## 1.2 Proof Techniques

This sub-unit describes some common techniques and procedures that can be used in proving mathematical theorems. In mathematics, a proposition denotes an important mathematical statement, by which we first informally mean a statement that can be either true or false. Examples are “seven is an even number” (this assertion is false) or “water is wet” (this assertion is correct). The correctness of a proposition is often not as obvious as

Proof in the previous examples and therefore usually must be proven with a proof. By proof we

Mathematicians use mean the faultless and comprehensible derivation of a statement which shows in an proofs as a tool to dem-

onstrate that a given unambiguous and unchallengeable way either the correctness or the inaccuracy of a

mathematical statement statement. is either true or false.

Because proofs play an extremely important role in mathematics, you will first learn about some of the most common proof techniques.

Proving mathematical theorems usually requires a lot of experience and creativity. After a proof has been given, i.e., the statement of a proposition has been successfully proved, a proof is often concluded with the words “quod erat demonstrandum” (q.e.d.). This is Latin and means “thus it has been demonstrated.” This phrase can be traced back to famous

instead of this expression, the sign □ is often used, which goes back to the American Greek mathematicians of antiquity such as Euclid and Archimedes, who completed their proofs in this way (with the corresponding Greek translation, of course). Nowadays,

mathematician Paul Halmos (1916—2006). We will also use this character in this workbook to indicate that a proof is fully completed.

First of all, a word about the importance of proofs in this book: With a few exceptions, we will prove all statements throughout. Frequently, working through a proof can strengthen the understanding of a statement or illustrate its message. You do not have to tediously work through every proof down to the smallest detail! However, you should at least understand some of the great and important proofs since proofs are extremely important in mathematics. Most of the methods presented below are based on the laws of logic. Please do not despair if you do not understand in detail all the techniques of each proof presented. Just read on first!