UNIVERSIDAD DE LOS ANDES

ALGORITHM ANALYSIS

ISIS 4208

Homework 3

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1 Planar graph generation

A process for generating simple, connected, planar graphs, i.e., graphs that can be drawn on a plane without their edges crossing, was defined. For this, we use triangulation and planarity validation methods. The function receives a target number of nodes and edges, and its operation is described below:

1. By default, if the desired number of edges is less than $n_{\text{nodes}} - 1$ an error is generated, since connectivity cannot be guaranteed with so few edges.
2. n_{nodes} random points are generated in two dimensions, each coordinate in $(0, 1)$. These will be used as nodes of the graph.
3. The Delaunay triangulation is calculated on these points to obtain an initial set of edges that, by construction, form a planar graph.
4. The edges generated by the triangulation simplexes are iteratively and randomly added to the graph until the desired number is reached.
5. If the number of edges is not yet the desired number, edges are added randomly between nodes as long as the graph remains planar. If after `max_trials` this is not achieved, a warning is displayed.
6. If the graph is asked to ensure connectedness (`ensure_connectivity = True`), the function verifies that the graph is connected. If it is not, edges are added between connected components to achieve this.

The resulting graphs are saved in `csv` format to ensure easy interoperability with *Cytoscape*.

1.1 Graph visualization

The visualization and coloring of the graphs was carried out in *Cytoscape*, for 20 nodes with a variable number of edges.

