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APM 2663

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# Optional Feedback #2

(3.4) 25. (a) ER or not? $xRy : \lfloor x \rfloor = \lfloor y \rfloor$ (Ground set — $\mathbb{R}$)

✓ Reflexive: Floor(x) is a function, so $\lfloor x \rfloor$ always equals $\lfloor x \rfloor$.

✓ Symmetric: There is no symmetric combo of real values which evaluate differently, $\lfloor 3.9 \rfloor = \lfloor 3 \rfloor = 3$.

This is a proof

✓ Transitive: Since only $x$ in the interval $[n, n+1)$ will evaluate to $n$ (where $\lfloor x \rfloor = n$, or $n \le x < n+1$) we know it is transitively closed over that interval.

Formally, we need to show that
$\lfloor x \rfloor = \lfloor y \rfloor$ and $\lfloor y \rfloor = \lfloor z \rfloor \Rightarrow \lfloor x \rfloor = \lfloor z \rfloor$.

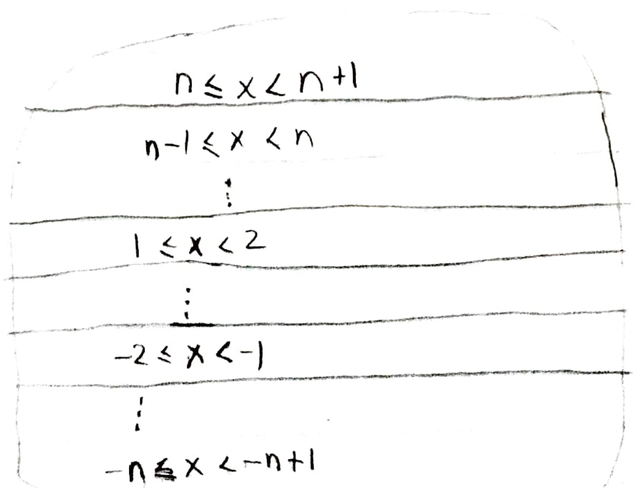
by transitivity of = over $\mathbb{R}$.

(see 3.1 #27) ↳

The Floor function implies $x = \lfloor x \rfloor + c$ $(x, c \in \mathbb{R},\ 0 \le c < 1)$.
So, $x - c_1 = y - c_2$ and $y - c_2 = z - c_3$.
But this means $x - c_1 = z - c_3$ and so $\lfloor x \rfloor = \lfloor z \rfloor$. □
So, similar to symmetry, this relation is transitive by the equality operator.
∴ R is an equivalence relation.

(b) Each partition is the continuous set of interrelated real numbers on the intervals $n \le x < n+1$ $(n \in \mathbb{Z})$.
This can also be written as $[n, n+1)$.

$[0] : \{ 0 \bullet\!\!-\!\!\circ 1 \}$, $[1] : \{ 1 \bullet\!\!-\!\!\circ 2 \}$ ... $[n] : \{ n \bullet\!\!-\!\!\circ n{+}1 \}$ ... $[-n] : \{ -n \bullet\!\!-\!\!\circ -n{+}1 \}$




✓

Partition illustration.