MHD

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1 Introduction

Magnetohydrodynamics or MHD is the motion of magnetized fluids. You could also say that it is the study of electrically conducting fluids and how the combination of fluid dynamics and electromagnetism govern it's motion. The magnetized fluid we discuss in this paper is the ferrofluid. There are other practical uses for MHD like the confinement of nuclear fusion, but we are not describing that today. A ferrofluid contains ferromagnetic particles in a carrier fluid. It is very easy to say that a ferrofluid will align and move with a magnetic field. If it were up to me, I would leave it at that. However, in order to understand this world more we need to delve deeper into MHD. To have a coherent understanding of MHD, we need to learn how fluid dynamics and electricity magnetism (EM) fit together to describe the motion of ferrofluids. In this paper we shall explore fluid dynamics, EM, how fluid dynamics and EM come together to model MHD, and specific properties of ferrofluids and how they are made.

We first need to define a few things for our equations.

1. $\rho =$ mass density
2. $\vec{u}$ = flow velocity
3. $P =$ pressure
4. $\sigma =$ charge density
5. $\vec{J}$ = current density
6. $\vec{E}$ = electric field
7. $\vec{B}$ = magnetic field

Essentially we will take a fluid motion equation that looks like $F = ma$ and an EM equation that looks like $F = ma$, and put them together to get an equation of motion for our ferrofluid. One way to approach this problem is by considering conservation of mass. We use the equation $d\rho/dt = -\dot{\nabla}(\rho\vec{u})$ to show that the changing mass density is dependent upon the divergence of the mass density and the flow velocity. We can then use conservation of momentum to get an equation of the form $\rho(d\vec{u}/dt + \vec{u}\dot{\nabla}\vec{u}) = -\nabla p - \nabla\Pi + \vec{F}$. This momentum depends on pressure, viscous stress, and any other forces like gravity. This is the Navier-Stokes eqation for fluid motion. Viscous stress is a frictional force inside the fluid from certain parts of the fluid flowing opposite directions.

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We will for right now move on the EM portion. The evolution of of a magnetic field can be shown by $d\vec{B}/dt = -\nabla \times \vec{E}$ This is one of maxwells equations. Usually we have $\nabla \times \vec{E} = 0$. But this is only the case with static charges. When the charges start moving, we get a current and our above equation.

From Coulomb's law we can see that the force on a stationary charge due to another charge is $F = kqQ/r^2$ where $k = 1/4\pi\epsilon_0$ and $\epsilon_0 = 8.85418782 * 10^{-12}$. This will yield a repulsive force between the two particles if $F > 0$, and an attractive force if $F < 0$. IF we add more test charges to our system, we can get the force by using the principle of superposition. We also can calculate the electric field from the force. this equation is $E = F/Q$. These are our initial backbone equations, but these are only useful for stationary discrete charges. We introduce and σ to account for a volume of charges. our electric field can then take the form $E = 1/k \int \sigma d\tau/r^2$. This is important because we can then use gauss's law to get one of maxwells equations.
$\nabla \cdot \vec{E} = \sigma/\epsilon_0$
Along those same lines, for a stationary charge we get that $\nabla \times \vec{E} = 0$. This is another one of maxwell's equations, but it is useless to us right now. We know that this is the case for stationary charges, but when we introduce moving charges, we get a different expression.

Before we move onto moving charges and their expressions, let us first describe "static" magnetic field. The force due to a magnetic field can be described as $F = Q(v \times B)$ where Q is some test charge, v is the velocity of the test charge, and B is the magnetic field the charge moves through. You can see that F must be perpendicular to both the velocity vector and the magnetic field vector. If the charge is in the presence of both a magnetic field and an electric field, the force it feel would just superpose the two forces together like so: $F = Q(E + v \times B)$ We can derive three important equations using a magnetic field and moving charges.

1. $\nabla \times \vec{B} = \mu_0 J$
2. $\nabla \cdot \vec{B} = 0$
3. $\nabla \times \vec{E} = -dB/dt$

1. tells us that the curl of a magnetic field is equal to the permeability of free space, which is $\mu_0 = 4\pi * 10^7$ multiplied by J which is the current density.

2. tells us that the divergence of a magnetic field is always zero. This makes sense because if you were to look at one straight magnetic field line due to a

magnet and measure its strength at the base compared to its strength some distance away, the strengths would be the same. You may wonder why the magnet gets stronger closer to the magnet, but that is because there are more field lines entering/exiting the magnet near the base of it.

3. This one is a bit tricky. We just showed you up above that $\nabla \times \vec{E} = 0$ and now we are changing that. It can be confusing, but this new equation holds for moving charges while our previous equation holds for static charges. We want to use this new equation because in a fluid we aren't dealing with static fluids, we are dealing with the motion of these fluids and therefore the motion of the charges within these fluids.

We have now described all four of maxwells equations. We shall use these and their force laws in combination with fluid motion force laws to get an expression for MHD.

fluids can have a multitude of forces acting on them. They can have friction, pressure, gravity, EM, etc. We need to unpack each force term and then superpose them. We already briefly unpacked EM. Now let's look at pressure. Pressure is force divided by area. We can say force due to pressure is pressure times area. the acceleration of our fluid multiplied by our mass will give us force. We say
$dv/dt(v \cdot \nabla)v = -1/\rho \nabla p - \nabla \phi + f_{ext}$
This is the navier stokes equation. This is used to calculate fluid motion. The left side of the equation is the velocity of the fluid. ρ is the mass density of the fluid, ∇p is the force pressure term, $\nabla \phi$ is the potential term, and f_{ext} is any force term that we happened to miss.

Temperature is a critical consideration when it comes to MHD. In order for the colloidal suspension to remain stable the magnetic particles generally have to be approximately 10 nm in diameter. when they are this size they have only one magnetic domain. These particles then feel an attraction to each other. To stop this attraction to each other we must add a repulsive interaction. We can then charge the particles to give them a repulsive electrostatic force.

Making ferrofluids. most common magnetic particles in ferrofluids are megnetite and maghemite.

Vorticity is a property of fluids. It essentially measures the curl of a fluid. Often times, vorticity arises from friction of the fluid against the fluid container. You could imagine a river that is flowing past you. If there are no impedences, the river will be flowing the fastest in the center and the slowest near a the riverbanks. If you were to throw a stick into the river, the stick would spin because the river has curl. Vorticity can be generated by thermomagnetic interaction even in the absence of viscosity (think of convection). ferrohydrodynamics- the fluid dynamic and heat transfer processes associated with the motion of incompressible, magnetically polarizable fluids in the presence of magnetic field and temperature gradients. strong coupling is occurs when the polarization is field and temperature dependent.

charged particles are introduced into a fluid. When this fluid is placed within a magnetic field, the particles slip relative to the fluid and transmit drag to it. This is what causes the whole fluid to move. This is of scientific interest because in zero gravity we cannnot prime rocket pumps. We can potentially add charged particles to the fuel and use a EM field to move the fluid.

not the best draft, but hopefully my final will be much better. do you know how to convert latex to a editable word file? or do you know how to insert a picture into latex? these will be helpful for pictures.