CHAPTER 1
LOGIC

SECTION 1.1 Propositions

1. **(a)** yes **(b)** yes **(c)** no (it is a command) **(d)** yes **(e)** yes
(f) yes **(g)** yes **(h)** no (it is a question) **(i)** yes
(j) no (it is a noun phrase) **(k)** yes **(l)** yes

3. **(b)** F **(f)** T (2 is even) **(i)** T (in fact he was over 6 feet tall) **(k)** T

5. **(a)** Paris is not the capital of Spain.

(b) $4 < 7$ or 13 is prime.

(c) If $4 < 7$, then both 13 is prime and Paris is the capital of Spain.

(d) $4 \not< 7$ or 13 is not prime.

(e) It is not the case that both $4 < 7$ and 13 is prime.

(f) Either $4 < 7$ implies that 13 is prime, or 13 is prime implies that Paris is the capital of Spain.

(g) $Q \wedge R$: 13 is prime and Paris is the capital of Spain.

(h) $\overline{P}$: $4 \not< 7$

(i) $Q \vee \overline{R}$: Either 13 is prime or Paris is not the capital of Spain.

7. In each case the final column of T's shows that the proposition is a tautology.

(a)

P	Q	$P \to Q$	$P \wedge (P \to Q)$	$(P \wedge (P \to Q)) \to Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

(b)

P	Q	$P \to Q$	$\overline{P}$	$Q \vee \overline{P}$	$(P \to Q) \leftrightarrow (Q \vee \overline{P})$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

(c)

P	Q	$\overline{P}$	$P \wedge \overline{P}$	$(P \wedge \overline{P}) \rightarrow Q$
T	T	F	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

9. **(a)** T if $x = 2$; F if $x = 3$

(d) T if $p = 17$; F if $p = 18$

(e) F if $x = 2$ and $y = 1$; never T since the right-hand side is always greater than the left-hand side

(g) F if He = Ronald Reagan; never T since that candidate was actually a "she"

(l) T if $N = 11$; F if $N = 10$

11. **(a)** $\mathrm{F} \rightarrow \mathrm{T}$ is T **(b)** $\overline{\mathrm{T}} \wedge \mathrm{T}$ is F **(c)** $\overline{\mathrm{T}} \vee \mathrm{F}$ is F

(d) $(\overline{\mathrm{T}} \vee \mathrm{F}) \wedge \overline{(\overline{\mathrm{T}} \wedge \mathrm{F})}$ is F **(e)** $\mathrm{T} \rightarrow \mathrm{T}$ is T **(f)** $\mathrm{T} \rightarrow \mathrm{T}$ is T

(g) $\mathrm{T} \rightarrow \mathrm{T}$ is T **(h)** $\mathrm{T} \leftrightarrow \mathrm{T}$ is T **(i)** $\mathrm{F} \rightarrow \mathrm{T}$ is T

13. If P is false, then $x \leq 2$, which implies that $x < 7$; this makes Q true. Therefore it is impossible to have both P and Q false. The other six possibilities are all actually possible. If $x = 5$ or $x = 6$, then P and Q are true, and R is true in the first case, false in the second. Similarly, if $x = 11$ or $x = 12$, then P is true and Q is false; and if $x = 1$ or $x = 2$, then P is false and Q is true.

15. **(a)** neither (the implication is T if and only if P is F)

(b) tautology ($\mathrm{T} \rightarrow \mathrm{T}$ is T, and $\mathrm{F} \rightarrow \mathrm{F}$ is T)

(c) tautology (if $P \wedge Q$ is T, then P and Q are both T, so $P \vee Q$ is T)

(d) neither (the proposition is F if P is T and Q is F; it is T if P is F)

(e) contradiction (if P is T, then $\neg(P \vee Q)$ is F; therefore $P \wedge \neg(P \vee Q)$ is always F)

(f) neither (the proposition is F if P is F and Q is T; it is T if P is T)

(g) neither (the proposition is F if P and Q have the same truth value; it is T if P and Q have opposite truth values)

(h) neither (the proposition is F if P and Q have opposite truth values; it is T if P and Q have the same truth value)

17. If Q is F, then $Q \wedge (P \vee \overline{R})$ is F. However $(Q \wedge P) \vee \overline{R}$ is T in this case as long as R is F.

