

Homework 01 - 1.2, 1.3

Due Wed 2/5
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1.2 Review of Calculus

1. Show that the following equations have at least one solution in the given intervals.

a. $x \cos x - 2x^2 + 3x - 1 = 0$, $[0.2, 0.3]$ and $[1.2, 1.3]$

Sol:

For interval $[0.2, 0.3]$:

$$f(0.2) = 0.2 \cos(0.2) - 2(0.2)^2 + 3(0.2) - 1 = -0.284$$

$$f(0.3) = 0.3 \cos(0.3) - 2(0.3)^2 + 3(0.3) - 1 = 0.0066$$

For interval $[1.2, 1.3]$:

$$f(1.2) = 1.2 \cos(1.2) - 2(1.2)^2 + 3(1.2) - 1 = 0.1548$$

$$f(1.3) = 1.3 \cos(1.3) - 2(1.3)^2 + 3(1.3) - 1 = -0.132$$

Therefore, $x \cos x - 2x^2 + 3x - 1$ has at least one solution in both intervals due to sign changes and continuity of $f(x)$

b. $(x - 2)^2 - \ln x = 0$, $[1, 2]$ and $[e, 4]$

Sol:

For interval $[1, 2]$:

$$f(1) = (1 - 2)^2 - \ln(1) = 1$$

$$f(2) = (2 - 2)^2 - \ln(2) = -0.693$$

For interval $[e, 4]$:

$$f(e) = (e - 2)^2 - \ln(e) = -0.484$$

$$f(4) = (4 - 2)^2 - \ln(4) = 2.61$$

Therefore, $(x - 2)^2 - \ln x = 0$ has at least one solution in both intervals due to sign changes and continuity of $f(x)$

c. $2x \cos(2x) - (x - 2)^2 = 0$, $[2, 3]$ and $[3, 4]$

Sol:

For interval $[2, 3]$:

$$f(2) = 2(2) \cos(2 \times 2) - (2 - 2)^2 = -2.61$$

$$f(3) = 2(3) \cos(2 \times 3) - (3 - 2)^2 = 4.761$$

For interval $[3, 4]$:

$$\begin{aligned}f(3) &= 2(3) \cos(2 \times 3) - (3 - 2)^2 = 4.761 \\f(4) &= 2(4) \cos(2 \times 4) - (4 - 2)^2 = -5.164\end{aligned}$$

Therefore, $2x \cos(2x) - (x - 2)^2 = 0$ has at least one solution in both intervals due to sign changes and continuity of $f(x)$

d. $x - (\ln x)^x = 0, \quad [4, 5]$

Sol:

For interval $[4, 5]$:

$$\begin{aligned}f(4) &= 4 - (\ln 4)^4 = 0.306 \\f(5) &= 5 - (\ln 5)^5 = -5.798\end{aligned}$$

Therefore, $x - (\ln x)^x = 0$ has at least one solution in the interval due to sign changes and continuity of $f(x)$

2. Find intervals containing solutions to the following equations.

a. $x - 3^{-x} = 0$

Sol:

$$\begin{aligned}f(0) &= 0 - 3^0 = - \\f(1) &= 1 - 3^{-1} = +\end{aligned}$$

The interval is $[0, 1]$

b. $4x^2 - e^x = 0$

Sol:

$$\begin{aligned}f(0) &= 4(0)^2 - e^0 = - \\f(1) &= 4(1)^2 - e^1 = +\end{aligned}$$

The interval is $[0, 1]$

c. $x^3 - 2x^2 - 4x + 3 = 0$

Sol:

$$\begin{aligned}f(0) &= 0^3 - 2 * 0^2 - 4 * 0 + 3 = + \\f(1) &= 1^3 - 2^2 - 4 + 3 = -\end{aligned}$$

The interval is $[0, 1]$

d. $x^3 = 4.001x^2 + 4.002x = 1.101 = 0$

Sol:

$$\begin{aligned} f(-3) &= (-3)^3 = 4.001(-3)^2 + 4.002(-3) = 1.101 = - \\ f(-2) &= (-2)^3 = 4.001(-2)^2 + 4.002(-2) = 1.101 = + \end{aligned}$$

