APM 2663 Quiz 2

Fall 2024

Instructor: Eddie Cheng
Time Limit: 45 minutes

Important:

- Please use the answer sheet to record your answer.
- Each question is worth 1 mark.
- Recall that the word *if* in a definition means *if and only if*.
- Recall that $\mathbb{N}$ is the set of positive integers.
- Recall that $\mathbb{Z}$ is the set of integers.
- Recall that $\mathbb{Q}$ is the set of rational numbers.
- Recall that $\mathbb{R}$ is the set of real numbers.
- Recall that $\emptyset$ is the empty set.
- Cheating is a serious academic misconduct. Oakland University policy requires that all suspected instances of cheating be reported to the Office of the Dean of Students/Academic Conduct Committee for adjudication. I have forwarded cases to the Office of the Dean of Students/Academic Conduct Committee before and I will not hesitate to do this again if I suspect academic misconduct has occurred. Anyone found responsible of cheating in this assessment will receive a course grade of F, in addition to any penalty assigned by the Academic Conduct Committee.
- Discussion with anyone about this assessment prior to the release of the answers to the questions on this assessment by me will be considered as academic misconduct.

(1) Write down your name and student number.

(2) Consider a proof of "$1+2+3+\cdots+n = n(n+1)/2$ for every integer $n \geq 1$" using mathematical induction. Which of the following step (a, b or c) is not necessary?
  (a) Check the formula for $n = 1$.
  (b) Check the formula for $n = 2$.
  (c) Assume the formula is correct for a fixed but arbitrary integer $k \geq 1$, and check that the formula is correct for the integer $k+1$.
  (d) The first three statements are all unnecessary.
  (e) The first three statements are all necessary.

(3) Which of the following count the number of surjections from a set of size m to a set of size n, assuming $m \geq n$?
  (a) m^n.
  (b) n^m.
  (c) $\binom{m}{n} n!$.
  (d) $\binom{m}{n}$.
  (e) None of the above.

(4) In a standard deck of cards, six cards are selected. A 3-pair is a set of 6 cards where two of them have "number" X, two of them have "number" Y, and two of them have "number" Z, where X, Y, Z are distinct. For example, a hand with 2 of clubs, 2 of spades, 5 of hearts, 5 of diamonds, Q of clubs and Q of hearts form a 3-pair. How many 3-pair hands are there?
  (a) $\frac{13\cdot 12\cdot 11}{6}\binom{4}{2}^3$.
  (b) $\frac{13\cdot 12\cdot 11}{3}\binom{4}{2}^3$.
  (c) $13\cdot 12\cdot 11\cdot 6\binom{4}{2}^3$.
  (d) $13\cdot 12\cdot 11\cdot 3\binom{4}{2}^3$.
  (e) None of the above.

(5) A pizza store has 32 ingredients available to make a pizza. Each pizza must have 5 different ingredients. The number of ways to make two thin crust pizzas and a thick crust pizza, where an ingredient may be selected for at most one pizza, is
  (a) $2\binom{32}{5}\binom{27}{5}\binom{22}{5}$.
  (b) $\binom{32}{5}\binom{27}{5}\binom{22}{5}/2$.
  (c) $2\binom{32}{5}^2\binom{27}{5}$.
  (d) $\binom{32}{5}^2\binom{27}{5}/2$.

(e) None of the above.

(6) Find the number of solutions to $x_1 + x_2 + \cdots + x_{2n} = m$ such that x_i and i have the same parity and x_i is a non-negative integer for all i, assuming $m \geq n$. (So x_1 is odd, x_2 is even, x_3 is odd, and so on.)
(a) $\binom{(m-n)/2+2n-1}{(m-n)/2}$.
(b) $\binom{(m-n)/2+2n}{(m-n)/2}$.
(c) $\binom{(m-n)/2+n-1}{(m-n)/2}$.
(d) $\binom{(m-n)/2+n}{(m-n)/2}$.
(e) None of the above.

(7) The number of ways to distribute 100 identical balls to 5 distinct boxes such that every box has at least 10 balls is
(a) $\binom{54}{3}$.
(b) $\binom{54}{4}$.
(c) $\binom{54}{5}$.
(d) 1.
(e) None of the above.

(8) The number of ways to distribute 15 identical balls to 5 distinct boxes with no box empty such that no two boxes have the same number of balls is
(a) $\binom{15}{4}$.
(b) $\binom{9}{4}$.
(c) 5!.
(d) 1.
(e) None of the above.

(9) The number of positive integers not exceeding 999 that are divisible by 7 or 13 is
(a) 208.
(b) 209.
(c) 210.
(d) 211.
(e) None of the above.

(10) If we use the Principle of Inclusion and Exclusion to expand $|A \cup B|$, we get 3 terms as $|A \cup B| = |A| + |B| - |A \cap B|$. How many terms will we get if we expand $|A_1 \cup A_2 \cup \ldots \cup A_n|$?

(a) $n^2 - 1$.
(b) $n + 1$.
(c) $2^n - 1$.
(d) $2^n - n + 1$.
(e) None of the above.

(11) Which of the following (a-d) is not $O(n^7)$?
(a) $2663n^4$.
(b) $2022n^3 \ln n$.
(c) $83689n^7 \ln n$.
(d) $n^7 + 62537n^2$.
(e) None of the above.