### Theorem: Division with Remainder

which We consider the set r = b − qa ⇔ b = qa + r be the smallest number in . Obviously, r ∈ ℕr ≥ |a|M0. forM. Proof:

r − |a| ∈ M. This leads to a contradicr as the smallest non−negative number in . . We prove this statement by a contradiction: suppose

< r′ − r < |a| r′ − r = b − q′a − b + qa = (q − q′) a remains to be shown. For this, let |a| r′ − rr′ ≠ r Mq′ ∈ ℤ0 must apply.

and with . We assume that . Then it applies

that as the smallest non-negative number in . Thus