APM 2663 Final Exam.
Fall 2020
Instructor: Eddie Cheng

Important:

- Recall that the word *if* in a definition means *if and only if*.
- Recall that $\mathbb{N}$ is the set of positive integers.
- Recall that $\mathbb{Z}$ is the set of integers.
- Recall that $\mathbb{Q}$ is the set of rational numbers.
- Recall that $\mathbb{R}$ is the set of real numbers.
- Recall that $\emptyset$ is the empty set.
- This is a closed book assessment. You may use a calculator.
- Cheating is a serious academic misconduct. Oakland University policy requires that all suspected instances of cheating be reported to the Office of the Dean of Students/Academic Conduct Committee for adjudication. I have forwarded cases to the Office of the Dean of Students/Academic Conduct Committee before and I will not hesitate to do this again if I suspect academic misconduct has occurred. Anyone found responsible of cheating in this assessment will receive a course grade of F, in addition to any penalty assigned by the Academic Conduct Committee.
- Discussion with anyone about this exam before December 16 will be considered as academic misconduct.
- This exam is worth 110 marks. If you receive x marks, your grade will be $\min\{x, 100\}\%$.

(1) Read the instructions and sign your name (in the space provided below) indicating that you have read the instructions. [1 mark]

(2) Write down your name and student number. [1 mark]

(3) Find the gcd of 3654 and 231. Write the gcd as $3654x + 231y$ for some $x, y \in \mathbb{Z}$. [8 marks]

(4) How many ways are there for Professor Aycil Cesmelioglu to distribute $4n$ distinct balls into n identical boxes such that every box has exactly 4 balls? [10 marks]

(5) Use mathematical induction to prove that

$$\sum_{i=1}^{n} \frac{1}{(3i-2)(3i+1)} = \frac{n}{3n+1} \text{ for } n \geq 1. \text{ [10 marks]}$$

(6) Let $n \geq 2$. The n-cube (or the hypercube of order n), H_n, is the graph whose vertices are the binary strings of length n. Two vertices are adjacent if and only if the two corresponding binary strings differ in exactly one bit position. Draw H_1, H_2, H_3. How many vertices and edges are there in H_n? For each value of n, determine whether H_n is Eulerian. For each value of n, determine whether H_n is Hamiltonian. [20 marks]

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(7) Evaluate

$$1^2\binom{n}{1}3 + 2^2\binom{n}{2}3^2 + 3^2\binom{n}{3}3^3 + \cdots + n^2\binom{n}{n}3^n$$

combinatorially. [15 marks].

(You may evaluate it algebraically for 10 marks.)

(8) How many ways are there for Professor Anna Spagnuolo to distribute 100 identical balls to 10 distinct boxes, 5 red boxes labelled 1 to 5 and 5 blue boxes labelled 1 to 5, such that the blue box with label i and the red box with label i receive the same number of balls, for every $i \in \{1, 2, 3, 4, 5\}$? [10 marks]

(9) Give an example of two non-isomorphic graphs with the same degree sequence. [5 marks]

(10) Let p be a prime number. Suppose a and b are integers. Prove that $p|ab$ implies $p|a$ or $p|b$ without using the Fundamental Theorem of Arithmeics. [15 marks]

(11) Define $f : \mathbb{Z} \longrightarrow \mathbb{Z}$ by $f(x) = 1219x^3 - 2020x$. Determine whether or not f is one-to-one and/or onto. [15 marks]

Estimate your grade in this test. Let x be your guess. If your grade is in the interval $[x-5, x+5]$, you will receive 2 bonus marks.