### Definition: Propositional Formula

cation) and (equivalence) logical statements can be combined into propositional logical formulas (also called propositional logical expressions). The set of propositional logical formulas is inductively defined as follows

By means of the logical operators A, B ∈ A⇔A A ∧ (conjunction), A ∈ 0A,1 ∈ A ∨ (disjunction), A ¬ (negation), ⇒ (impli-

For each statement it applies that

If then the following expressions are also in

AA¬AAAA∧∨BBBB The ( ) operator influences—as usual in mathematics—the binding strength of the operators. Bracketed expressions are always resolved from the inside out.

Atomic statements We call the individual logical statements of such a formula atomic statements, statement These are statements variables, or just variables of the formula.

which are either true or false and which cannot be

Let A, B and C be logical statements. Then we can combine them by logical operators to a broken down into further Example: propositional formula parts.

form a propositional logical formula:

The propositional formula B A ∧ B(¬A) ∧ B) ∨ C contains the statements A A and AB B, which are linkedA B C C

by a conjunction. We could also abbreviate this expression as .

The propositional formula contains the statements , and . The brackets are resolved from the inside out, i.e., first is negated by the negation operator and then linked to AAB∨C by a conjunction. This partial expression is then linked to A by a disjunction. We could also abbreviate this expression as

is also a (very simple) propositional logical formula, in which only the variable and no logical operators occur. We call such expressions atomic. Formulas in which at least one logical operator occurs are called compound formulas.

Please note that in the following examples we will often use the abbreviated notation for conjunction and negation for a better overview.

Two propositional logical expressions P and P ≡ QQ are called logically equivalent if P and Q Definition: Logically Equivalent

Please note the difference between = and ≡! Two propositional logical expressions do not always have the same truth value. One writes .

have to be the same to be logically equivalent. The following example should illustrate this.

There are three statements C := A, B and C with A := “7 is an even number,” A B := “It is dark atB C Example: Logically equivalent

night,” and “Water is wet.” Obviously statement is wrong, and statements and are correct.

It applies that B ≡ C, because both statements can be regarded as atomic exprese.g., P := A ∧ B and Q := A ∧ C. Then P ≡ QA ∧ BP ≠C is Q, because the expressions containAP Q B sions and both are correct.

With these statements we can also form compound propositional logical expressions,

and A ∧ C is also false (because A is false and ≡ is true). different statements. Nevertheless, applies, because and have the same truth value. Both are false because is false (because is false and is true)

The previous example shows why we chose the operator also for statements, which have the same truth value: Since we can regard any statement as an atomic expression, the definition of the logical equivalence of statements is thus consistent with the definition of the logical equivalence of expressions. This fact makes many things easier, because we no longer have to distinguish meticulously between statements and expressions, but can show many things more generally—namely for expressions—and simply transfer them to statements.