

APM 2663 Quiz 2

Fall 2024

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Time Limit: 45 minutes

Important:

- Please use the answer sheet to record your answer.
- Each question is worth 1 mark.
- Recall that the word *if* in a definition means *if and only if*.
- Recall that \mathbb{N} is the set of positive integers.
- Recall that \mathbb{Z} is the set of integers.
- Recall that \mathbb{Q} is the set of rational numbers.
- Recall that \mathbb{R} is the set of real numbers.
- Recall that \emptyset is the empty set.
- This is a closed book assessment. You may use a calculator.
- Cheating is a serious academic misconduct. Oakland University policy requires that all suspected instances of cheating be reported to the Office of the Dean of Students/Academic Conduct Committee for adjudication. I have forwarded cases to the Office of the Dean of Students/Academic Conduct Committee before and I will not hesitate to do this again if I suspect academic misconduct has occurred. Anyone found responsible of cheating in this assessment will receive a course grade of F, in addition to any penalty assigned by the Academic Conduct Committee.
- Discussion with anyone about this assessment prior to the release of the answers to the questions on this assessment by me will be considered as academic misconduct.

- (1) Write down your name and student number.
- (2) Consider a proof of “ $1 + 2 + 3 + \cdots + n = n(n + 1)/2$ for every integer $n \geq 1$ ” using mathematical induction. Which of the following step (a, b or c) is not necessary?
- (a) Check the formula for $n = 1$.
 - (b) Check the formula for $n = 2$.
 - (c) Assume the formula is correct for a fixed but arbitrary integer $k \geq 1$, and check that the formula is correct for the integer $k + 1$.
 - (d) The first three statements are all unnecessary.
 - (e) The first three statements are all necessary.
- (3) Which of the following count the number of surjections from a set of size m to a set of size n , assuming $m \geq n$?
- (a) m^n .
 - (b) n^m .
 - (c) $\binom{m}{n}n!$.
 - (d) $\binom{m}{n}$.
 - (e) None of the above.
- (4) In a standard deck of cards, six cards are selected. A 3-pair is a set of 6 cards where two of them have “number” X , two of them have “number” Y , and two of them have “number” Z , where X, Y, Z are distinct. For example, a hand with 2 of clubs, 2 of spades, 5 of hearts, 5 of diamonds, Q of clubs and Q of hearts form a 3-pair. How many 3-pair hands are there?
- (a) $\frac{13 \cdot 12 \cdot 11}{6} \binom{4}{2}^3$.
 - (b) $\frac{13 \cdot 12 \cdot 11}{3} \binom{4}{2}^3$.
 - (c) $13 \cdot 12 \cdot 11 \cdot 6 \binom{4}{2}^3$.
 - (d) $13 \cdot 12 \cdot 11 \cdot 3 \binom{4}{2}^3$.
 - (e) None of the above.
- (5) A pizza store has 32 ingredients available to make a pizza. The number of ways to choose five different ingredients to make two thin crust pizzas and five different ingredients to make a thick crust pizza, where an ingredient may be selected for at most one pizza, is
- (a) $2 \binom{32}{5} \binom{27}{5} \binom{22}{5}$.
 - (b) $\binom{32}{5} \binom{27}{5} \binom{22}{5} / 2$.
 - (c) $2 \binom{32}{5}^2 \binom{27}{5}$.

- (d) $\binom{32}{5}^2 \binom{27}{5} / 2$.
- (e) None of the above.
- (6) Find the number of solutions to $x_1 + x_2 + \cdots + x_{2n} = m$ such that x_i and i have the same parity and x_i is an integer for all i , assuming $m \geq n$. (So x_1 is odd, x_2 is even, x_3 is odd, and so on.)
- (a) $\binom{(m-n)/2+2n-1}{(m-n)/2}$.
- (b) $\binom{(m-n)/2+2n}{(m-n)/2}$.
- (c) $\binom{(m-n)/2+n-1}{(m-n)/2}$.
- (d) $\binom{(m-n)/2+n}{(m-n)/2}$.
- (e) None of the above.
- (7) The number of ways to distribute 100 identical balls to 5 distinct boxes such that every box has at least 10 balls is
- (a) $\binom{54}{3}$.
- (b) $\binom{54}{4}$.
- (c) $\binom{54}{5}$.
- (d) 1.
- (e) None of the above.
- (8) The number of ways to distribute 15 identical balls to 5 distinct boxes with no box empty such that no two boxes have the same number of balls is
- (a) $\binom{15}{4}$.
- (b) $\binom{9}{4}$.
- (c) $5!$.
- (d) 1.
- (e) None of the above.
- (9) The number of positive integers not exceeding 999 that are divisible by 7 or 13 is
- (a) 208.
- (b) 209.
- (c) 210.
- (d) 211.
- (e) None of the above.

- (10) If we use the Principle of Inclusion and Exclusion to expand $|A \cup B|$, we get 3 terms as $|A \cup B| = |A| + |B| - |A \cap B|$. How many terms will we get if we expand $|A_1 \cup A_2 \cup \dots \cup A_n|$?
- (a) $n^2 - 1$.
 - (b) $n + 1$.
 - (c) $2^n - 1$.
 - (d) $2^n - n + 1$.
 - (e) None of the above.
- (11) Which of the following (a-d) is not $O(n^7)$?
- (a) $2663n^4$.
 - (b) $2022n^3 \ln n$.
 - (c) $83689n^7 \ln n$.
 - (d) $n^7 + 62537n^2$.
 - (e) None of the above.