19. We present this proof with a truth table.

P	Q	$P \to Q$	$\overline{P}$	$\overline{P} \to Q$	$(P \to Q) \wedge (\overline{P} \to Q)$
T	T	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

Observe that the second column has a T whenever the last column has a T. Thus $(P \to Q) \wedge (\overline{P} \to Q) \Longrightarrow Q$. In fact, these two columns are identical, so $(P \to Q) \wedge (\overline{P} \to Q) \Longleftrightarrow Q$. In everyday terms, we are saying: "If Q is true when P holds, and if Q is true when P does not hold, then Q is true."

21. This is a tautology. We will prove it using Theorems 1 and 3. (This fact can also be proved by using truth tables.) First, using various parts of Theorem 1 we have

$$\begin{aligned} P \to (Q \to R) &\Longleftrightarrow P \to (\overline{Q} \vee R) && \text{(implication)} \\ &\Longleftrightarrow \overline{P} \vee (\overline{Q} \vee R) && \text{(implication)} \\ &\Longleftrightarrow (\overline{P} \vee \overline{Q}) \vee R && \text{(associative law)} \\ &\Longleftrightarrow \overline{P \wedge Q} \vee R && \text{(DeMorgan's law).} \end{aligned}$$

Thus by Theorem 3b, $(P \to (Q \to R)) \leftrightarrow (\overline{P \wedge Q} \vee R)$ is a tautology.

23. **(a)** Let S be the proposition that you may swim here, let L be the proposition that you are less than six years old, and let P be the proposition that your parent is present. The lifeguard said $S \to (L \wedge P)$ and then $(\overline{L} \wedge \overline{P}) \to \overline{S}$.

(b) As we see from the following truth table, in every case in which $S \to (L \wedge P)$ is true, $(\overline{L} \wedge \overline{P}) \to \overline{S}$ is also true. However, in the third and fifth lines of the table, $(\overline{L} \wedge \overline{P}) \to \overline{S}$ is true, but $S \to (L \wedge P)$ is false.

L	P	S	$L \wedge P$	$S \to (L \wedge P)$	$\overline{L} \wedge \overline{P}$	$(\overline{L} \wedge \overline{P}) \to \overline{S}$
T	T	T	T	T	F	T
T	T	F	T	T	F	T
T	F	T	F	F	F	T
T	F	F	F	T	F	T
F	T	T	F	F	F	T
F	T	F	F	T	F	T
F	F	T	F	F	T	F
F	F	F	F	T	T	T

(c) She should have said: "If you're not less than six years old *or* your parent isn't present, then you may not swim in the pool." This is $(\overline{L} \vee \overline{P}) \to \overline{S}$. By the contrapositive law, this is logically equivalent to $\overline{\overline{S}} \to \overline{\overline{L} \vee \overline{P}}$, which is in turn equivalent to $S \to (L \wedge P)$ by DeMorgan's law and the double negative law.

25. **(a)** P is always true. It says the same thing as "If x and y are both positive, then $x \cdot y$ is positive."

(b) For $x \cdot y$ to be positive, it is sufficient that x and y both be positive.

(c) For $x \cdot y$ to be positive, it is necessary that x and y both be positive. For x and y both to be positive, it is sufficient that $x \cdot y$ be positive.

(d) The converse is false if, for example, $x = y = -1$, since in this case it is not true that x and y are both positive, but it is true that $x \cdot y$ is positive.

27. **(a)** First $42548 \div 3 = 14182\frac{2}{3}$, so 42548 is not divisible by 3; and $4+2+5+4+8=23$ is not divisible by 3. The proposition holds: F $\leftrightarrow$ F is T. Similarly, $121551 \div 3 = 40517$, so 121551 is divisible by 3; and $1+2+1+5+5+1=15$ is also divisible by 3. Again the proposition holds: T $\leftrightarrow$ T is T.

