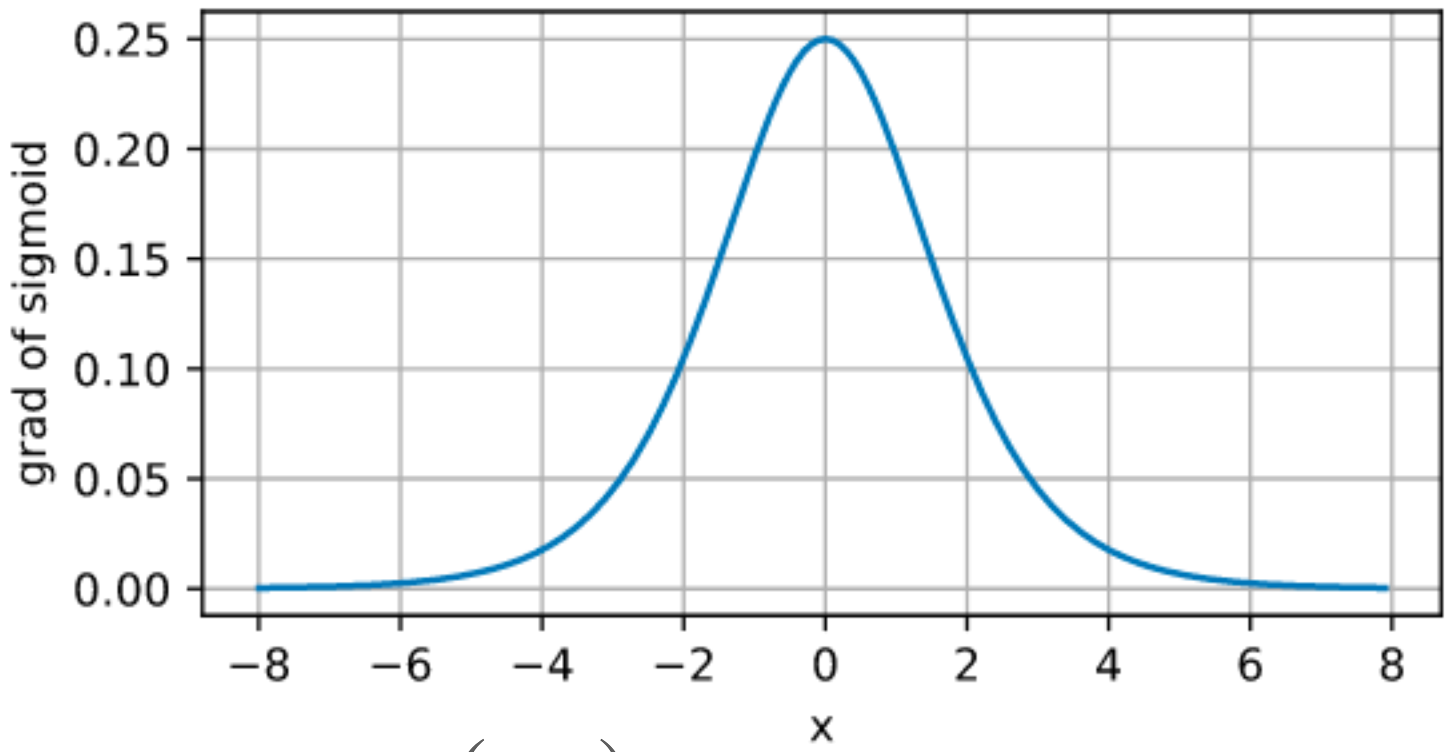


- When attention shifted to gradient based learning, the sigmoid funciton was a natural choice because it is a smooth, differentiable approximation to a thresholding unit.

