APM 2663

Fall 2024

Instructor: Eddie Cheng

Date: December 10, 2024

Important:

- Recall that the word *if* in a definition means *if and only if*.
- **To receive full credit for a question, you should provide all logical steps. All answers must be justified unless the questions stating otherwise.**
- Recall that $\mathbb{N}$ is the set of positive integers. The definition in the book includes 0.
- Recall that $\mathbb{Z}$ is the set of integers.
- Recall that $\mathbb{Q}$ is the set of rational numbers.
- Recall that $\mathbb{R}$ is the set of real numbers.
- This is a closed book examination. No external aids are allowed, except a calculator.
- Cheating is a serious academic misconduct. Oakland University policy requires that all suspected instances of cheating be reported to the Office of the Dean of Students/Academic Conduct Committee for adjudication. I have forwarded cases to the Office of the Dean of Students/Academic Conduct Committee before and I will not hesitate to do this again if I suspect academic misconduct has occurred. Anyone found responsible of cheating in this assessment will receive a course grade of F, in addition to any penalty assigned by the Academic Conduct Committee.
- I may ask for a meeting for you to explain your solutions.
- This test is worth 110 marks. If you receive x marks, your grade will be $\min\{x, 100\}\%$.

(1) Read the instructions and sign your name (in the space provided below) indicating that you have read the instructions. [1 mark]

1

(2) Write down your name. [1 mark]

1

(3) Find the gcd of ??? and !!! using the Euclidean Algorithm. Write the gcd as $???x+!!!y$ for some $x, y \in \mathbb{Z}$. [10 marks]

10

(4) Use mathematical induction to prove blah blah blah. [10 marks]

8

(5) Prove the following combinatorially. [15 marks] 10

(6) A relation question. [8 marks] 4

(7) A proof (not in graph theory) that I have done in class. [10 marks] 0

(8) A counting problem. [10 marks] 8

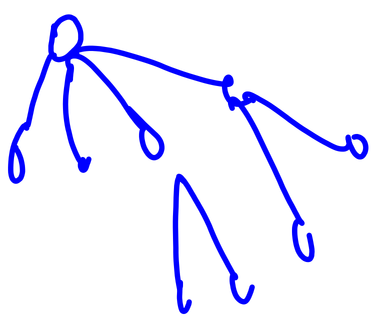
(9) A counting problem. [10 marks] 6

(10) Define $f : \mathbb{Z} \longrightarrow \mathbb{Z}$ by ???. Determine whether or not f is one-to-one and/or onto. [10 marks] 8

(11) A graph theory question with several parts. [10 marks] 6

(12) A proof in graph theory that I have done in class [15 marks] 12

Estimate your grade in this test. Let x be your guess. If your grade is in the interval $[x-5, x+5]$, you will receive 2 bonus marks.

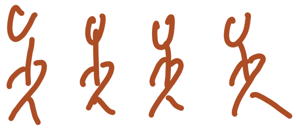


~~74~~ 68

C

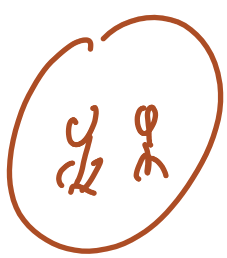
6n players

2 games



$$\underbrace{\text{4 people} \quad \text{4 people} \quad - \; - \quad \text{4 people}}_{n}$$

no group distinction



2 - - — 2

court 1, court 2 - -- court n

$$\frac{1}{n!}\underbrace{\binom{6n}{4}4!}_{\text{group "1"}}\underbrace{\binom{6n-4}{4}4!}_{\text{group "2"}}\cdots\underbrace{\binom{2n+4}{4}4!}_{\text{group "n"}}$$

$$\cdot\underbrace{\binom{2n}{2}}_{\text{group 1}}\underbrace{\binom{2n-2}{2}}_{\text{group 2}}\cdots\underbrace{\binom{2}{2}}_{\text{group n}}$$

$$\frac{1}{n!}\cancel{24^n}\frac{(6n)!}{\cancel{4!}\,\cancel{(6n-4)!}}\frac{\cancel{(6n-4)!}}{\cancel{4!}\,\cancel{(6n-8)!}}\cdots\frac{\cancel{(2n+4)!}}{\cancel{4!}\,\cancel{(2n)!}}$$

$$\cdot\frac{\cancel{(2n)!}}{2!\,\cancel{(2n-2)!}}\frac{\cancel{(2n-2)!}}{2!\,\cancel{(2n-4)!}}\frac{\cancel{(2n-4)!}}{2!\,\cancel{(2n-6)!}}\cdots\frac{\cancel{2!}}{2!\,0!}$$

