Examples of binary relations involving various properties

The following table gives examples of relations of various combinations in terms of reflexive, symmetric, transtitive and anti-symmetric. All examples are on the set $\{1, 2, 3\}$. To simplify the entries, we let $Z = \{(1, 1), (2, 2), (3, 3)\}$.

Reflexive	Symmetric	Transitive	Anti-symmetric	
Y	Y	Y	Y	Z
Y	Y	Y	N	$Z \cup \{(1, 2), (2, 1)\}$
Y	Y	N	Y	Not possible
Y	Y	N	N	$Z \cup \{(1, 2), (2, 1), (2, 3), (3, 2)\}$
Y	N	Y	Y	$Z \cup \{(1, 2)\}$
Y	N	Y	N	$Z \cup \{(1, 2), (2, 1), (1, 3), (2, 3)\}$
Y	N	N	Y	$Z \cup \{(1, 2), (2, 3)\}$
Y	N	N	N	$Z \cup \{(1, 2), (2, 3), (2, 1)\}$
N	Y	Y	Y	$\emptyset$
N	Y	Y	N	$\{(1, 2), (2, 1), (1, 1), (2, 2)\}$
N	Y	N	Y	Not possible
N	Y	N	N	$\{(1, 2), (2, 1)\}$
N	N	Y	Y	$\{(1, 2)\}$
N	N	Y	N	$\{(1, 2), (2, 1), (1, 1), (2, 3), (1, 3), (2, 2)\}$
N	N	N	Y	$\{(1, 2), (2, 3)\}$
N	N	N	N	$\{(1, 2), (2, 1), (2, 3)\}$

The following proposition covers the two "not possible" cases.

Proposition:

Let R be a relation on A. If R is symmetric and anti-symmetric, then R is transitive.

Proof: Let $(a, b), (b, c) \in R$. We want to show that $(a, c) \in R$. Since R is symmetric, $(b, a), (c, b) \in R$. Now $(a, b), (b, a) \in R$ implies $a = b$ as R is anti-symmetric. Hence $(a, c) = (b, c) \in R$ as required. $\square$