- widely used as activation functions on the output units, when we want to interpret the outputs as probabilities for binary classification problems (special case of the softmax).



$$sigmoid(x) = \frac{1}{1 + exp(-x)}$$



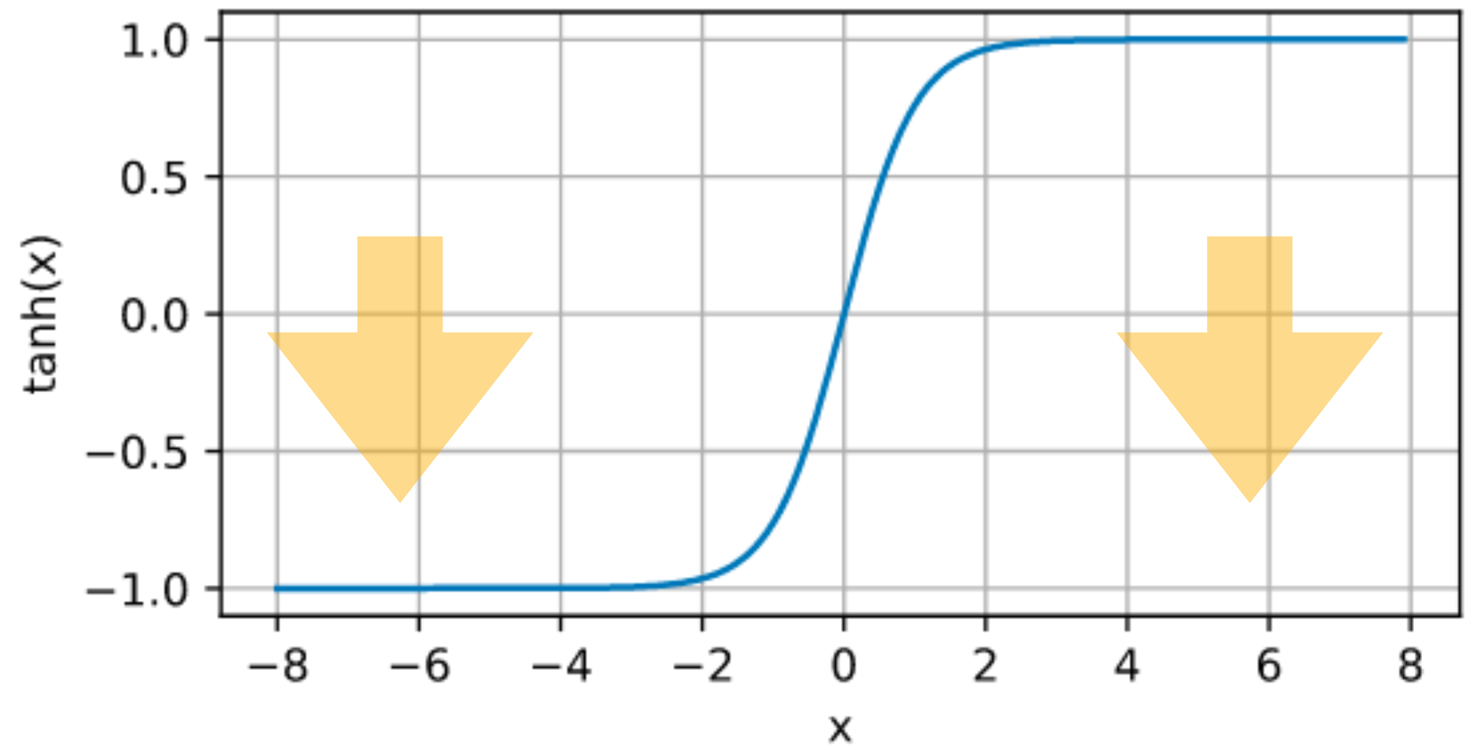
vanishing gradient

$$\frac{d}{dx} sigmoid(x) = \frac{exp(-x)}{(1 + exp(-x))^2} = sigmoid(x)(1 - sigmoid(x))$$

