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A Note on the Parameter of Evaporation in the Ant Colony Optimization Algorithm

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Abstract

Ant colony optimization is a very popular evolutionary algorithm. As we know the parameter of pheromone evaporation plays a considerable role in the performance of ACO. In this paper, we present some important results on runtime of SACO(Simple Ant Colony Optimization Algorithm). We also discuss about the selection of the evaporation factor, which is very crucial in directing the ants movements.

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1 Introduction

The Ant Colony Optimization (ACO) algorithm is a popular nature-inspired cybernetic method, which is being used for solving various combinatorial optimization problems. ACO [3] is a cognitive informatics (CI) model of social animals like ants. The idea of ACO comes from the ants behavior, which is different from traditional mathematics-based cybernetic techniques.

The ACO algorithm was first proposed by Dorigo and his colleagues to solve the combinatorial optimization problems like the travelling salesman problem

(TSP) [1], [3]. This algorithm is presented under the inspiration that an ant colony could build the shortest path from a food source to their nest by using some chemical substance called pheromone. Ants lay down pheromone trails when passing paths. The more ants choose a path, the more pheromone is laid down on it. The latter ants tend to choose the path with a higher pheromone intensity. However, it is very interesting that ants do not always choose the path with the highest pheromone intensity. Otherwise, the shortest path will hardly be built up. Dorigo abstracted the ACO model from the biologic phenomenon and carried out the simulation of ant behavior to solve the TSP problems. The tremendous computation power of ACO attracts increasingly more and more researchers to study its theoretical foundations, including mathematical model construction, convergence, and runtime analysis. In this paper, some theoretical results on analyzing how fast ACO algorithms converge based on the quantum of pheromone deposited and the amount of its evaporation are discussed.

2 Preliminaries

Before moving on to the main results, some description on the preliminary notions on the 'theory of pheromone' is necessary. It has been observed that a colony of ants is able to find the shortest path to a food source. As an ant moves and searches for food, it lays down a chemical substance called pheromone along its path. As the ant travels, it looks for pheromone trails on its path and prefers to follow trails with higher levels of pheromone deposits. If there are multiple paths to reach a food source, an ant will lay the same amount of pheromone at each step regardless of the path chosen . However, it will return to its starting point quicker when it takes the shorter path which contains more pheromone. It is then able to return to the food source to collect more food. Thus, in an equal amount of time, the ant would lay a higher concentration of pheromone over its path if it takes the shorter path, since it would complete more trips in the given time. The pheromone is then used by other ants to determine the shortest path to find food as described.

During the process, another factor affects on the amount of pheromone deposition namely, evaporation of pheromone, which can be seen as an exploration mechanism that delays faster convergence of all ants towards a suboptimal path. The decrease in pheromone intensity favors the exploration of different paths during the whole search process. In real ant colonies, pheromone trail also evaporate, but as we have seen, evaporation does not play an important role in real ant's shortest path finding. But on the contrary, the importance of pheromone evaporation in artificial ants is probably due to the fact that the optimization problems tackled by artificial ants are much more complex than those real ants can solve. A mechanism like evaporation that by favoring the forgetting of errors or of poor choices done in the past plays the important function of bounding the maximum value achievable by pheromone trails. In S-ACO the pheromone evaporation is interleaved with pheromone deposit of ants. As pheromone evaporation plays some role in the efficiency of the algorithm, an effective formula for finding the rate at which the evaporation occurs is needed. We have derived a new formula [9], given in the following theorems.

Theorem 2.1 *Let the pheromone evaporation at time t be ρ_t, where the value of ρ_t, lies in the closed interval $[0, 1]$. Now the recurrence relation for the evaporation of pheromone at time $t + 1$ is given by*

$$\rho_{t+1} = \alpha\ \rho_t + \beta(1 - \rho_t) = k\rho_t + \beta \tag{1}$$

where α, β are two constants such that $0 \leq \alpha, \beta \leq 1$ and $k = \alpha - \beta$.

Theorem 2.2 *Let the pheromone evaporation at time t be ρ_t, where the value of ρ_t, lies in the closed interval $[0, 1]$ and the rate at which the evaporation occurs be given by the formula (1) with the additional condition $\alpha \geq \beta$. Then*

$$\begin{aligned} \rho_t &= \frac{\beta(1-k^t)}{(1-k)} \quad if \quad k \neq 1 \\ &= \rho_0; \qquad\qquad if \quad k = 1. \end{aligned} \tag{2}$$

3 Results

We now do comparative runtime analysis between the 1-ANT and a simple evolutionary algorithm called (1 + 1) EA, which has extensively been studied with respect to its runtime distribution. Even though it is already done for a particular range of the value of the evaporation ρ we verify it for the new formula given in the paper[9]. The (1 + 1) EA starts with a solution x_0 that is chosen uniformly at random and produces in each iteration a new solution x from a currently best solution x_0 by flipping each bit of x_1 with probability $1/n$. Hence, the probability of producing a certain solution x with Hamming distance $H(x, x_1)$ to x_0 is $(1/n)^{H(x,x_0)}(1 - 1/n)^{n-H(x,x_0)}$. In the following, we consider the 1-ANT with values of $\rho_t = \frac{\beta(1-k^t)}{(1-k)} \quad if \quad k \neq 1$ in the theorem 2.2. Here one can see that the 1-ANT behaves as the (1+1) EA on each function. This also means that the 1-ANT has the same expected optimization time as the (1 +1) EA on each function. Before going to our prime theorem we state an important lemma due to Neumann and Witt [8].

