## 4.2 Binary System

The binary system, also called dual system, is a number system which is of enormous relevance, especially in computer science. Today’s computer systems calculate almost exclusively in the binary system. Without this number system, computer science as we know it today would be unthinkable. It is therefore one of the most important number systems besides the decimal system.

In contrast to the decimal system, where ten different digits are used to represent the numbers, the binary system uses only the two digits: 0 and 1. These digits are also called binary digits. The name of this number system is also derived from this. Binary numbers are represented using base 2.

As you know, computers calculate with bits and bytes. A bit is a binary digit, i.e., 0 or 1, and a byte is a value consisting of eight consecutive bits. The representation of the numwith b −1 1 0 −1 −2 −n

In contrast to the representation in the decimal system, however, the leading sign is omitted here. Binary numbers are therefore initially always unsigned. The reason for this is that computers only calculate with bits, i.e., with 0 and 1, and there is no sign provided there at first. The representation of negative binary numbers is discussed below.

mal system, the significance of each digit is no longer a power of ten, but a power of twoThe index 2 . i again indicates the importance of the respective digit b bi. Different to the decii

Accordingly, the (decimal) value of a binary number is determined from the sum of powers of base 2. Just as in the calculation of the value of decimal numbers, the digits before the decimal point are multiplied by powers of two with non-negative exponents and the digits after the decimal point are multiplied by powers of two with negative exponents

and summed up. The value of each binary number can thus be calculated as follows:To indicate that it is a number in binary notation, we use a 2 as the subscript index.b = i =∑m−nbi ·2i

Example: binary system 11012=1·8+1·4+0·2+1·1=13=1·23 +1·22 +0·21 +1·20

This example shows very well why it is important to distinguish between the representa-10 Numbers with decimal places can also be represented in the binary system without prob-1101We call two numbers equal when they have the same numerical value. So 10. 1101 tion of a number and its actual value (which is always given as a decimal number, as described above): The binary number understood as a decimal number, it would have a completely value with .

In today’s computer systems, 32 or 64 bit numbers are =17310 often used for calculations. As already mentioned, these are binary numbers with a length of 32 or 64 bits, or in other words with a length of four or eight bytes. For better readability, blanks can be inserted between the bits for very long binary numbers. Often the bits are divided into packets of four digits each (called nibbles or half-bytes) and a short space is inserted between each of them. For example, the 32-bit number101001101011010110101101011011112 is slightly clearer when written1010 0110 1011 0101 1010 1101 0110 11112

This notation has the advantage that you can quickly determine the length of the number by counting the nibbles without actually having to count each individual digit.

In digital technology, bits are used to store states. A 1 generally means that the state is set, a 0 that the state is not set. Often these values are equated with the truth values “true” and “false” known from logic. For computer scientists it is often important to know how many states can be stored in a binary number with a certain length. The number of possible states for binary numbers always corresponds to a power of two. These powers of two are so important and are used so frequently that most computer scientists know at least the first ten to twelve powers from the following table by heart and do not need to calculate them. They occur in a wide variety of areas, e.g., in the specification of hard disk space, working memory sizes, data types for programming languages, or transmission speeds.

Table 2: Number of possible states, which can be represented by binary numbers

Source: Brückmann, 2013.