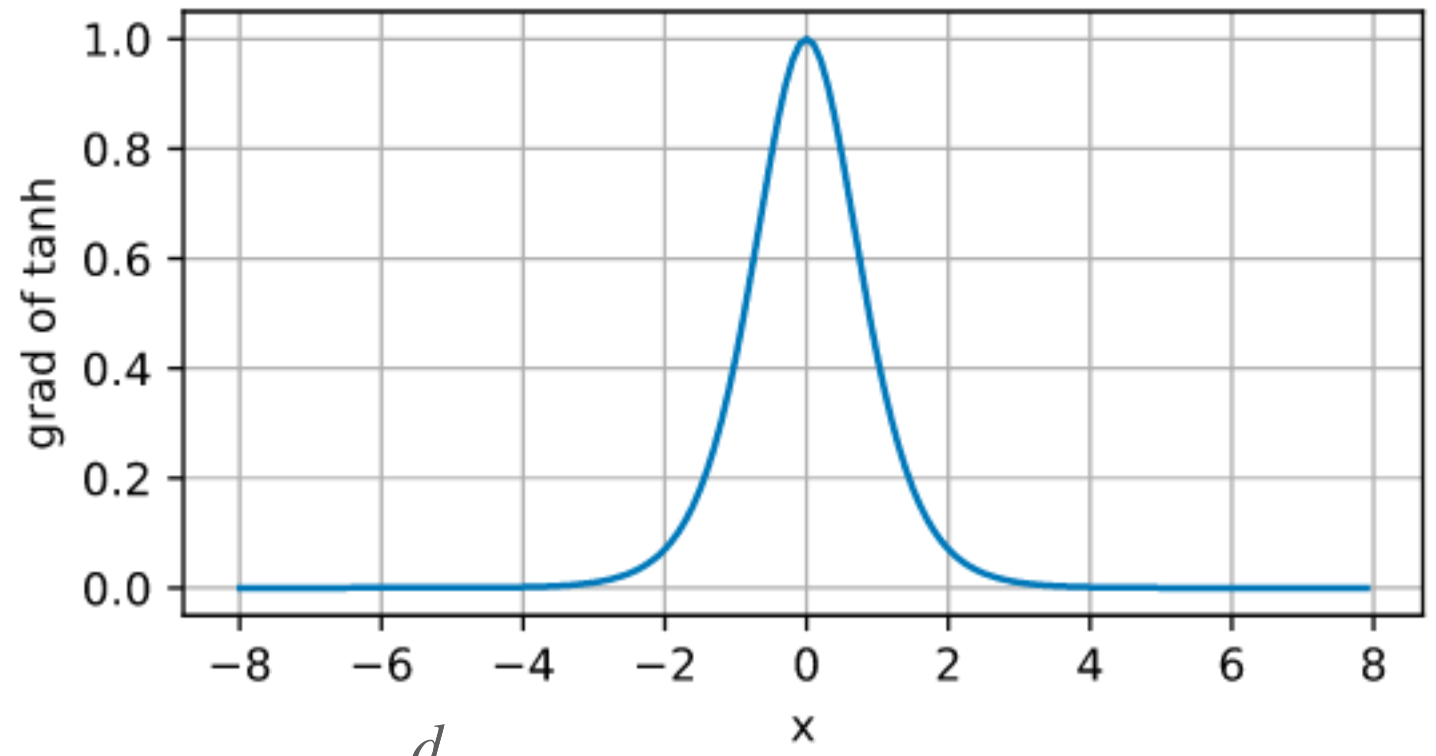
Multilayer Perceptrons

Tanh (hyperbolic tangent)

- Like the sigmoid function, the tanh function also squashes its inputs, transforming them into elements on the interval between -1 and 1.
- Although the shape of the function is similar to that of the sigmoid function, the tanh function exhibits point symmetry about the origin of the coordinate system.