Lemma 3.1 *For all $\rho \geq \frac{n-2}{3n-2}$, the 1-ANT has the same runtime distribution as the (1+1) EA on each function.*

For large values of n lemma is true for $\rho \geq \frac{1}{3}$. With the help of this we show that the new formula proposed in theorem 2.2 for ρ_t also makes 1-ANT algorithm to have the probability to produce a specific solution that has a Hamming distance as same as in the case of (1+1)EA.

Theorem 3.2 *Choosing the values of α, β in ρ_t such that $\alpha + 2\beta \geq 1$ the 1-ANT has the same runtime distribution as the (1+1) EA on each function.*

Proof. Note that $\rho_t \geq \frac{\beta}{(1-k)}$. In view of the lemma 3.1, it suffices to show that

$$\frac{\beta}{(1-k)} \geq \frac{1}{3}. \tag{3}$$

We have $\alpha + 2\beta \geq 1 \Rightarrow 2\beta \geq 1 - \alpha \Rightarrow 3\beta \geq 1 - \alpha + \beta = (1-k)$. Simple rearrangement yields the desired inequality.

The below theorem is on the relative change of pheromone values before and after the pheromone updation. The part of this theorem is appeared in [8] and we acknowledge the paper for motivating us to come up with this theorem.

Theorem 3.3 *Let e_1 and e_2 be two edges of connected graph of a combinatorial problem and let τ_1 and τ_2 respectively be their current pheromone values in the 1-ANT. Let τ_1' and τ_2' respectively be their updated pheromone values for the next accepted solution x. If e_1 and e_2 are in the path $P(x)$ of the accepted solution x then*

$$|\tau_1' - \tau_2'| = |\tau_1 - \tau_2|[1 - \frac{\beta}{1 - \alpha + 2n\beta}]. \tag{4}$$

Proof. The pheromone values in 1-ANT are updated depending on whether edge (u, v) is contained in the path $P(x)$ of the accepted solution x. In fact the pheromone value updation formulae are given as follows;

$$\tau_1' = \frac{(1-\rho)\tau_1 + \rho}{1 - \rho + 2n\rho}$$

$$\tau_2' = \frac{(1-\rho)\tau_2 + \rho}{1 - \rho + 2n\rho}.$$

But then,

$$\tau_1' - \tau_2' = \frac{(1-\rho)(\tau_1 - \tau_2)}{1 - \rho + 2n\rho}.$$

Thus by taking $\rho = \frac{\beta}{1-k}$ and considering both the possibilities $\tau_1 \geq \tau_2$ and $\tau_2 \geq \tau_1$ we will arrive at the expression (4).

4 Conclusion

In this small article an investigation on pheromone update mechanism in 1-ANT algorithm is discussed. The modification we suggested for the pheromone evaporation definitely refines the existing algorithm. But in any case our results show that the efficiency of the 1-ANT is vulnerable with respect to the choice of ρ_t. Further analysis can be done on some classic combinatorial problems for update parameters that do not allow the pheromone values to reach the saturation level.

References

[1] M. Dorigo and C. Blum, Ant colony optimization theory: A survey, Theor. Comput. Sci., 344(2005), 243-278.

[2] A. Badr and A. Fahmy, A proof of convergence for ant algorithms, Inform. Sci., 160 (2004), 267-279.

[3] M. Dorigo and T. Stutzle, Ant Colony Optimization. Cambridge, MA: MIT Press, 2004.

[4] T. Stutzle and H. H. Hoos, MAX-MIN ant system, Future Gener. Comput. Syst., 16 (2000), 889-914.

[5] W. J. Gutjahr, A graph-based ant system and its convergence, Future Gener. Comput. Syst., 16 (2000) 873-888.

[6] W. J. Gutjahr, ACO algorithms with guaranteed convergence to the optimal solution, Inform. Process. Lett., 82 (2002) 145-153.

[7] H. Huang, Z. F. Hao, C. G. Wu, and Y. Qin, The convergence speed of Ant Colony Optimization, Chin. J. Comput., 30 (2007) 1343-1353.

[8] F. Neumann and C. Witt, Runtime Analysis of a Simple Ant Colony Optimization Algorithm, Algorithmica, 54 (2009), 243-255.

[9] Prasanna Kumar and G. S.Raghavendra , On the Evaporation Mechanism in the Ant Colony Optimization Algorithms, Ann.Comp. Science Ser., 9 (2011), 51-56.

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