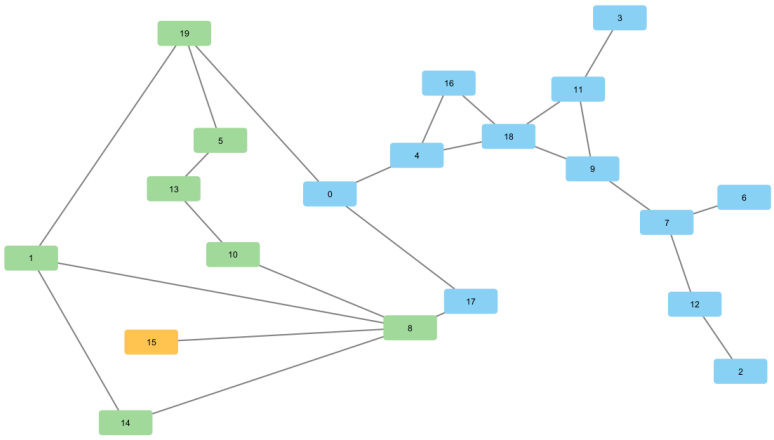

Figure 1: Graph with 24 edges

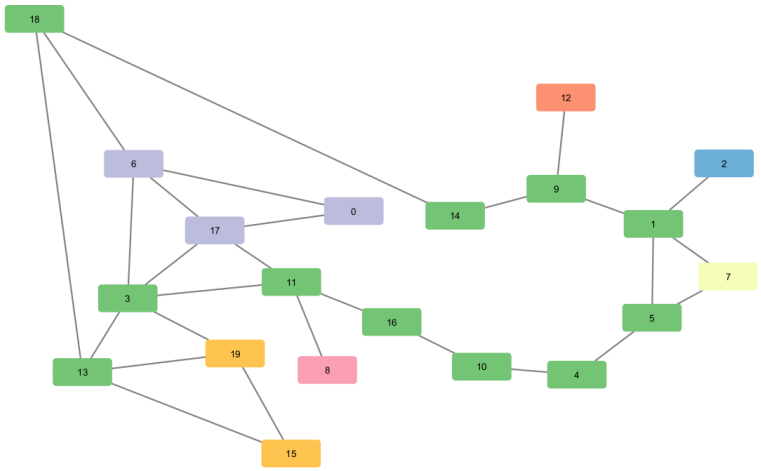

Figure 2: Graph with 27 edges

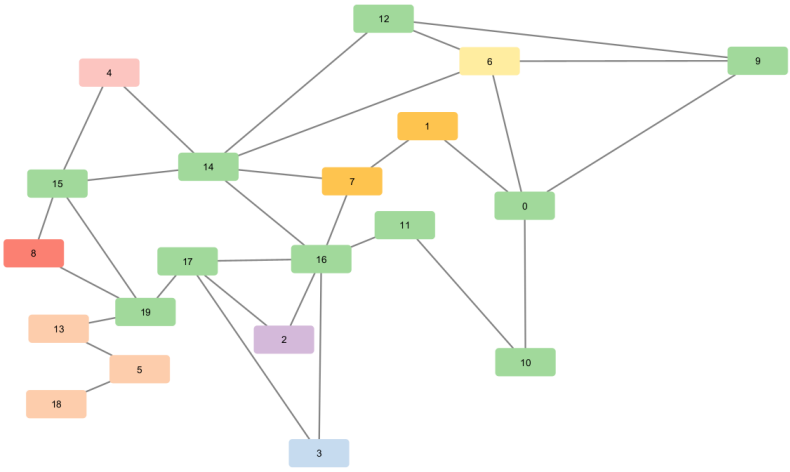

Figure 3: Graph with 30 edges

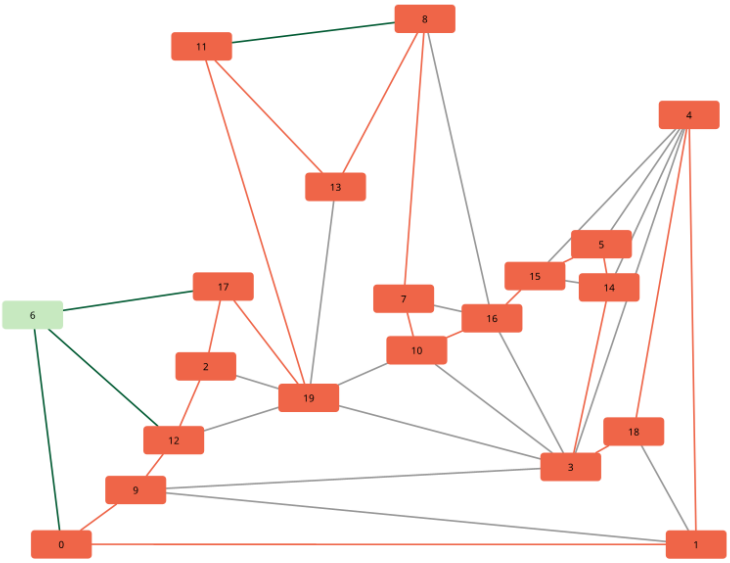

Figure 4: Graph with 40 edges

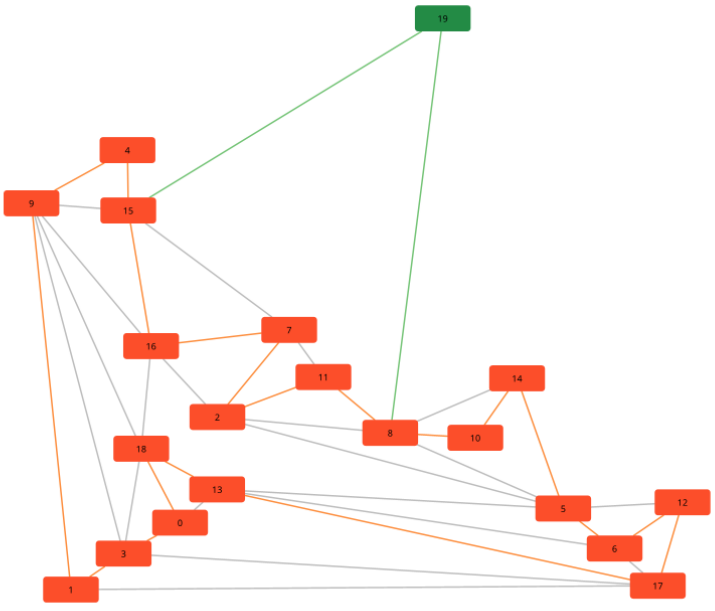

Figure 5: Graph with 41 edges

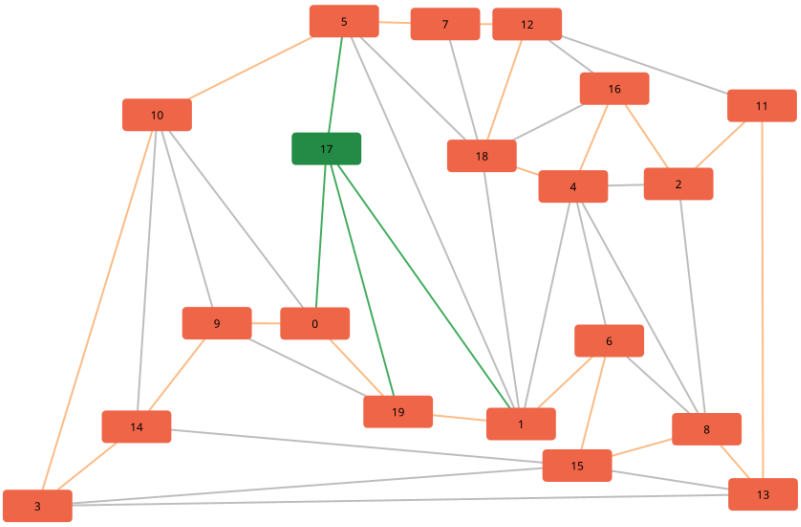

Figure 6: Graph with 44 edges

The visualization and coloring of the graphs was also carried out for graphs with more than 20 nodes. In particular, here