(b) A necessary and sufficient condition for a natural number to be divisible by 3 is that the sum of its digits be divisible by 3.

29. **(a)** The following table shows that Q (in column 2) is true whenever $P \wedge (P \to Q)$ (in column 4) is true, namely in line 1.

P	Q	$P \to Q$	$P \wedge (P \to Q)$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

(b) The following table shows that $P \vee Q$ (in column 3) is true whenever P (in column 1) is true, namely in lines 1 and 2.

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

(c) The following table shows that P (in column 1) is true whenever $P \wedge Q$ (in column 3) is true, namely in line 1.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

(d) The following table shows that P (in column 1) is true whenever $\overline{P} \to P$ (in column 3) is true, namely in line 1. In fact, these two propositions are logically equivalent.

P	$\overline{P}$	$\overline{P} \to P$
T	F	T
F	T	F

(e) The following table shows that P (in column 1) is true whenever $\overline{P} \to \mathrm{F}$ (in column 3) is true, namely in line 1. In fact, these two propositions are logically equivalent.

P	$\overline{P}$	$\overline{P} \to \mathrm{F}$
T	F	T
F	T	F

(f) Since F is never true, it holds vacuously that P is true whenever F is true.

(g) Since T is always true, it holds trivially that T is true whenever P is true.

(h) The following table shows that $P \to R$ (in column 7) is true whenever $(P \to Q) \wedge (Q \to R)$ (in column 6) is true, namely in lines 1, 5, 7, and 8.

P	Q	R	$P \to Q$	$Q \to R$	$(P \to Q) \wedge (Q \to R)$	$P \to R$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	F	T
T	F	F	F	T	F	F
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

31. Only (d) and (e) are logical equivalences; the truth tables shown in the solution to Exercise 29 show that in these two parts (and only in these two parts) the columns in question are identical.

33. **(a)** We need to have either that P is true and Q is false, or that P is false and Q is true. The most straightforward way to assert this is $(P \wedge \overline{Q}) \vee (\overline{P} \wedge Q)$.

(b) We simply assert the conjunction of the negations of P and Q: $\overline{P} \wedge \overline{Q}$.

(c) This time we want to assert that either P is false or Q is false; in symbols, $\overline{P} \vee \overline{Q}$.

35. $P \Longleftrightarrow Q$ means that every assignment of truth values to the propositional variables in P and Q results in the same truth value for P and for Q. By the definition of $\leftrightarrow$, this occurs if and only if $P \leftrightarrow Q$ is always true, that is, $P \leftrightarrow Q$ is a tautology.

37. **(a)** $(P \wedge Q) \vee R$; $\overline{P \vee Q} \wedge (\mathrm{T} \wedge Q)$

(b) If we make these replacements twice, then each of $\wedge$, $\vee$, T, and F returns to what it was originally.

(c) We show that $(P \wedge Q) \vee (P \wedge \overline{Q}) \iff P$ by the following argument using Theorem 1.

$$\begin{aligned} (P \wedge Q) \vee (P \wedge \overline{Q}) &\iff P \wedge (Q \vee \overline{Q}) && \text{(distributive law)} \\ &\iff P \wedge \mathrm{T} && \text{(complement law)} \\ &\iff P && \text{(identity law)} \end{aligned}$$

Dualizing this argument shows that the dual propositions are logically equivalent.

$$\begin{aligned} (P \vee Q) \wedge (P \vee \overline{Q}) &\iff P \vee (Q \wedge \overline{Q}) && \text{(distributive law)} \\ &\iff P \vee \mathrm{F} && \text{(complement law)} \\ &\iff P && \text{(identity law)} \end{aligned}$$

39. **(a)** The obvious thing to try first, since there is only one propositional variable here, is $P \uparrow P$. This works, since it follows immediately from the truth table definition that $P \uparrow P \iff \overline{P}$.