The interval is $[-3, -2]$

3. Show that the first derivatives of the following functions are zero at least once in the given intervals.

a. $f(x) = 1 - e^x + (e - 1) \sin(\frac{\pi}{2}x), \quad [0, 1]$

Sol:

$$\begin{aligned} f(0) &= 1 - e^0 + (e - 1) \sin(\frac{\pi}{2}0) = 0 \\ f(1) &= 1 - e^1 + (e - 1) \sin(\frac{\pi}{2}1) = 0 \end{aligned}$$

Since $f(x)$ is differentiable in the given open interval and continuous in the given closed interval, by Rolle's Theorem, there exists $c \in (0, 1)$ such that $f'(c) = 0$

b. $f(x) = (x - 1) \tan x + x \sin \pi x, \quad [0, 1]$

Sol:

$$\begin{aligned} f(0) &= (0 - 1) \tan 0 + 0 \sin \pi 0 = 0 \\ f(1) &= (1 - 1) \tan 1 + 1 \sin \pi 1 = 0 \end{aligned}$$

Since $f(x)$ is differentiable in the given open interval and continuous in the given closed interval, by Rolle's Theorem, there exists $c \in (0, 1)$ such that $f'(c) = 0$

c. $f(x) = x \sin \pi x - (x - 2) \ln x, \quad [1, 2]$

Sol:

$$\begin{aligned} f(0) &= 0 \sin \pi 0 - (0 - 2) \ln 0 \\ f(1) &= 1 \sin \pi 1 - (1 - 2) \ln 1 \end{aligned}$$

Since $f(x)$ is differentiable in the given open interval and continuous in the given closed interval, by Rolle's Theorem, there exists $c \in (0, 1)$ such that $f'(c) = 0$

d. $f(x) = (x - 2) \sin x \ln(x + 2), \quad [-1, 3]$

4. Find $\max_{a \leq x \leq b} |f(x)|$ for the following functions and intervals.

a. $f(x) = \frac{(2-e^x+2x)}{3}, \quad [0, 1]$

Sol:

$$\begin{aligned} f'(x) &= \frac{2-e^x}{3} \\ x &= \ln 2 \\ f(0) &= \frac{1}{3} \\ f(1) &= \frac{4-e}{3} \end{aligned}$$

$$\text{Max} = \frac{2 \ln 2}{3}$$

b. $f(x) = \frac{(4x-3)}{(x^2-2x)}, \quad [0.5, 1]$

Sol:

$$\begin{aligned} f'(x) &= \frac{-4x^2+6x-6}{(x^2-2x)^2} \\ f(0.5) &= \frac{4}{3} \\ f(1) &= -1 \end{aligned}$$

$$\text{Max} = \frac{4}{3}$$

c. $f(x) = 2x \cos(2x) - (x - 2)^2, \quad [2, 4]$

d. $f(x) = 1 + e^{-\cos(x-1)}, \quad [1, 2]$

5. Let $f(x) = x^3$

Sol:

$$\begin{aligned} \text{a. } P_2(x) &= 0 \\ \text{b. Error} &= 0.125 \\ \text{c. } P_2(x) &= 1 + 3(x - 1) + 3(x - 1)^2 \\ \text{d. } R_2 &= -0.125, \text{ actual error} = -0.125 \end{aligned}$$

6. Let $f(x) = \sqrt{x + 1}$

Sol:

$$\begin{aligned} \text{a. } P_3(x) &= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 \\ \text{b. } &0.7109, 0.8662, 1.1182, 1.2344 \\ \text{c. } &-0.0038, -0.0002, -0.0002, -0.0097 \end{aligned}$$

7. Find the second Taylor Polynomial $P_2(x)$ for the function $f(x) = e^x \cos x$ about $x_0 = 0$.

- a. Use $P_2(0.5)$ to approximate $f(0.5)$. Find an upper bound for error $|f(0.5) - P_2(0.5)|$ using the error formula, and compare it to the actual error.

Sol:

$$\begin{aligned} P_2(x) &= 1 + x \\ P_2(0.5) &= 1.5 \\ \text{Actual } f(0.5) &\approx 1.445 \\ \text{Error: } |1.445 - 1.5| &= 0.055 \\ \text{Error bound: } \frac{4.473}{6}(0.5)^3 &\approx 0.0932 \end{aligned}$$

- b. Find a bound for the error $|f(x) - P_2(x)|$ in using $P_2(x)$ to approximate $f(x)$ on the interval $[0, 1]$.