we have a planar graph with 25 nodes and 40 edges, and one with 30 nodes and 41 edges.

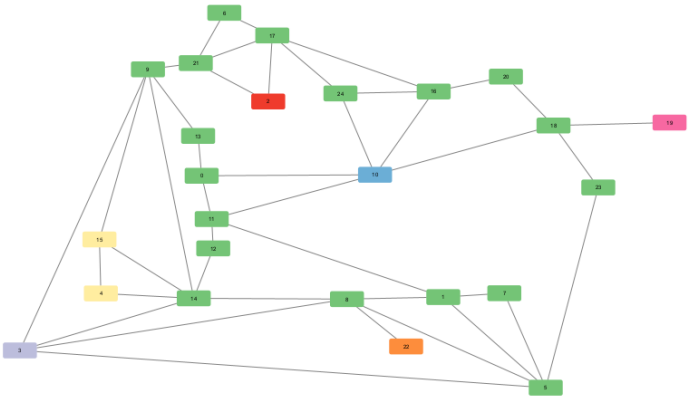

Figure 7: Graph with 25 nodes and 40 edges

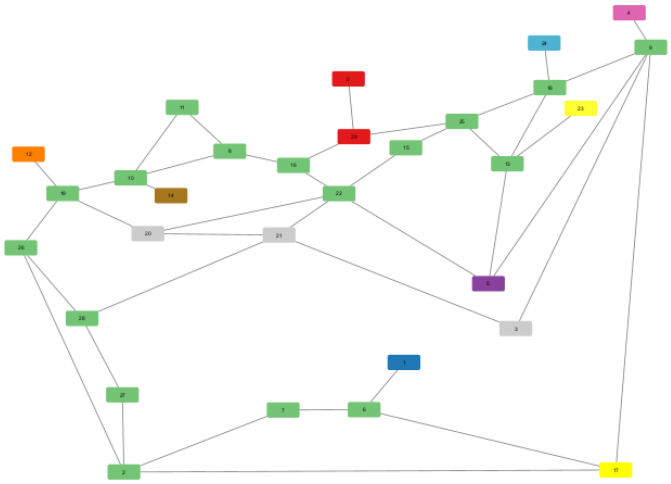

Figure 8: Graph with 30 nodes and 41 edges

2 Planarity experiment

The proposed experiment is designed to analyze the threshold of planarity for graphs by generating planar graphs using our proposed method with varying edge counts. The goal is to determine the probability that a generated graph remains planar as the number of edges increases.

