Ex: Suppose $A, B \neq \phi$.

Prove that $A \times B = B \times A \iff A = B$

Proof: ($\implies$) Assume $A \times B = B \times A$

WTS $A = B$

(i) Show $A \subseteq B$

Let $a \in A \neq \phi$

Now $B \neq \phi$, $\therefore \exists b \in B$

$\therefore$ By def'n, $(a, b) \in A \times B = B \times A$

$\therefore (a, b) \in B \times A$

$\implies a \in B$ and $b \in A$

$\therefore a \in B$

(ii) Show $B \subseteq A$

Similar to (i)

($\Leftarrow$) Assume $A = B$

WTS $\quad A \times B = B \times A$

(i) Show $\quad A \times B \subseteq B \times A$

Let 🙂 $\in A \times B$

Now 🙂 $= (a, b)$ where $a \in A$ and
$$b \in B$$

But $A = B$

$\therefore \quad a \in A = B$ and $b \in B = A$

$\therefore \quad a \in B$ and $b \in A$

$\implies (a, b) \in B \times A \quad$ (by def'n)

$\therefore$ 🙂 $\in B \times A$

(ii) Show $\quad B \times A \subseteq A \times B$

Similar to (i)

Different method for ($\Leftarrow$)

WTS $A = B \implies A \times B = B \times A$

L.H.S. $A \times B$

$= B \times \boxed{B}$ since $A = B$

$= B \times \boxed{A}$ since $A = B$

$= $ R.H.S.