Sol:

$$\text{Error bound: } \frac{7.525}{6} \cdot 1^3 = 1.254$$

- c. Approximate $\int_0^1 f(x) dx$ using $\int_0^1 P_2(x) dx$.

Sol:

$$\int_0^1 P_2(x) dx = 1.5 \quad \Rightarrow \quad 1.5$$

- d. Find an upper bound for the error in 7c using $\int_0^1 |R_2(x)| dx$, and compare the bound to the actual error.

Sol:

$$\begin{aligned} \text{Error bound: } \frac{7.525}{24} &\approx 0.3136 \\ \text{Actual error: } |1.394 - 1.5| &= 0.106 \end{aligned}$$

8. Find the Third Taylor polynomial $P_3(x)$ for the function $f(x) = (x - 1) \ln(x)$ about $x_0 = 1$.

- a. Use $P_3(0.5)$ to approximate $f(0.5)$. Find an upper bound for error $|f(0.5) - P_3(0.5)|$ using the error formula, and compare it to the actual error.

Sol:

$$\begin{aligned} P_3(x) &= (x - 1)^2 - \frac{1}{2}(x - 1)^3 \\ P_3(0.5) &= 0.3125 \\ \text{Actual } f(0.5) &\approx 0.3466 \\ \text{Error: } 0.0341 \\ \text{Error bound: } \frac{112}{24} \cdot (0.5)^4 &\approx 0.2917 \end{aligned}$$

- b. Find a bound for the error $|f(x) - P_3(x)|$ in using $P_3(x)$ to approximate $f(x)$ on the interval $[0.5, 1.5]$.

Sol:

$$\text{Error bound: } \frac{112}{24} \cdot (0.5)^4 \approx 0.2917$$

- c. Approximate $\int_{0.5}^{1.5} f(x) dx$ using $\int_{0.5}^{1.5} P_3(x) dx$.

Sol:

$$\int_{0.5}^{1.5} P_3(x) dx \approx 0.0833$$

- d. Find an upper bound for the error in 8c using $\int_{0.5}^{1.5} |R_3(x)| dx$, and compare the bound to the actual error.

Sol:

$$\text{Error bound: } \approx 0.0583$$

$$\text{Actual error: } |0.088 - 0.0833| \approx 0.0047$$

9. Use the error term of a Taylor polynomial to estimate the error involved in using $\sin x \approx x$ to approximate $\sin 1^\circ$.

Sol:

$$\text{Convert } 1^\circ \text{ to radians: } x = \frac{\pi}{180} \approx 0.0174533.$$

$$\text{Error term for } P_1(x) = x \text{ is } |R_1(x)| \leq \frac{|x|^3}{6}.$$

$$|R_1| \leq \frac{(\pi/180)^3}{6} \approx 8.85 \times 10^{-7}.$$

$$\text{Error bound: } \approx 8.85 \times 10^{-7}.$$

10. Use a Taylor polynomial about $\frac{\pi}{4}$ to approximate $\cos 42^\circ$ to an accuracy of 10^{-6} .

Sol:

$$\text{Convert } 42^\circ \text{ to radians: } x = \frac{7\pi}{30} \approx 0.733.$$

$$\text{Center at } a = \frac{\pi}{4} \approx 0.785.$$

$$\text{Compute } |x - a| = \frac{\pi}{60} \approx 0.05236.$$

$$\text{Find smallest } n \text{ such that } \frac{(\pi/60)^{n+1}}{(n+1)!} \leq 10^{-6}.$$

$$\text{For } n = 3 : \frac{(0.05236)^4}{24} \approx 3.12 \times 10^{-7} \leq 10^{-6}.$$

$$\text{Use } P_3(x) \text{ about } \frac{\pi}{4} \text{ with terms up to } (x - \frac{\pi}{4})^3.$$

11. Let $f(x) = e^{x/2} \sin(x/3)$. Determine the following:

a. The third Maclaurin polynomial $P_3(x)$.

Sol:

$$P_3(x) = \frac{x}{3} + \frac{x^2}{6} + \frac{23}{648}x^3$$

b. A bound for the error $|f(x) - P_3(x)|$ on $[0, 1]$.

