## 5.2 Special Features of Functions

There are certain characteristics that make some functions stand out. In the following we would like to briefly introduce the most important of these.

Let f : A → B be a function. f is called injective if for all f a = f a′ a = a′a, a ∈ A the following applies:B Definition: Injectivity

In other words, a function is called injective if each element of the range has at most one preimage, i.e., if there are no two different input values that map to the same function value.

The following figure graphically illustrates the principle of injectivity.

Figure 10: Left: In an injective mapping, each element of the range B has at most one preimage in A. Right: If an element has several possible preimages, the function is not injective.

Example: Injectivity1. The function ple that f(−1) = 1 = f(1)f : ℝ → ℝ+, then f is injective, because for with . f(x) ∶= x + 1f(x) ∶= 2xf(x) ∶= xa = a'a, a' ≥ 02a = a' is not injective, because it applies for exam-. a, a' ∈ ℝ+ with a, a' ∈ ℕ = f(a') = a' → ℝf(a) =f(a) 2. On the other hand, if we restrict the domain and consider the function a = a' because of 1+ +2

The function f : ℝ → ℝf : ℕ → ℕ with is injective, because for with

a + 1 = f(a') = a' + it follows that .

a = f(a') = 2a' it follows that .

The function with is injective, because for with

Let b f : A → B be a function. f is surjective if there is an a ∈ A for each b ∈ B with f(a) = Definition: Surjectivity

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ment of the range B. For a surjective function f : A → B it applies that f(A) = B. In other words, a function is called surjective if there is at least one preimage for each ele-

The following figure graphically illustrates the principle of surjectivity.

Figure 11: Left: In a surjective mapping, each element of the range B has at least one preimage in A. Right: If there are elements in the range for which no preimage exists, then the function is not surjective.

1. The function f : ℕ → ℕ with f(x) ∶= 2x is not surjective, because the image of f are Example: Surjectivity ages under f. f : ℝ → ℝ

3. The identity function there exists a preimage, namely ℝ the preimage is just idb (b − 1 ∈ ℝb) = bid. ℝ(:x) ∶= x It is f(b − 1) = (b − 1) + 1 = b is surjective, because for every . b ∈ ℝb ∈ the even natural numbers. The odd natural numbers therefore do not have any preim-

2. The function is surjective, because for every