Ground set $= \mathbb{R}$

Let $a, b \in \mathbb{R}$. aRb iff $\lfloor a \rfloor = \lfloor b \rfloor$

Claim: R is reflexive.

Let $a \in \mathbb{R}$. Since $\lfloor a \rfloor = \lfloor a \rfloor$, ✓

R is reflexive.

Claim: R is symmetric.

Suppose aRb. Then $\lfloor a \rfloor = \lfloor b \rfloor$

$\Longrightarrow$ $\lfloor b \rfloor = \lfloor a \rfloor$ $\therefore bRa$

by properties of real numbers and $=$ sign.

Claim: R is transitive

Suppose aRb and bRc. Then

$\lfloor a \rfloor = \lfloor b \rfloor$ and $\lfloor b \rfloor = \lfloor c \rfloor$

$\Longrightarrow$ $\lfloor a \rfloor = \lfloor c \rfloor \therefore aRc$

since $=$ sign is transitive on real numbers

$\therefore$ R is an equivalence relation.

For the equiv. classes, $\lfloor a \rfloor$ is an integer say n. We want to know which a will gives $\lfloor a \rfloor = n$.

By the def'n of $\lfloor \; \rfloor$ function,

$$\lfloor a \rfloor = n \quad \text{iff} \quad a \in [n, n+1)$$

$\therefore$ The equiv. classes are

$$[n, n+1) \quad \forall \, n \in \mathbb{Z}$$