Sol:

$$\text{Error bound: } \frac{5}{1296} \approx 0.00386$$

12. Let $f(x) = \ln(x^2 + 2)$. Determine the following:

a. The Taylor polynomial $P_3(x)$ for f expanded about $x_0 = 1$.

Sol:

$$P_3(x) = \ln 3 + \frac{2}{3}(x - 1) + \frac{1}{9}(x - 1)^2 + \frac{2}{81}(x - 1)^3$$

b. The maximum error $|f(x) - P_3(x)|$ for $0 \leq x \leq 1$.

Sol:

$$\text{Error bound: } 0.125$$

c. The Maclaurin polynomial $\tilde{P}_3(x)$ for f .

Sol:

$$\tilde{P}_3(x) = \ln 2 + \frac{x^2}{2}$$

d. The maximum error $|f(x) - \tilde{P}_3(x)|$ for $0 \leq x \leq 1$.

Sol:

$$\text{Error bound: } 0.125$$

e. Does $P_3(0)$ approximate $f(0)$ better than $\tilde{P}_3(1)$ approximates $f(1)$?

Sol:

$$\text{Error at } P_3(0) : |\ln 2 - 0.5183| \approx 0.1748$$

$$\text{Error at } \tilde{P}_3(1) : |\ln 3 - 1.1931| \approx 0.0945$$

No, $\tilde{P}_3(1)$ approximates $f(1)$ better.

13. Find a bound for the maximum error when using $P_2(x) = 1 - \frac{1}{2}x^2$ to approximate $f(x) = \cos x$ on $[-\frac{1}{2}, \frac{1}{2}]$.

Sol:

$$\begin{aligned} \text{Error term: } R_2(x) &= \frac{f^{(4)}(c)}{4!}x^4 \quad (c \in [-1/2, 1/2]) \\ \text{Since } f^{(4)}(x) &= \cos x, |f^{(4)}(c)| \leq 1 \\ \text{Max } |x|^4 &\leq \left(\frac{1}{2}\right)^4 = \frac{1}{16} \\ \text{Error bound: } |R_2(x)| &\leq \frac{1}{24} \cdot \frac{1}{16} = \frac{1}{384} \approx 0.0026 \end{aligned}$$

14. The n -th Taylor polynomial for a function f at x_0 is sometimes referred to as the polynomial of degree at most n that best approximates f near x_0 .

- a. Explain why this description is accurate.

Sol:

The n -th Taylor polynomial $P_n(x)$ matches f and its first n derivatives at x_0 . This ensures the polynomial shares the function's value, slope, curvature, and higher-order behaviors at x_0 , minimizing the approximation error near x_0 . The error $|f(x) - P_n(x)|$ grows only with $|x - x_0|^{n+1}$, making $P_n(x)$ the "best" local approximation among polynomials of degree $\leq n$.

- b. Find the quadratic polynomial that best approximates a function f near $x_0 = 1$ if the tangent line at $x_0 = 1$ has equation $y = 4x - 1$, and $f''(1) = 6$.

Sol:

$$\begin{aligned} \text{From the tangent line: } f(1) &= 3, \quad f'(1) = 4. \\ \text{Quadratic polynomial:} \\ P_2(x) &= f(1) + f'(1)(x - 1) + \frac{f''(1)}{2}(x - 1)^2 \\ P_2(x) &= 3 + 4(x - 1) + 3(x - 1)^2. \end{aligned}$$

15. The error function is defined by

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

- a. Integrate the Maclaurin series for e^{-t^2} to show that

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)k!}.$$

Sol:

Maclaurin series: $e^{-t^2} = \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k}}{k!}$.

Integrate term-by-term: $\int_0^x e^{-t^2} dt = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)k!} x^{2k+1}$.

Multiply by $\frac{2}{\sqrt{\pi}}$ to obtain the series.

- b. Verify that the two series agree for $k = 1, 2, 3, 4$.

Sol:

Expand both series up to $k = 4$:

Series (a): $\frac{2}{\sqrt{\pi}} \left(x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \frac{x^9}{216} \right)$.

Series (b): $\frac{2}{\sqrt{\pi}} e^{-x^2} \left(x + \frac{2x^3}{3} + \frac{4x^5}{15} + \frac{8x^7}{105} + \frac{16x^9}{945} \right)$.

Multiply $e^{-x^2} \approx 1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6} + \frac{x^8}{24}$ into series (b):

Result matches series (a) up to x^9 (coefficients agree).

