### Theorem: Uniqueness of the Inverse Element

Let (M, ∗) be a semigroup with identity element m e. Let m ∈ M be an invertible element.

Then the inverse element to is uniquely determined. Let ′ ∗ m = em′, m ′ ∈ M′ ∗ e = m be inverse elements to m ′ ∗ m = emm′ = m, i.e., m ∗ m′ = m′ ∗ m = e′ □ m′ = e ∗ m′ = (m(m ′ ∗ m) ∗ m′ = m and m ∗ m′ ∗ m) ∗ m′′ = m′′ ∗ (m

Proof:

∗ m′) = m. Then because of ′ it follows that .

With the associativity of the operation it further follows that

ment with 0. If In semigroups with the operation with −a. is an invertible element, we also denote the element inverse to a0,+), we denote the identity ele-

In semigroups with the linkage a ∈ M ·, i.e., (M, ·−), e.g., (ℤ, ·), we denote the identity element with 1. If is an invertible element, we also denote the inverse element to a with a−1.

The following designation for commutative semigroups goes back to the Norwegian mathematician Niels Henrik Abel (1802—1829).

Let be a semigroup. is commutative, or abelian, if the commutative law2 = m2 ∗ m1. In this case one speaks of a commuta-

#### Example: Abelian semigroup

The semigroups are abelian.

M ∶= {1, 2, 3}f(1) = 1. Furthermore, let us take the functions f(2) = 3 f(3) = 2 g(1) = 3f : M → Mg(2) = 2 and g : M → Mg(3) = 1

The semigroup is not abelian, as can be seen from the following example: Let

given by as well as , and .

Then

and ggg

fff gg Thus g is ∘ f ≠ f ∘ g. g