### Definition: Bijectivity

Let f : A → B be a function. f is bijective if f is injective and surjective. B

In other words, a function is called bijective if each element of the range has exactly one preimage.

The figure below graphically illustrates the principle of bijectivity. Bijective functions play an extremely important role in many areas of mathematics, as we will see below.

Figure 12: In a bijective mapping, each element of the range B has exactly one preimage in A.

The function f : ℝ → ℝf : ℝ → ℝ with f(x) ∶= 2x is bijective because it is injective and sur-

jective.

The function with is not bijective because it is neither injective nor surjective. surjective.

The function f : ℕ → ℕ with is not bijective, because it is injective but not

Definition: Invertible Function and b ∈ B.

Let and f : A → Bf ∘ g = id be a function. B. f is invertible if there is a function f, (g ∘ f)(a) = a and (f ∘ g)(b) = bg : B → A with for all g ∘ f = ida ∈ AA

In particular, for an invertible function + 3 and f (g ∘ f)(a) = g(f(af : A → B) = g(a + 3) = a + 3 − 3 = ag : B → A Example: Invertable functions