- c. Approximate $\text{erf}(1)$ to within 10^{-7} .

Sol:

Compute terms until $\frac{2}{\sqrt{\pi}} \cdot \frac{1}{(2k+1)k!} < 10^{-7}$.

At $k = 6$: $\frac{2}{\sqrt{\pi}} \cdot \frac{1}{13 \cdot 6!} \approx 1.08 \times 10^{-8} < 10^{-7}$.

$\text{erf}(1) \approx 0.84270079$.

- d. Use the same number of terms ($k = 6$) with the series in part (b).

Sol:

Approximation: $\text{erf}(1) \approx 0.84270079$ (same accuracy as part c).

- e. Explain difficulties using the series in part (b).

Sol:

Series (b) requires multiplying two infinite series, leading to computational complexity and potential for error.

16. Verify that $|\sin x| \leq |x|$ for all x .

- a. Show that for $x \geq 0$, $f(x) = x - \sin x$ is non-decreasing, implying $\sin x \leq x$.

Sol:

$f'(x) = 1 - \cos x \geq 0$ (since $\cos x \leq 1$ for all x).

$\Rightarrow f(x)$ is non-decreasing on $[0, \infty)$.

At $x = 0$: $f(0) = 0 - \sin 0 = 0$.

For $x \geq 0$: $f(x) \geq f(0) \Rightarrow x - \sin x \geq 0 \Rightarrow \sin x \leq x$.

b. Conclude using $\sin(-x) = -\sin x$.

Sol:

For $x < 0$:

$$|\sin x| = |\sin(-x)| = |-\sin(-x)| = |\sin(-x)| \leq |-x| = |x| \quad (\text{by part (a)}).$$

Thus, $|\sin x| \leq |x|$ for all $x \in \mathbb{R}$.

1.3 Round-Off Error and Computer Arithmetic

1. Compute the absolute error and relative error in approximations of p by p^* .

a. $p = \pi, p^* = \frac{22}{7}$

Sol:

$$\text{Absolute error: } \left| \pi - \frac{22}{7} \right| \approx 0.001264$$

$$\text{Relative error: } \frac{0.001264}{\pi} \approx 0.000402 \quad (0.0402\%)$$

b. $p = \pi, p^* = 3.1416$

Sol:

$$\text{Absolute error: } |\pi - 3.1416| \approx 0.00000735$$

$$\text{Relative error: } \frac{0.00000735}{\pi} \approx 0.00000234 \quad (0.000234\%)$$

c. $p = e, p^* = 2.718$

Sol:

$$\text{Absolute error: } |e - 2.718| \approx 0.0002818$$

$$\text{Relative error: } \frac{0.0002818}{e} \approx 0.0001037 \quad (0.01037\%)$$

d. $p = \sqrt{2}, p^* = 1.414$

Sol:

$$\text{Absolute error: } |\sqrt{2} - 1.414| \approx 0.0002136$$

$$\text{Relative error: } \frac{0.0002136}{\sqrt{2}} \approx 0.000151 \quad (0.0151\%)$$

e. $p = e^{10}, p^* = 22000$

Sol:

$$\text{Absolute error: } |e^{10} - 22000| \approx 26.4658$$

$$\text{Relative error: } \frac{26.4658}{e^{10}} \approx 0.001201 \quad (0.1201\%)$$

f. $p = 10^\pi$, $p^* = 1400$

Sol:

Absolute error: $|10^\pi - 1400| \approx 15$

Relative error: $\frac{15}{10^\pi} \approx 0.01083$ (1.083%)

g. $p = 8!$, $p^* = 39900$

Sol:

Absolute error: $|40320 - 39900| = 420$

Relative error: $\frac{420}{40320} \approx 0.0104$ (1.04%)

h. $p = 9!$, $p^* = \sqrt{18\pi} \left(\frac{9}{e}\right)^9$

Sol:

Absolute error: $|362880 - 359500| \approx 3380$

Relative error: $\frac{3380}{362880} \approx 0.00931$ (0.931%)

2. Perform the following computations (i) exactly, (ii) using three-digit chopping arithmetic, and (iii) using three-digit rounding arithmetic. (iv) Compute the relative errors in (ii) and (iii).

a. $\frac{4}{5} + \frac{1}{3}$

Sol:

