№ 844*

$$\left(\frac{ax+b}{cx+d}\right)' = \frac{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}{(cx+d)^2}$$

$$\left(\frac{ax+b}{cx+d}\right)' = \frac{(ax+b)'(cx+d)-(ax+b)(cx+d)'}{(cx+d)^2} =$$

$$= \frac{a(cx+d)-c(ax+b)}{(cx+d)^2} = \frac{ad-cb}{(cx+d)^2}$$

$$\frac{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}{(cx+d)^2} = \frac{ad-cb}{(cx+d)^2}$$

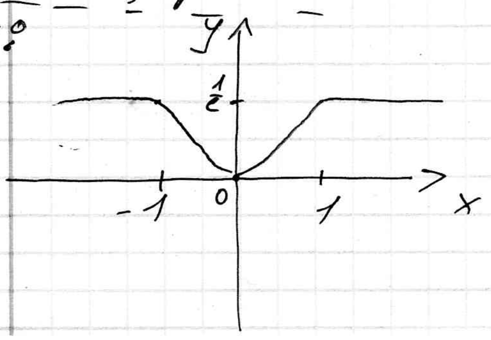
№ 983*

$$y = \begin{cases} x^2e^{-x^2}, & |x| \le 1 \\ \frac{1}{e}, & |x| > 1 \end{cases}$$

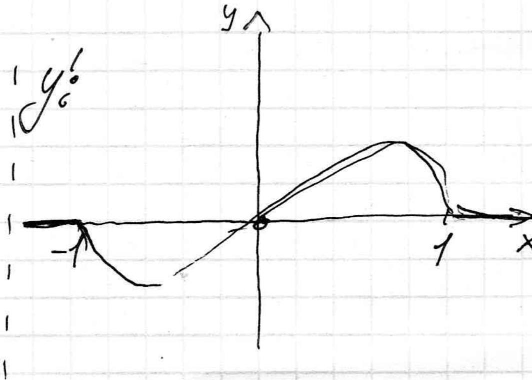
$$y' = 2xe^{-x^2}(1-x^2), \quad |x| \le 1$$

$y' = 0$, при $|x| > 1$

y:



y':



844. Доказать формулу $\left(\frac{ax+b}{cx+d}\right)' = \frac{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}{(cx+d)^2}$.

983. $y = \begin{cases} x^2e^{-x^2} & \text{при } |x| \le 1; \\ \frac{1}{e} & \text{при } |x| > 1. \end{cases}$

№ 839

$$y=(x+1)(x+2)^2(x+3)^3 = u'vw+uv'w+uvw'$$

$$y'=2(x+2)\cdot 3(x+3)^2(3x^2+11x+9)$$

№ 840

$$y=(x\sin\alpha+\cos\alpha)(x\cos\alpha-\sin\alpha)=$$

$$y'=\sin\alpha(x\cos\alpha-\sin\alpha)+\cos\alpha(x\sin\alpha+\cos\alpha)$$

$$=\cancel{x\cos\alpha-\sin^2\alpha+x\sin\alpha+\cos^2\alpha}$$

$$=\cancel{\sin^2\alpha}+x\sin\alpha\cos\alpha-\sin^2\alpha+x\sin\alpha\cos\alpha$$

$$+\cos^2\alpha=2x\sin\alpha\cos\alpha-\sin^2\alpha+\cos^2\alpha$$

$$=x\sin2\alpha+\cos2\alpha$$

№ 843

$$y=\frac{1}{x}+\frac{2}{x^2}+\frac{3}{x^3}=u^4+v^3+w^3$$

$$y'=\frac{-1}{x^2}+\frac{-4x}{x^4}+\frac{-9x^2}{x^6}=$$

$$=-\left(\frac{1}{x^2}+\frac{4}{x^3}+\frac{9}{x^4}\right)$$

839. $y=(x+1)(x+2)^2(x+3)^3$.

840. $y=(x\sin\alpha+\cos\alpha)(x\cos\alpha-\sin\alpha)$.

843. $y=\frac{1}{x}+\frac{2}{x^2}+\frac{3}{x^3}$.

№ 848

$$y=\frac{(2-x^2)(2-x^3)}{(1-x)^2}$$

$$y'=\frac{((2-x^2)(2-x^3))'(1-x)^2-((1-x)^2)'(2-x^2)(2-x^3)}{(1-x)^4}=$$

$$=\frac{(5x^4-6x^2-4x)(1-x)^{\cancel{2}}+2\cancel{(1-x)}(2-x^2)(2-x^3)}{(1-x)^{\cancel{4}}}$$

$$=\frac{-3x^5+2x^3-6x^2-4x-8+5x^4}{(1-x)^3}$$

№ 854

$$y=x\sqrt{1+x^2}$$

$$y'=x'\cdot(1+x^2)^{\frac{1}{2}}+x\left((1+x^2)^{\frac{1}{2}}\right)'=$$

$$=\sqrt{1+x^2}+x\cdot\frac{1}{2}\cdot 2x\,(1+x^2)^{-\frac{1}{2}}=$$

$$=\frac{1+2x^2}{\sqrt{1+\cancel{2}x^2}}$$

№ 858

$$y=\sqrt[3]{\frac{1+x^3}{1-x^3}}=\left(\frac{1+x^3}{1-x^3}\right)^{\frac{1}{3}}$$

