### Definition: Tuples

ments listed in a tuple areA tuple is a finite, orderedlist of elements. The ele-Tuple mLet Unlike sets, the elements of a tuple do not necessarily have to be n ∈ MM be a set. By an in a list. We write such a tuple as (n-tuple (or tuple for short) we mean an sequence of m1, ., mn). differentn objects from eachm1, .,

non-negative. other, i.e., the same element can occur several times in a tuple. Furthermore, the order of the elements in a tuple is important. A 2-tuple is also known as an ordered pair, a 3-tuple as a triple and a 4-tuple as a quadruple. Let M be a non-empty set. A mapping M ∗ from M × M to M, i.e., ∗ : M × M → M, is called Definition: Operation Instead of ∗ (m1, m2 ) for m1, m2 ∈ M we also write m1 ∗ m2 for short.

an operation on .

#### 1. b for all

b for all

∘ the set of all

with and . Then

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∗) to which the associative law applies,(M,

i.e., , then we call the tuple

#### Example: semigroup

and · : ℤ × ℤ → ℤ be defined as in the previous

that the composition of functions is associative. Thus (ℳ, ∘) is a semigroup. example. Then and are semigroups.

Let be defined as in the previous example. We have already shown