### the expression is Theorem: Exchanging the Sum Symbols in Double Sumsmn aij

If the associative and commutative laws apply to the summands, then the sum symbols of Proof:the double sums may be swapped, i.e., it applies that:∑11≤≤ij≤≤mn aij = i = 1∑m j = 1∑n aij = j = 1∑n i = 1∑m aij = ∑11≤≤ij≤≤mn aij

The assertion can be proved by dissolving the sum and then changing the summands. Since the associative law applies according to the precondition, we may omit the brackets around the sums in the first step. Since the commutative law is still valid, we can swap the summands at will in the second step. In the third step we are allowed to put new brackets due to the current associative law. This results in

i = 1∑m ∑= 1n aij=i = 1∑m ai1 +ai2 +⋯+ain +amn

which is the made assertion .The previous statement applies in the same way to double sums of products, i.e., sums of===aj = 1∑an11+a1a □j21++a2⋯j ++⋯am1+amj+ a=12j = 1∑+n a22i = 1∑m+a⋯ij+am2 +⋯+ a1n +a2n +⋯+amn