$$\sum_{n=1}^{\infty} \frac{(x+2)^{n^2}}{n^2} \quad ①$$

$$\lim_{n\to\infty} \left| \frac{(x+2)^{(n+1)^2} \cdot n^n}{(n+1)^{(n+1)} (x+2)^{n^2}} \right| = \lim_{n\to\infty} \left| \frac{(x+2)^{2n+1} n^n}{(n+1)^{(n+1)}} \right| =$$

$$= \lim_{n\to\infty} \left| \frac{(x+2)^{(2n+1)} n^n}{(n+1)(n+1)^n} \right| = \lim_{n\to\infty} \left| (x+2)^{2n+1} \left(\frac{n}{n+1}\right)^n \left(\frac{1}{n+1}\right) \right| = 0$$

Сходится на $(-\infty; \infty)$

$$\sum_{n=1}^{\infty} \frac{n!}{x^n} \quad ②$$

$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)!}{x^{(n+1)}}}{\frac{n!}{x^n}} \right| = \lim_{n\to\infty} \left| \frac{n+1}{x} \right|$$

расходится $\forall x$

$$y'' + 4y' + 4y = \sin x \quad (6)$$

$$y'' + 4y' + 4y = 0$$

$$y = e^{kx}$$

$$k^2 + 4k + 4 = 0$$

$$(k+2)^2 = 0$$

$$k_{1,2} = -2 \Rightarrow \begin{matrix} y_1 = e^{-2x} \\ y_2 = xe^{-2x} \end{matrix} \Rightarrow y_0 = C_1 e^{-2x} + C_2 x e^{-2x} = e^{-2x}(C_1 + C_2 x)$$

Част. решение:

$$y = A\cos x + B\sin x$$

$$y'' + 4y' + 4y = -A\cos x - B\sin x + 4(-A\sin x + B\cos x) +$$
$$+ 4(A\cos x + B\sin x) = (3A + 4B)\cos x + (3B - 4A)\sin x = \sin x$$

$$\begin{cases} 3A + 4B = 0 \\ 3B - 4A = 1 \end{cases} \Rightarrow \begin{matrix} A = -\frac{4}{25} \\ B = \frac{3}{25} \end{matrix}$$

$$y = (C_1 + C_2 x)e^{-2x} - \frac{4}{25}\cos x + \frac{3}{25}\sin x$$

$$x^2 y''' = 2y' \quad (7)$$

$$x^2 y''' - 2y' = 0$$

$$y = x^k$$

$$x^2 x^{k-3} k(k-1)(k-2) - 2x^{k-1} k = 0$$

$$x^{k-1} k^2 (k-3) = 0$$

$$\begin{matrix} k_{1,2} = 0 \\ k_3 = 3 \end{matrix} \Rightarrow \begin{matrix} y_1 = x^0 \\ y_2 = x^0 \log x \\ y_3 = x^3 \end{matrix} \Rightarrow y = C_1 x^3 + C_2 \log x + C_3$$

$(x y' - 1) \ln x = 2y$ 5

$x \ln x \, y' - 2y = 0$

$$\frac{dy}{2y} = \frac{dx}{x \ln x}$$

$$\frac{1}{2} \ln y = \ln(\ln x) + c$$

$y^{1/2} = c \ln x$

$y = c \ln^2 x$

$$y' = c' \ln^2 x + \frac{2 \ln x}{x} + c$$

$x \ln x \, y' = \ln x$ $-2y$

$x \ln^3(x) c' + 2 \ln^2(x) c - 2 \ln^2(x) c = \ln x$

$x \ln^2(x) \cdot c' = 1$

$$c' = -\frac{1}{x \ln^2 x}$$

$$c = \frac{1}{\ln x} + \bar{c}$$

$y = \ln^2 x \cdot c = \bar{c} \ln^2 x - \ln x$

$$xy' = y - xe^{y/x}$$ (3)

$$y = tx$$

$$y' = t + t'x$$

$$x(t + t'x) = tx - xe^t$$

$$t'x^2 = -xe^t$$

$$\frac{x\,dt}{dx} = -e^t$$

$$e^{-t}dt = -\frac{dx}{x}$$

$$-e^{-t} + \ln|c| = -\ln|x|$$

$$e^{-t} = \ln cx$$

$$t = -\ln\ln cx$$

$$y = tx = -x\ln\ln cx$$

$$8y'^3 = 27y$$ (4)

$$y'^3 = \frac{27}{8}y$$

$$\frac{dy}{dx} = \frac{3}{2}\sqrt[3]{y}$$

$$\frac{dy}{\sqrt[3]{y}} = \frac{3}{2}dx$$

$$\frac{3}{2}\sqrt[3]{y^2} = \frac{3}{2}x + c$$

$$\sqrt[3]{y^2} = x + c$$

$$y^2 = (x + c)^3$$