### Theorem: Euclid’s Theorem

Euclid’s Theorem postulates that there are more prime numbers than there are number of prime numbers (in other words, that the number of prime number is infinite).

Proof: Euclid proved the statement of this theorem with a proof by contradiction: Suppose thereproduct of all these prime numbers. Then there are two possibilities for First possibility: were only finitely many prime numbers m + 1 would be another prime number. This contradicts the assumption.m + 1m + 1 is a prime number. But since is not a prime number. Then it must be divisible by a primeq is also a divisor of i.e., 1. However, this is not possible, since 1 does notpq1 must then be one of the prime numbers , ..., pnm. Let . Since m + 1m := pq is a divisor of both 1 · p2 · m + 1. · pn−1:m. and ppn be the, ..., pm +p then is greater than all 1 n,

Second possibility: 1pnumber , n. However, this means that q must also divide the q. According to the assumption, difference, 1, ...,

have a prime divider. This results in a contradiction to the assumption. numbers is obviously wrong. □ As there are no further possibilities, the assumption that there is a finite number of prime Let m, n ∈ ℕn . We call m and n coprime or mutually prime if the prime factorizations of m