APM 2663
Fall 2024
Instructor: Eddie Cheng
Date: December 10, 2024

Important:

- Recall that the word *if* in a definition means *if and only if*.
- **To receive full credit for a question, you should provide all logical steps. All answers must be justified unless the questions stating otherwise.**
- Recall that $\mathbb{N}$ is the set of positive integers. The definition in the book includes 0.
- Recall that $\mathbb{Z}$ is the set of integers.
- Recall that $\mathbb{Q}$ is the set of rational numbers.
- Recall that $\mathbb{R}$ is the set of real numbers.
- This is a closed book examination. No external aids are allowed, except a calculator.
- Cheating is a serious academic misconduct. Oakland University policy requires that all suspected instances of cheating be reported to the Office of the Dean of Students/Academic Conduct Committee for adjudication. I have forwarded cases to the Office of the Dean of Students/Academic Conduct Committee before and I will not hesitate to do this again if I suspect academic misconduct has occurred. Anyone found responsible of cheating in this assessment will receive a course grade of F, in addition to any penalty assigned by the Academic Conduct Committee.
- I may ask for a meeting for you to explain your solutions.
- This test is worth 110 marks. If you receive x marks, your grade will be $\min\{x, 100\}\%$.

(1) Read the instructions and sign your name (in the space provided below) indicating that you have read the instructions. [1 mark]

(2) Write down your name. [1 mark]

(3) Find the gcd of ??? and !!! using the Euclidean Algorithm. Write the gcd as $???x+!!!y$ for some $x, y \in \mathbb{Z}$. [10 marks]

(4) Use mathematical induction to prove blah blah blah. [10 marks]

(5) Prove the following combinatorially. [15 marks]

(6) A relation question. [8 marks]

(7) A proof (not in graph theory) that I have done in class. [10 marks]

(8) A counting problem. [10 marks]

(9) A counting problem. [10 marks]

(10) Define $f : \mathbb{Z} \longrightarrow \mathbb{Z}$ by ???. Determine whether or not f is one-to-one and/or onto. [10 marks]

(11) A graph theory question with several parts. [10 marks]

(12) A proof in graph theory that I have done in class [15 marks]

Estimate your grade in this test. Let x be your guess. If your grade is in the interval $[x - 5, x + 5]$, you will receive 2 bonus marks.