$$y'=\frac{1}{3}\cdot\left(\frac{1+x^3}{1-x^3}\right)^{-\frac{2}{3}}\cdot\frac{(1+x^3)'(1-x^3)-(1+x^3)(1-x^3)'}{(1-x^3)^2}=$$

$$=\sqrt[3]{\frac{1-x^3}{1+x^3}}\cdot\frac{2x^2}{1-x^6}$$

848. $y=\dfrac{(2-x^2)(2-x^3)}{(1-x)^2}$.

854. $y=x\sqrt{1+x^2}$.

858. $y=\sqrt[3]{\dfrac{1+x^3}{1-x^3}}$.

№ 860

$$y=\sqrt{x+\sqrt{x+\sqrt{x}}}=\left(x+\left(x+x^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$y'=\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\cdot\left(1+\frac{1}{2\sqrt{x+\sqrt{x}}}\cdot\left(1+\frac{1}{2\sqrt{x}}\right)\right)=$$

$$=\frac{4\sqrt{x+\sqrt{x}}\cdot\sqrt{x}+2\sqrt{x}+1}{8\sqrt{x+\sqrt{x+\sqrt{x}}}\cdot\sqrt{x+\sqrt{x}}\cdot\sqrt{x}}$$

№ 864

$$y=\sin(\cos^2x)\cdot\cos(\sin^2x)$$

$$y'=-2\sin x\cos x\cos(\sin^2x)\cos(\cos^2x)-$$

$$-2\sin x\cos x\sin(\sin^2x)\sin(\cos^2x)=$$

$$=-\sin 2x\left(\cos(\cos^2x)(\sin^2x)+\sin(\cos^2x)(\sin\right.$$

$$=-\sin 2x\cos(\cos^2x-\sin^2x)=-\sin 2x\cos(\cos 2$$

860. $y=\sqrt{x+\sqrt{x+\sqrt{x}}}$.

864. $y=\sin(\cos^2 x)\cdot\cos(\sin^2 x)$.

№ 870

$$y=\frac{\sin x - x\cos x}{\cos x + x\sin x}$$

$$y'=\frac{(\cos x-\cos x+x\sin x)(\cos x+x\sin x)-}{(\cos x+x\sin x)^2}$$

$$\frac{-(\sin x-x\cos x)(-\sin x+\sin x+x\cos x)}{}=$$

$$=\frac{x\sin x\cos x+x^2\sin^2 x-x\cos x\sin x+x^2\cos^2 x}{(\cos x+x\sin x)^2}$$

$$=\frac{x^2(\sin^2 x+\cos^2 x)}{(\cos x+x\sin x)^2}=\frac{x^2}{(\cos x+x\sin x)^2}$$

№ 876

$$y=e^{-x^2}$$

$$y'=-2xe^{-x^2}$$

№ 880

$$y=e^x\left(1+\operatorname{ctg}\frac{x}{2}\right)$$

$$y'=e^x+e^x\operatorname{ctg}\frac{x}{2}-\frac{e^x}{2\sin^2\frac{x}{2}}=e^x\left(1+\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}-\frac{1}{2\sin^2\frac{x}{2}}\right)$$

$$=e^x\left(\frac{2\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}-1}{2\sin^2\frac{x}{2}}\right)=$$

$$=\frac{e^x(\sin x-\cos x)}{2\sin^2\frac{x}{2}}$$

870. $y=\dfrac{\sin x - x\cos x}{\cos x + x\sin x}$.

876. $y=e^{-x^2}$.

880. $y=e^x\left(1+\operatorname{ctg}\dfrac{x}{2}\right)$.

№ 883

$$y = e^x + e^{e^x} + e^{e^{e^x}}$$

$$y' = e^x + e^x \cdot e^{e^x} + e^{e^{x+1}} \cdot e^{x+1} \cdot 1 =$$

$$= e^x\left(1 + e^{e^x} + e^{e^{e^x}} \cdot e\right)$$

№ 887

$$y = \ln(\ln(\ln(x)))$$

$$y' = \frac{1}{\ln(\ln(x))} \cdot \frac{1}{\ln(x)} \cdot \frac{1}{x} =$$

$$= \frac{1}{x\ln(x) \cdot \ln(\ln(x))}$$

№ 895

$$y = \ln\left(x + \sqrt{x^2+1}\right)$$

$$y' = \frac{1}{x+\sqrt{x^2+1}} \cdot \left(1 + \frac{1}{2\sqrt{x^2+1}} \cdot 2x\right) =$$

$$= \frac{1}{x+\sqrt{x^2+1}} \cdot \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}}$$

883. $y = e^x + e^{e^x} + e^{e^{e^x}}$.

887. $y = \ln(\ln(\ln x))$.

895. $y = \ln\left(x + \sqrt{x^2+1}\right)$.