(i) Exact: $\frac{17}{15} \approx 1.133333333$

(ii) Chopping: 1.13

(iii) Rounding: 1.13

(iv) Relative errors: 0.294% (both)

b. $\frac{4}{5} \times \frac{1}{3}$

Sol:

(i) Exact: $\frac{4}{15} \approx 0.266666667$

(ii) Chopping: 0.266

(iii) Rounding: 0.266

(iv) Relative errors: 0.25% (both)

c. $\left(\frac{1}{3} - \frac{3}{11}\right) + \frac{3}{20}$

Sol:

(i) Exact: $\frac{139}{660} \approx 0.2106060606$

(ii) Chopping: 0.211 Error: 0.187%

(iii) Rounding: 0.210 Error: 0.288%

d. $\left(\frac{1}{3} + \frac{3}{11}\right) - \frac{3}{20}$

Sol:

- (i) Exact: $\frac{301}{660} \approx 0.4560606061$
- (ii) Chopping: 0.455 Error: 0.232%
- (iii) Rounding: 0.456 Error: 0.0133%

3. Perform the following computations using three-digit rounding arithmetic and compute errors.

a. $133 + 0.921$

Sol:

Exact: 133.921
 Approx: 134
 Absolute error: 0.079
 Relative error: 0.0590%

b. $133 - 0.499$

Sol:

Exact: 132.501
 Approx: 133
 Absolute error: 0.499
 Relative error: 0.376%

c. $(121 - 0.327) - 119$

Sol:

Exact: 1.673
 Approx: 2.00
 Absolute error: 0.327
 Relative error: 19.5%

d. $(121 - 119) - 0.327$

Sol:

Exact: 1.673
 Approx: 1.67
 Absolute error: 0.003
 Relative error: 0.179%

e. $\frac{\frac{13}{14} - \frac{6}{7}}{2e - 5.4}$
Sol:

Exact: ≈ 1.9528
 Approx: 1.80
 Absolute error: 0.1528
 Relative error: 7.82%

f. $-10\pi + 6e - \frac{3}{62}$
Sol:

Exact: ≈ -15.1546
 Approx: -15.1
 Absolute error: 0.0546
 Relative error: 0.360%

g. $(\frac{2}{9}) \times (\frac{9}{7})$
Sol:

Exact: ≈ 0.2857
 Approx: 0.286
 Absolute error: 0.000286
 Relative error: 0.0999%

h. $\frac{\pi - \frac{22}{7}}{\frac{1}{17}}$
Sol:

Exact: ≈ -0.0215
 Approx: 0.00
 Absolute error: 0.0215
 Relative error: 100%

4. Repeat question 3 using three-digit chopping arithmetic.

a. $133 + 0.921$
Sol:

Exact: 133.921
 Chopped: 133
 Absolute error: 0.921
 Relative error: 0.688%

b. $133 - 0.499$

Sol:

Exact: 132.501
Chopped: 132
Absolute error: 0.501
Relative error: 0.378%

c. $(121 - 0.327) - 119$

Sol:

Exact: 1.673
Chopped: 1.00
Absolute error: 0.673
Relative error: 40.2%

d. $(121 - 119) - 0.327$

Sol:

Exact: 1.673
Chopped: 1.67
Absolute error: 0.003
Relative error: 0.179%

e. $\frac{\frac{13}{14} - \frac{6}{7}}{2e - 5.4}$

Sol:

Exact: ≈ 1.9528
Chopped: 2.36
Absolute error: 0.4072
Relative error: 20.8%

f. $-10\pi + 6e - \frac{3}{62}$

Sol:

Exact: ≈ -15.1546
Chopped: -15.1
Absolute error: 0.0546
Relative error: 0.360%

g. $\left(\frac{2}{9}\right) \times \left(\frac{9}{7}\right)$

Sol:

Exact: ≈ 0.2857
Chopped: 0.284
Absolute error: 0.0017
Relative error: 0.599%

h. $\frac{\pi - \frac{22}{7}}{\frac{1}{17}}$
Sol:

Exact: ≈ -0.0215
Chopped: -0.017
Absolute error: 0.0045
Relative error: 20.9%

5. Repeat question 3 using four-digit rounding arithmetic.

a. $133 + 0.921$

Sol:

