APM 2663 Test 2
Fall 2024
Instructor: Eddie Cheng
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Important:

- Recall that the word *if* in a definition means *if and only if.*
- **To receive full credit for a question, you should provide all logical steps. All answers must be justified unless the questions stating otherwise.**
- Recall that $\mathbb{N}$ is the set of positive integers. The definition in the book includes 0.
- Recall that $\mathbb{Z}$ is the set of integers.
- Recall that $\mathbb{Q}$ is the set of rational numbers.
- Recall that $\mathbb{R}$ is the set of real numbers.
- This is a closed book examination. No external aids are allowed, except a calculator.
- Cheating is a serious academic misconduct. Oakland University policy requires that all suspected instances of cheating be reported to the Office of the Dean of Students/Academic Conduct Committee for adjudication. I have forwarded cases to the Office of the Dean of Students/Academic Conduct Committee before and I will not hesitate to do this again if I suspect academic misconduct has occurred. Anyone found responsible of cheating in this assessment will receive a course grade of F, in addition to any penalty assigned by the Academic Conduct Committee.
- I may ask for a meeting for you to explain your solutions.
- Until the solution to this test is posted/discussed by me, you may not discuss this test with others.
- This test is worth 110 marks. If you receive x marks, your grade will be $\min\{x, 100\}\%$.
- Solutions must be uploaded to Moodle unless otherwise arranged.

(1) Read the instructions and sign your name (in the space provided below) indicating that you have read the instructions. [1 mark]

(2) Write down your name and student number. [1 mark]

(3) Give a polynomial time algorithm, using the arithmetic model, to solve the following problem: Input n numbers, blah blah blah. [10 marks]

(4) Find the gcd of $\square$ and $\spadesuit$. Write the gcd as $\square x + \spadesuit y$ for some $x, y \in \mathbb{Z}$. [13 marks]

(5) Use mathematical induction to prove that

$$???. \quad [\text{15 marks}]$$

(6) A proof given in class. [10 marks]

(7) A type of problem that I said will be on the test. [10 marks]

(8) Evaluate

$$\sum ???$$

combinatorially. [20 marks].
(You may evaluate it algebraically for 12 marks.)

Note that: Here you do not know what one side is going to be. So you have to determine the "simpler" side and prove the identity. For example, instead of asking you to show

$$2^n = \sum_{k=0}^{n} \binom{n}{k},$$

the question is evaluate

$$\sum_{k=0}^{n} \binom{n}{k}.$$

(9) A counting problem. [15 marks]

(10) A counting problem. [15 marks]

(11) Estimate your grade in this test. Let x be your guess. If your grade is in the interval $[x - 5, x + 5]$, you will receive 2 bonus marks.