№ 904

$$y = \ln\sqrt{\frac{1-\sin x}{1+\sin x}}$$

$$y' = \frac{1}{\sqrt{\frac{1-\sin x}{1+\sin x}}}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{\frac{1-\sin x}{1+\sin x}}}\cdot\frac{-2\cos x}{(1+\sin x)^2}$$

$$= \frac{1+\sin x}{1-\sin x}\cdot\frac{-\cos x}{(1+\sin x)^2} = \frac{-\cos x}{1-\sin^2 x} = \frac{-1}{\cos x}$$

№ 911

$$y = x(\sin(\ln x) - \cos(\ln x))$$

$$y' = (\sin x(\ln x) - \cos(\ln x)) + x\left(\frac{1}{x}\cos(\ln x) + \sin(\ln x)\right.$$

$$= 2\sin(\ln x)$$

№ 938

$$y = \frac{\arccos x}{x} + \frac{1}{2}\ln\frac{1-\sqrt{1-x^2}}{1+\sqrt{1-x^2}}$$

$$y' = \frac{-\arccos x - \frac{x}{\sqrt{1-x^2}}}{x^2} + \frac{1}{2}\cdot\frac{\frac{2x}{\sqrt{1-x^2}}}{(1+\sqrt{1-x^2})^2}\cdot$$

$$\cdot\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}} = \frac{-\arccos x - \frac{x}{\sqrt{1-x^2}}}{x^2} + \frac{\frac{x}{\sqrt{1-x^2}}}{x^2}$$

$$= \frac{-\arccos x}{x^2}$$

904. $y = \ln\sqrt{\frac{1-\sin x}{1+\sin x}}$.

911. $y = x[\sin(\ln x) - \cos(\ln x)]$.

938. $y = \frac{\arccos x}{x} + \frac{1}{2}\ln\frac{1-\sqrt{1-x^2}}{1+\sqrt{1-x^2}}$.

№ 944

$$y = \operatorname{arctg}\frac{x}{1+\sqrt{1-x^2}}$$

$$y' = \frac{1}{1+\frac{x^2}{(1+\sqrt{1-x^2})^2}} \cdot \frac{1+\sqrt{1-x^2}+\frac{x^2}{\sqrt{1-x^2}}}{(1+\sqrt{1-x^2})^2} =$$

$$= \frac{1}{(1+\sqrt{1-x^2})^2+x^2} \cdot \frac{\sqrt{1-x^2}+1}{\sqrt{1-x^2}} =$$

$$= \frac{1}{2+2\sqrt{1-x^2}} \cdot \frac{\sqrt{1-x^2}+1}{\sqrt{1-x^2}} = \frac{1}{2\sqrt{1-x^2}}$$

№ 951

$$y = \ln(e^x+\sqrt{1+e^{2x}})$$

$$y' = \frac{1}{e^x+\sqrt{1+e^{2x}}} \cdot \left(e^x + \frac{e^{2x}}{\sqrt{1+e^{2x}}}\right) =$$

$$= \frac{1}{e^x+\sqrt{1+e^{2x}}} \cdot \frac{e^x(\sqrt{1+e^{2x}}+e^x)}{\sqrt{1+e^{2x}}} =$$

$$= \frac{e^x}{\sqrt{1+e^{2x}}}$$

944. $y = \operatorname{arctg}\frac{x}{1+\sqrt{1-x^2}}$.

951. $y = \ln\left(e^x+\sqrt{1+e^{2x}}\right)$.

№ 962

$$y = x^{x^a} + x^{a^x} + a^{x^x}$$

$$y' = x^{x^a}\left(ax^{a-1}\cdot\ln x + \frac{x^a}{x}\right) +$$

$$+ x^{a^x}\left(a^x\ln a\cdot\ln x + \frac{a^x}{x}\right) +$$

$$+ a^{x^x}\cdot\ln a\cdot x^x(\ln x + 1) =$$

$$= x^{x^a}\cdot x^{a-1}(a\ln x + 1) + x^{a^x}\cdot a^x\left(\ln a\cdot\ln x + \frac{1}{x}\right)$$

$$+ a^{x^x}\cdot\ln a\cdot x^x(\ln x + 1)$$

№ 972

$$y = \ln\left(\cos^2 x + \sqrt{1+\cos^4 x}\right)$$

$$u = \cos^2 x$$

$$y = \ln\left(u + \sqrt{1+u^2}\right)$$

$$y' = \frac{1}{u+\sqrt{1+u^2}}\cdot\frac{\sqrt{1+u^2}+u}{\sqrt{1+u^2}} = \frac{1}{\sqrt{1+u^2}}$$

$$u' = -2\cos x\sin x = -\sin 2x$$

$$y' = \frac{-\sin 2x}{\sqrt{1+\cos^4 x}}$$

962. $y = x^{x^a} + x^{a^x} + a^{x^x}$ $(a > 0,\ x > 0)$.

972. Найти производную функции

$$y = \ln(\cos^2 x + \sqrt{1 + \cos^4 x}),$$

вводя промежуточное переменное $u = \cos^2 x$.