$$tanh(x) = \frac{1 - exp(-2x)}{1 + exp(-2x)}$$



$$\frac{d}{dx} tanh(x) = 1 - tanh^2(x)$$

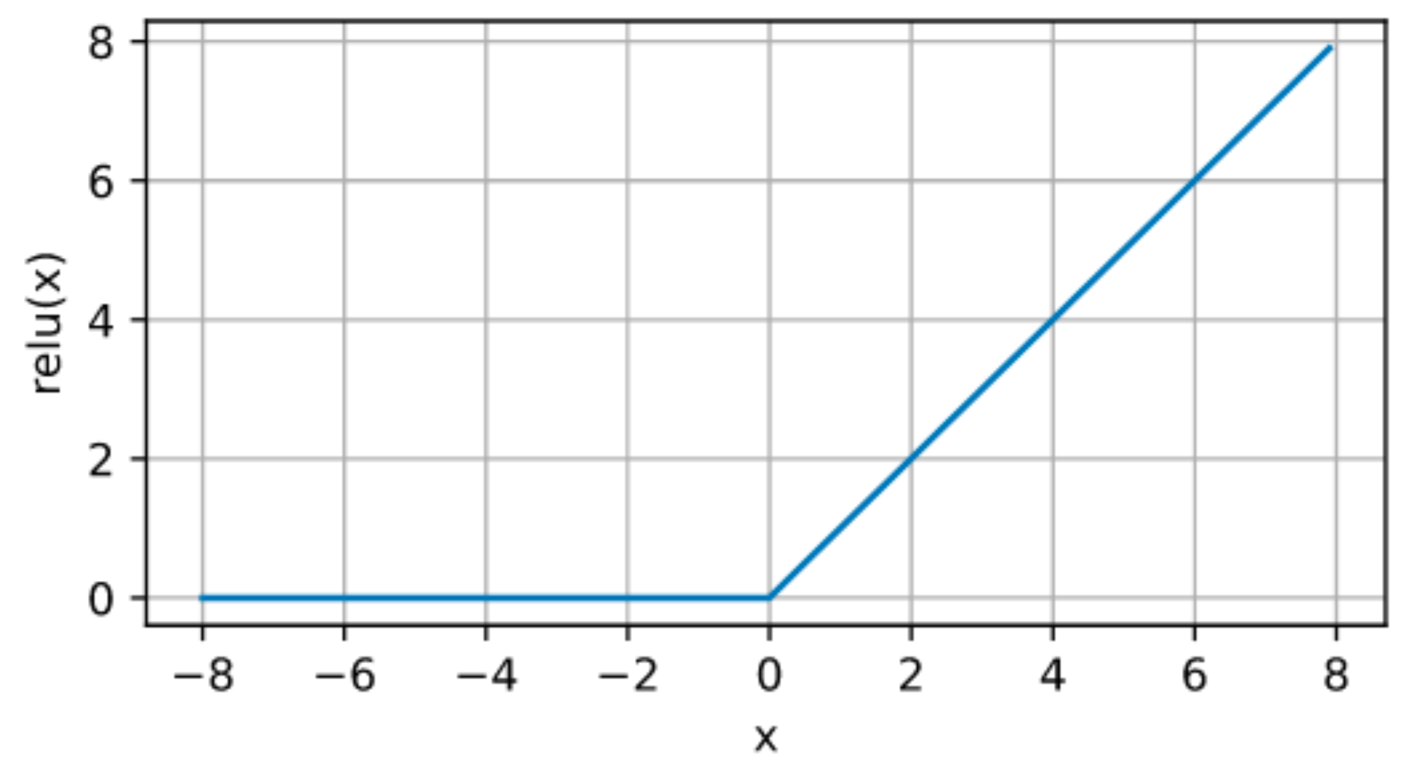
vanishing gradient

Multilayer Perceptrons

ReLU (rectified linear unit)

- The reason for using ReLU is that its derivates are particularly well behaved: either they vanish or they just let the argument through.
- This makes optimization better behaved and it mitigated the well-documented problem of vanishing gradients that plagued previous versions of neural networks.

$$pReLU(x) = max(0,x) + \alpha min(0,x)$$



$$ReLU(x) = max(x,0)$$