(b) Note from the given truth table that "nand," as its name suggests, is "not and" (the negation of the conjunction). By DeMorgan's law, $P \vee Q$ is the negation of the conjunction of $\overline{P}$ and $\overline{Q}$—in other words, it is the nand of $\overline{P}$ and $\overline{Q}$. Combining this observation with our result from part (a), we obtain $P \vee Q \iff (P \uparrow P) \uparrow (Q \uparrow Q)$.

(c) By the double negative law (and our observation in part (b) that nand is the negation of conjunction), we know that we want the negation of $P \uparrow Q$. But from part (a), this means $P \wedge Q \iff (P \uparrow Q) \uparrow (P \uparrow Q)$.

41. **(a)** The discussion of disjunctive normal form in Section 1.4 provides a solution (it follows the hint given here).

(b) Any use of $\vee$ in a proposition involving $\vee$, $\wedge$, and $\overline{}$ can be replaced by substituting $\overline{\overline{P} \wedge \overline{Q}}$ whenever we see an expression of the form $P \vee Q$. This follows from parts (h) and (i) of Theorem 1. Repeated applications of this substitution, from the inside of the expression outward, eliminates all uses of $\vee$. Thus $\{\wedge, \overline{}\}$ is complete. The dual reasoning (see Exercise 37) applies to eliminating $\wedge$, so $\{\vee, \overline{}\}$ is also complete.

(c) We know that $\{\vee, \overline{}\}$ is complete by part (b). From Exercise 39 we can replace each occurrence of $\overline{P}$ by $P \uparrow P$, one at a time, working from the inside out. This results in a proposition using only $\vee$ and $\uparrow$. Then by replacing each occurrence of $P \vee Q$ (one at a time, from the inside out) with $(P \uparrow P) \uparrow (Q \uparrow Q)$, we obtain a logically equivalent expression involving only $\uparrow$. A dual argument applies to $\downarrow$.

(d) $P \to Q \iff \overline{P} \vee Q \iff (P \uparrow P) \vee Q \iff [(P \uparrow P) \uparrow (P \uparrow P)] \uparrow (Q \uparrow Q)$

(e) $P \to Q \iff \overline{P} \vee Q \iff (P \downarrow P) \vee Q \iff [(P \downarrow P) \downarrow Q] \downarrow [(P \downarrow P) \downarrow Q]$

SECTION 1.2 Logical Quantifiers

1. **(a)** $\forall x\colon \big(x^2 > 4 \leftrightarrow (x > 2 \vee x < -2)\big)$

(b) $\exists x\colon x = x^2$

(c) $\forall x{>}1\colon \exists y\colon (y > x \wedge y < 2x)$

(d) $\exists x\colon \forall y\colon y^2 > x$

3. **(a)** There exists an x such that for every y, $x + y = y$.

(b) For every x there exists a y such that $x + y = y$.

(c) For every x and for every y, $x + y = y$.

(d) There exist x and y such that $x - y = y$.

(e) There exists an x such that for every y, $x - y = y$.

(f) For every x there exists a y such that $x - y = y$.

5. **(a)** There exist a and b, both greater than 1, whose product is x (i.e., x is composite).

(b) For every x there exists a y such that $x < y^2$.

(c) For every x there exists a y such that either $x = 3y$ or $x = 3y + 1$ or $x = 3y + 2$.

(d) For every x, if $x < 2$, then $x^2 < 4$.

(e) For every x, if $x^2 < 4$, then $x < 2$.

(f) There exists an x such that $x < 5$ implies $x < 3$.

(g) There exists an x such that $x^2 - 2x - 120 = 0$.

(h) For every x, $x^2 > a$.

7. **(a)** $\exists x\colon \exists y\colon (x \neq y \wedge M(\text{Diana}, x) \wedge M(\text{Diana}, y) \wedge F(\text{Charles}, x) \wedge F(\text{Charles}, y))$

(b) Assuming that we are interested in their joint offspring, we can express this with the unique existential quantifier symbol: $\exists! x\colon (M(\text{Suzanne}, x) \wedge F(\text{Jerry}, x))$.