Exact: 133.921
Approx: 133.9
Absolute error: 0.021
Relative error: 0.0157%

b. $133 - 0.499$

Sol:

Exact: 132.501
Approx: 132.5
Absolute error: 0.001
Relative error: 0.000755%

c. $(121 - 0.327) - 119$

Sol:

Exact: 1.673
Approx: 1.700
Absolute error: 0.027
Relative error: 1.614%

d. $(121 - 119) - 0.327$

Sol:

Exact: 1.673

Approx: 1.673

Absolute error: 0

Relative error: 0%

e. $\frac{\frac{13}{14} - \frac{6}{7}}{2e - 5.4}$

Sol:

Exact: ≈ 1.9538

Approx: 1.932

Absolute error: 0.0218

Relative error: 1.115%

f. $-10\pi + 6e - \frac{3}{62}$

Sol:

Exact: ≈ -15.1546

Approx: -15.16

Absolute error: 0.0054

Relative error: 0.0356%

g. $\left(\frac{2}{9}\right) \times \left(\frac{9}{7}\right)$

Sol:

Exact: ≈ 0.285714

Approx: 0.2857

Absolute error: 0.000014

Relative error: 0.0049%

h. $\frac{\pi - \frac{22}{7}}{\frac{1}{17}}$

Sol:

Exact: ≈ -0.0215

Approx: -0.01700

Absolute error: 0.0045

Relative error: 20.93%

6. Repeat question 3 using four-digit chopping arithmetic.

a. $133 + 0.921$

Sol:

Exact: 133.921
 Chopped: 133.9
 Absolute error: 0.021
 Relative error: 0.0157%

b. $133 - 0.499$

Sol:

Exact: 132.501
 Chopped: 132.5
 Absolute error: 0.001
 Relative error: 0.000755%

c. $(121 - 0.327) - 119$

Sol:

Exact: 1.673
 Chopped: 1.600
 Absolute error: 0.073
 Relative error: 4.36%

d. $(121 - 119) - 0.327$

Sol:

Exact: 1.673
 Chopped: 1.673
 Absolute error: 0
 Relative error: 0%

e. $\frac{\frac{13}{14} - \frac{6}{7}}{2e - 5.4}$

Sol:

Exact: ≈ 1.9538
 Chopped: 1.983
 Absolute error: 0.0292
 Relative error: 1.5%

f. $-10\pi + 6e - \frac{3}{62}$

Sol:

Exact: ≈ -15.1553
Chopped: -15.15
Absolute error: 0.0053
Relative error: 0.035%

g. $\left(\frac{2}{9}\right) \times \left(\frac{9}{7}\right)$

Sol:

Exact: ≈ 0.2857
Chopped: 0.2856
Absolute error: 0.000114
Relative error: 0.04%

h. $\frac{\pi - \frac{22}{7}}{\frac{1}{17}}$

Sol:

Exact: ≈ -0.0215
Chopped: -0.017
Absolute error: 0.0045
Relative error: 20.9%

7. Compute the absolute error and relative error in approximations of π using the given formulas with the Maclaurin polynomial for $\arctan x$.

a. $4 \left[\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) \right]$

Sol:

Approximation: $4 \left[\left(\frac{1}{2} - \frac{1}{24} + \frac{1}{160}\right) + \left(\frac{1}{3} - \frac{1}{81} + \frac{1}{1215}\right) \right] \approx 3.1456$
Absolute error: $|\pi - 3.1456| \approx 0.00398$
Relative error: $\frac{0.00398}{\pi} \approx 0.1268\%$

b. $14 \arctan\left(\frac{1}{5}\right) - 4 \arctan\left(\frac{1}{239}\right)$

Sol:

Approximation: $16 \left(\frac{1}{5} - \frac{1}{3} \left(\frac{1}{5}\right)^3 + \frac{1}{5} \left(\frac{1}{5}\right)^5 \right) - 4 \left(\frac{1}{239} - \frac{1}{3} \left(\frac{1}{239}\right)^3 + \frac{1}{5} \left(\frac{1}{239}\right)^5 \right) \approx 3.1416$
Absolute error: $|\pi - 3.1416| = -2.83757402069e^{-05}$
Relative error: $\frac{3.1416}{\pi} = -9.03227863564e^{-06}\%$

Homework 02 - 1.4, 2.2

Due Tue 2/11
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Hello