### Definition: Identity Element

Let M(M, ∗) be a semigroup. An element m ∈ M m ∗ e = e ∗ m = me ∈ M is an identity element (or neutral element)

in , if for all it holds that .

1. The semigroup ( , as defined in the previous example, is given: Then there is no Example: identity element

3.2. The semigroup that The semigroup identity element in this semigroup, because for all m ∈ Ma ∈ ℤa + 0 = 0 + a = a. (((ℕℤ, ·ℕ, ℳ, ∘0, +)+)) has the 1 as an identity element. It is that ) , on the other hand, has an identity element, namely 0. It is for all a ∈ ℕ0. a, b ∈ ℕ is a + b ≠ aa · 1 = 1 · a = a.(m) = m for

all

4. The semigroup has the identity function for all as an identity element. It is that .

Let (M, ∗) be a semigroup with an identity element. Then this neutral element is Identity elementThis element has the speTheorem: Uniqueness of the Identity Element

uniquely determined. cial property that it has

no effect on the effect or

Proof: purpose of the operation.

tity element that Let = e and be identity elements in and thus the uniqueness of the neutral element., because applies, because is an identity element as well. From this follows overall that (M, ∗e)2. Then, according to the definition of the iden- is an identity element. Furthermore, it appliese1

Let (M, ∗) be a semigroup with identity element m′ ∈ Mm m′m ∗ m′ = m′ ∗ m = eem. An element m ∈ M is called invertiblem′

if there is an element , with . The element is then called an inverse element to . One says is inverse to .

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#### Example: Invertability

In the semigroup there are no invertible elements, because there is no identity element in this semigroup. and −1. The inverse element of 1 is 1, with 1 · 1= 1. The inverse element of −1 is −1,(ℳ, ∘) element. The inverse element to 0 is 0 and .

In the semigroup , +) 0 is the identity element. The 0 is also the only invertible0 + 0 = 0

In the semigroup 1 is the identity element. The only invertible elements are 1

with .

We have already determined the invertible elements in . These are the bijective functions.