(c) $\exists x\colon (M(x, \text{Pam}) \wedge F(\text{Sam}, x))$

(d) $\exists x\colon \exists y\colon \exists z\colon (M(x,y) \wedge M(x,z) \wedge F(y, \text{Pam}) \wedge F(z, \text{Conrad}))$

(e) $\exists x\colon \forall y\colon (\overline{M(x,y)} \wedge \overline{F(x,y)})$

9. **(a)** $\exists n\colon 100 = 5 \cdot n$

(b) $\neg \exists n\colon 1000 = 8 \cdot n$

(c) $\forall x\colon \big((\exists y\colon x = 6 \cdot y) \leftrightarrow \exists y\colon x = 2 \cdot y\big)$

(d) Note that this does not say, "Not every multiple of 3 is even." It says something stronger; in symbols, $\forall x\colon \big((\exists y\colon x = 3 \cdot y) \rightarrow \neg \exists y\colon x = 2 \cdot y\big)$.

(e) $\exists x\colon \forall y\colon \overline{x = y \cdot y}$

11. **(a)** T $(100 = 5 \cdot 20)$

(b) F $(1000 = 8 \cdot 125)$

(c) F (4 is a multiple of 2 but not a multiple of 6)

(d) F (6 is a multiple of 3 that is even)

(e) T (look at 2 or -25)

13. **(a)** neither (it depends on x)

(b) T (take $y = |x| + 1$)

(c) T (divide x by 3 and ignore the remainder to obtain y)

(d) F (take $x = -3$)

(e) T (if $x^2 < 4$, then $-2 < x < 2$)

(f) T (take $x = 7$)

(g) T (take $x = 12$)

(h) neither (it depends on a)

15. **(a)** 100 is not a multiple of 5.

(b) 1000 is a multiple of 8.

(c) Some multiple of 6 is not a multiple of 2, or some multiple of 2 is not a multiple of 6.

(d) Some multiple of 3 is even.

(e) Every number has a square root.

17. **(a)** Some perfect square is not less than 500.

(b) Every perfect square is not less than 500.

(c) Some perfect square is less than 500.

(d) Every perfect square is less than 500.

(e) Some perfect square is less than 500.

(f) Every perfect square is less than 500.

19. **(a)** F $(1^2 \neq 2)$

(b) F $(4^2 \neq 2)$

(c) T $(3^2 = 9)$

(d) F ($\sqrt{6}$ is not an integer; there is no positive integer solution to $x^2 = 6$)

(e) T (take $y = 36$)

(f) T (take $x = 36$; note that it is the position of x in the symbol $P(6, x)$ and not the fact that it is the letter x that is relevant)

(g) T (for each x, take $y = x^2$)

(h) F (if $y = 6$, no such x exists)

(i) F (no matter what y is, look at $x = y+1$; there are no real solutions to $(y+1)^2 = y$)

21. **(a)** $7921 > 1 \wedge \neg\exists x : \exists y : (7921 = x \cdot y \wedge x > 1 \wedge y > 1)$. This is false, since $7921 = 89 \cdot 89$.

(b) $\forall x : \left[(\exists y : x = y^2) \rightarrow \left(x \leq 1 \vee \exists a : \exists b : (x = a \cdot b \wedge a > 1 \wedge b > 1)\right)\right]$. This is true. If x is a perfect square, then either $x = 0$ or $x = 1$, or else x factors as $y \cdot y$, with $y > 1$. In the latter case, we let $a = y$ and $b = y$ in the second part of the proposition.

23. **(a)** $P(1,1) \wedge P(1,2) \wedge P(2,1) \wedge P(2,2)$

(b) $P(1,1) \vee P(1,2) \vee P(2,1) \vee P(2,2)$

(c) $\left(P(1,1) \vee P(1,2)\right) \wedge \left(P(2,1) \vee P(2,2)\right)$

(d) $\left(P(1,1) \wedge P(1,2)\right) \vee \left(P(2,1) \wedge P(2,2)\right)$

25. Most people would probably interpret this to mean $\forall x : \exists y : \exists t : L(x, y, t)$.

27. **(a)** We need to say that x and y are not the same, but x and y have the same parents: $x \neq y \wedge \exists m : \exists f : \left(C(x, m, f) \wedge C(y, m, f)\right)$.