The core function takes three parameters: the number of nodes, the maximum number of edges, and the number of trials per edge count. The theoretical limit for planarity is computed using the formula $3n_{\text{nodes}} - 6$. This serves as a benchmark for evaluating the results.

The experiment iterates over a range of edge counts, starting from the minimum required for connectivity up to the specified maximum. For each edge count, multiple trials are conducted where graphs are generated and checked for planarity. The proportion of planar graphs over the total trials is recorded as the probability of planarity for that edge count.

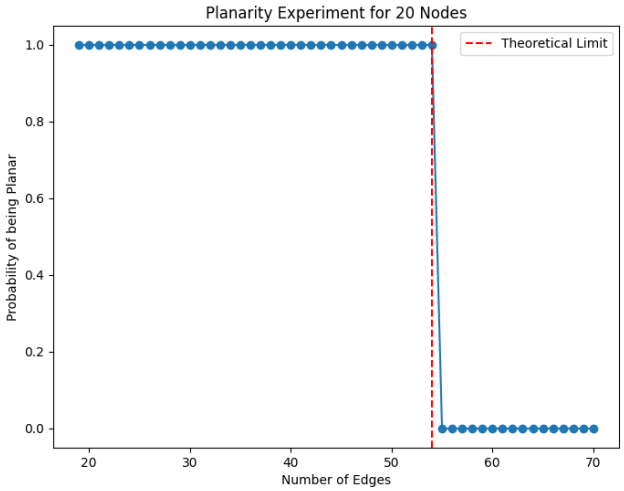

Figure 9: Planar experiment results

As Figure 9 demonstrates, our method always generates planar graphs until the theoretical threshold is surpassed.

3 Planar graph with $3n_{\text{nodes}} - 6$ edges

Using our proposed methods, it's possible to generate graphs with a number of edges equal to the theoretical limit.

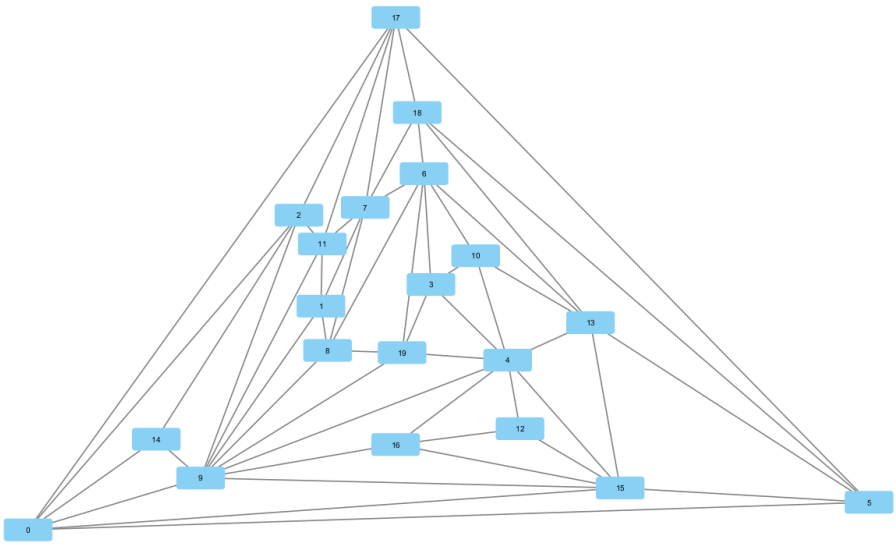

Figure 10: Graph with 20 nodes and 54 edges