### Definition: Graph of a Function

f(xLet x f(x)A)|x ∈ A and B} be sets and let x ∈ A f : A → Bf f : A → B be a mapping from A to B. The set G(f) ∶= {(x,

is called the graph of .

To illustrate the graph of a function graphically, proceed as follows: Draw a coordinate system with a horizontal x-axis and a vertical y-axis. There you enter all points ( , ) with . The following six figures outline the graphs for the functions from the

Figure 4: Graph of function f:1,2,3>4,5,6 with f x : = x+3 examples above.

Source: Brückmann, 2013.

square root of a number x and then add a value m. This can be defined for functions in Occasionally, you may want to perform several functions in succession, e.g., first take the general.

Let A, B, C and f(x) ∈ CD be non-empty sets and let x ∈ A f : A → B and g : C → D be two figures.f g Definition: Composition

Furthermore, applies to all . Then the composition of the functions and

One pronounces g ∘ f as “g composed (with) g∘f:A D, g∘f” or “f x g: = afterg f f.x” is defined by

Example: Composition 3. We consider the absolute value function tion of f : ℕ → ℕf and g (g ∘ f)(x) ∶= 3(x + + and the root function5). tion of and

Let f : ℝ → ℝf g with . Then the composi-

Let . Then the composi-

:R+ R+

Theorem: Associativity of the CompositionBecause of as ∘ :|x| ≥ 0R R for all + with x ∈ ℝ∘ we can construct the composition of x ≔ x . | and

Proof:composition of functions is associative, i.e., Let A, B, C and D be sets and let f : A → Bh ∘ (g ∘ f) = (h ∘ g) ∘ f, g : B → C and h : C → D. be functions. The

First, one considers that the compositions on both sides of the equals sign are actually

gfrom valid and have the same domain and range:from mapping from Both compositions are thus valid and each has the same domain and range. It remains to is a map from is a map from to AB. to to h is a map from DD. . f is a mapping from AAB to to to DBC. and and C to gh is a map from is a map from D. Thus, A to Bh ∘ (g ∘ f). This means that BC to to CD is still defined, and overall is a mapping. Thus . Thus g ∘ fh ∘ g(h ∘ g) ∘ f is defined and is a map from is defined and is a mapping is still defined and is a fA C

We define be shown that the functions are in fact the same:and hg ∶= h ∘ g and gf ∶= g ∘ fh∘ g for all ∘f aa ∈ A==hh it applies thatfgf∘afaf aa

h∘g ∘f a ===

Thus it applies that (g ∘ f) = (h ∘ g) ∘ f( applies also. h ∘ (g ∘ f )(a) = □ (h ∘ g) ∘ f)(a=hhhhggg∘f∘gfaf)a for all faa a ∈ A and thus the claim h ∘