### Definition: Function

BLet is a relation that assigns an element from A and B be non-empty sets. Then a function f : A → Bf : A → B B to each element from ff with domain (of definition) f A BA A in a unique way.B A and range

For this we write , which is spoken as “ is a mapping from to .” The expression is often simply written as , if and are obvious from the

If an x ∈ A is given, then the element from x B, which is uniquely determined by the assign-f f x x ff(x) context.

ment rule, is called the image or function value of at the position . One writes to express this. The is then also called the argument or input value of the function . Note that we may speak of “the” image of the function at the position because of its unam-

If preimage.an image) need not be unique (because there could well be same function value f(x) = y, then x is called a preimage of y, i.e., f(x1) = y = f(xy under 2) with f. Note that an preimage (as opposed tox1 ≠ x2)different. That is why we speak of “a” x1, x2that map to the biguousness.

The set f(A) ∶= {y ∈ B|∃ x ∈ A : f(x) = y} ⊆ BB is called the image (or image set) of f(A) = Bf(A) ≠ B f.

Note that (depending on the function under consideration) it can be that , since not every element of must have a preimage. Functions for which have a spe-

Two figures f : A → BA and C g : C → D have the same name if B A = CD and f B = Dg and if cial name. each element from (or ) is assigned the same element in (or ) by and .

#### Example: Functions

Let A ∶= {1, 2, 3} and B ∶= {4, 5, 6}. We can define a function f(1) ∶= 4f : A → Bf(2) ∶= 5 by explic-f(3)

itly defining the function value for each argument, e.g., , and

3}∶= 6. However, this only makes sense for sets with few elements. Usually, therefore, af f(x) ∶= x + 3 x ∈ {1, 2,f : ℝ

function is more likely to be defined by specifying a calculation or assignment rule. We could define the previous function for example as for

We can define the square of real numbers as a power of two function as follows:

A very important function is the identity map or identity function . A

5.4. constdefined as This is defined asAnother popular example of a function is the square root function (or root function) 7(3) =const 7 and c : A → Aconst7(32) = with const7. c(x) ∶= c. With A ∶= ℝ and c ∶= 7, for example,c ∈ A . with :ℝ ℝ

With this definition, for examplex

and | : ℝ → ℝ+ 49 = 7 | value function measures

6. Another important function is the absolute value function (or modulus) . This is Absolute value function

|5| = 5∥ |− 32, 4| = 32, 4x : = x : = −x, ifxx, ifx < 0|0| = 0≥ 0 from zero on the number defined as with Put simply, the absolute

the distance a number is

line.

For example, , and .

In order to illustrate function or their values, graphs can be used.