### Theorem: Associative Laws for Sets

Let L, M, and N be sets. Then the following two rules apply:LL MM NN == LL MM NN

Proof:

First, we prove that L ∪ (M ∪ N) = (L ∪ M ) ∪ N. For this purpose we follow the defini-

tion of equality of sets: If we can show that the set on the left side is a subset of the set on the right side and vice versa, then it follows that the two sets are equal. This is exactly how

We first show that L ∪ M) ∪ N L ∪ (M ∪ N)L ∪ (M ∪ N) is ⊆ (L ∪ M) ∪ Nx (L ∪ M) ∪ N. For this we must show that eachL M ∪ Nx ∈xN Lx ∈ Nx x ∈ L ∪x ∈x ∈ we want to proceed:

this set, so x ∈ L ∪ (M ∪ N). Since lies in the union of and , must lie in at element from is also located in . So let be any element of

is , then

x ∈ (L ∪ M) ∪ N. If , then

or in or in both. If

(this is left to you for

practice). This leads to the first part of the claim, namely L ∪ (M ∪ N) = (L ∪ M) ∪ N.

Let ∩ M) ∩ NN □x ∈ L ∩ (M ∩ N)x M . Then N x L ∩ (M ∩ N) ⊇ (L ∩ M) ∩ NL ∩ (M ∩ N) = (L ∩ M) ∩ The second part of the assertion is shown in a similar way:

that is in both and . It follows further that

. Thus it follows that

Analogously, one can show that (this is left to you for practice). This leads to the second part of the claim, namely

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#### Example: associative laws for sets

and

L∩ M∩N

Let The proof of this statement is Theorem: Distribution Laws for SetsL, M and N be sets. Then the following two rules apply:LL leftMM to you for practice. The procedure is very similar to theNN = L M LL NN

previous proof.

The following important rules were written by the British mathematician Augustus de Morgan (1806—1871) and were also named after him. Theorem: Rules of de MorganLet Proof:L, M and N be sets. Then the following two rules apply:L\L\ MM NN == L\ML\M L\NL\N

contained in only one of the two sets then Thus Let We first prove the first rule of de Morgan and proceed analogously to the previous proofs.because of N, then even x ∈ L\ML\(M ∩ N)⊆ (L\M) ∪ (L\N) x ∈ Lx ∈ L\M and therefore , x ∈. Then and is L\Nx ∈ L\Nxx ∈ (L\M) ∪ (L\N) is and therefore ∈ Lfollows. and and thus in particular M, Nx ∉ M ∩ N or in none of them. If x ∈ (L\M) applies accordingly. If . The latter means that is x ∈ (L\M) ∪ (L\N)∪ (L\N)x ∈ M∧ x ∉ N. If x ∉ M∧ x ∈ Nx ∉ Mx is is either applies.∧ x ∉, then, x ∈ L\(M ∩ N)

then x ∈ (L\M) ∪ (L\N)L L\(M ∩ N)⊇ (L\M) ∪ (L\N)

The second rule of de Morgan is proved in an analogous way (we leave this to you for prac-tice). The overall conclusion is therefore that the allegation is correct. □ The following theorem shows some more interesting properties of subsets with respect to union and intersection.