$$\frac{1}{n!}\frac{(6n)!}{2^n}$$

80 numbers

$\binom{80}{10}$

80

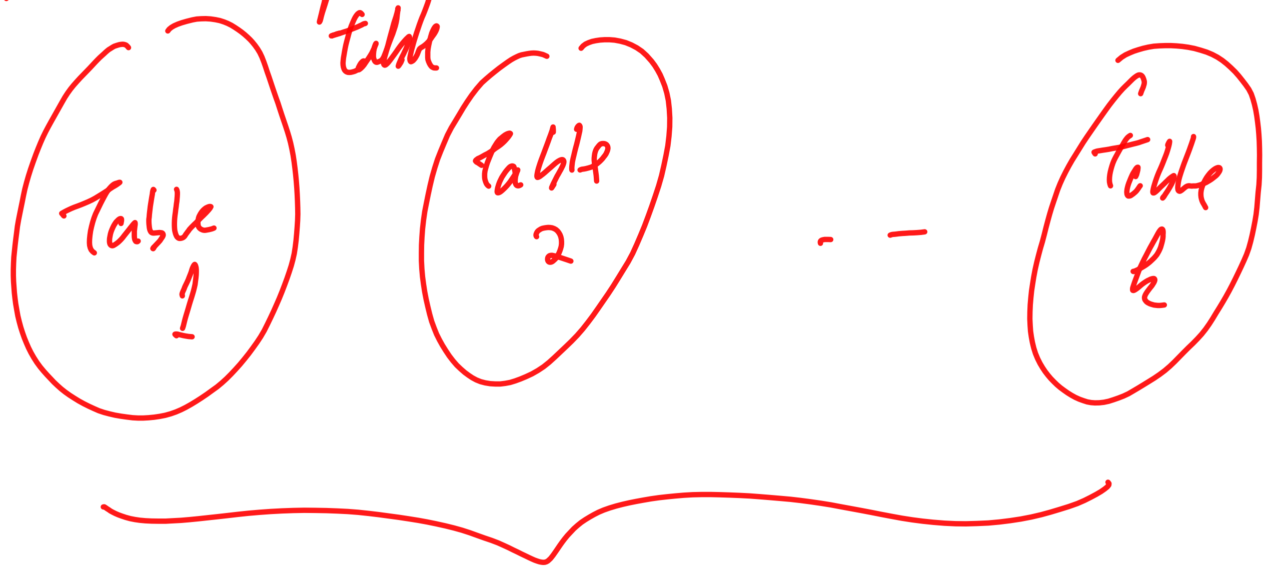
$\binom{80}{20}$

$\binom{20}{10}$

$$\binom{20}{8}\binom{60}{2} + \binom{20}{9}\binom{60}{1} + \binom{20}{10}$$

$\underbrace{\text{HWK}}_{\text{family}}$ n families

α families per table



k tables

HkW

WKH

$\frac{n}{k} = \alpha$

$\boxed{k \mid n}$

$$\binom{n}{\alpha}\underbrace{(\alpha-1)!\,2^{\alpha}}\binom{n-\alpha}{\alpha}\underbrace{(\alpha-1)!\,2^{\alpha}}$$

for table 1

$$\cdots\binom{\alpha}{\alpha}\underbrace{(\alpha-1)!\,2^{\alpha}}$$

$$\binom{n}{\alpha}\binom{n-\alpha}{\alpha}\cdots\binom{\alpha}{\alpha}\cdot(\alpha-1)!^{k}\,2^{n}$$

$$=\frac{n!}{\alpha!\,\cancel{(n-\alpha)!}}\;\frac{\cancel{(n-\alpha)!}}{\alpha!\,\cancel{(n-2\alpha)!}}\;\cancel{}\cdots\frac{\cancel{\alpha!}}{\alpha!\,0!}\,(\alpha-1)!^{k}\,2^{n}$$

$$=\frac{n!\,(\alpha-1)!^{k}\,2^{n}}{\alpha!^{k}}$$

$$\sum_{k=1}^{n} (k-1)^2 \binom{n}{k} 4^{k-2}$$

$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$$

$$\Rightarrow (1+x)^n = 1 + \sum_{k=1}^{n} \binom{n}{k} x^k$$

$$\Rightarrow (1+x)^n - 1 = \sum_{k=1}^{n} \binom{n}{k} x^k$$

$$\Rightarrow \frac{(1+x)^n - 1}{x} = \sum_{k=1}^{n} \binom{n}{k} x^{k-1}$$

$$\Rightarrow \frac{d}{dx}\left[\frac{(1+x)^n-1}{x}\right] = \sum_{k=1}^{n} (k-1)\binom{n}{k} x^{k-2}$$

$$\Rightarrow x\frac{d}{dx}\left[\frac{(1+x)^n-1}{x}\right] = \sum_{k=1}^{n} (k-1)\binom{n}{k} x^{k-1}$$

$$\Rightarrow \frac{d}{dx}\left[x\frac{d}{dx}\left[\frac{(1+x)^n-1}{x}\right]\right] = \sum_{k=1}^{n} (k-1)^2\binom{n}{k} x^{k-2}$$

$$\Rightarrow \frac{d}{dx}\left[x\frac{d}{dx}\left[\frac{(1+x)^n-1}{x}\right]\right]\Bigg|_{x=4} = \text{garbage that you want}$$

NOT ON EXAM

$$\sum_{k=1}^{n} k\binom{n}{k}4^k$$

$$(1+x)^n = \sum_{k=0}^{n}\binom{n}{k}x^k$$

diff wrt x

$$n(1+x)^{n-1} = \sum_{k=1}^{n} k\binom{n}{k}x^{k-1}$$

$$n\overset{4}{x}\,(\overset{5}{\cancel{1+x}})^{n-1} = \boxed{\sum_{k=1}^{n} k\binom{n}{k}4^k}$$

pick one to win a car

$x = 4$

n students pick some $[1,\ldots,n]$ to DW 4 t-shirts to pick from