(b) We need to make up existentially quantified variables for y's mother (p), father (q), and paternal grandmother (r) in order to express this: $\exists p : \exists q : \exists r : \left(C(y, p, q) \wedge C(q, r, x)\right)$.

(c) There are four ways in which x might have a grandchild, and we rule out each of them. In the following proposition, p is the (nonexistent) grandchild, q and r are its parents, and s is x's spouse: $\neg\exists p : \exists q : \exists r : \exists s : \left[C(p, q, r) \wedge \left(C(q, x, s) \vee C(q, s, x) \vee C(r, x, s) \vee C(r, s, x)\right)\right]$.

(d) Here we need to assert the existence of y's parents (m and f), one of whom is a sibling of x: $\exists m : \exists f : \exists a : \exists b : \left[\left(C(y, m, f) \wedge C(x, a, b) \wedge C(m, a, b) \wedge m \neq x\right) \vee \left(C(y, m, f) \wedge C(x, a, b) \wedge C(f, a, b) \wedge f \neq x\right)\right]$.

29. **(a)** These are equivalent. Both state that P and Q are always true, no matter what value x has.

(b) These are not equivalent. For example, let $P(x)$ be "x is odd," and let $Q(x)$ be "x is even." Then $\forall x : (P \vee Q)$ is true, but neither $\forall x : P$ nor $\forall x : Q$ is true.

(c) These are not equivalent. The same counterexample as in part (b) applies. The point is that the x's for the second proposition could be different.

(d) These are equivalent. Both state that there is some x for which either P or Q is true.

(e) These are not equivalent. Again, the same counterexample as in part (b) applies.

(f) These are not equivalent. For example, let $P(x)$ be $x = 3$, and let $Q(x)$ be $x \neq x$. Then the first proposition is true, but the second is false.

31. **(a)** $\forall n{>}2: \forall x: \forall y: \forall z: (x^n + y^n \neq z^n)$

(b) positive integers x, y, z, and n, with $n > 2$, such that $x^n + y^n = z^n$

33. The quantified proposition has no free variables, so it cannot mean that $x = 0$ (a statement about x). The proposition *means* that every number has an additive identity (possibly depending on the number). One can say that for every y, the one and only x that makes $x + y = y$ is $x = 0$.

35. We need to assert the existence of an x that makes P true and the fact that any y that makes P true is in fact this x. In symbols we have $\exists x: [P(x) \wedge \forall y: (P(y) \rightarrow y = x)]$.

37. This is false, since $41^2 - 41 + 41 = 41^2$ is not prime.

39. **(a)** $\forall f: \forall a: \big(f \text{ is continuous at } a \leftrightarrow \forall \epsilon{>}0: \exists \delta{>}0: \forall x: (|x - a| < \delta \rightarrow |f(x) - f(a)| < \epsilon)\big)$

(b) $\forall x: \forall p: (x \text{ is a quadratic residue modulo } p \leftrightarrow \exists y: \exists m: x - y^2 = pm)$

(c) $\forall f: \forall a: \forall b{>}a: \big[\big(f \text{ is continuous on } [a,b] \wedge f \text{ is differentiable on } (a,b) \wedge f(a) = 0 \wedge f(b) = 0\big) \rightarrow \exists c: \big(a < c < b \wedge f'(c) = 0\big)\big]$

(d) $\forall p: \forall q: \big(p \neq q \rightarrow \exists! l: (p \text{ is on } l \wedge q \text{ is on } l)\big)$

SECTION 1.3 Proofs

1. Let $2n$ be the given even number. Then $(2n)^2 = 4n^2 = 2(2n^2)$. Since this is 2 times some number, it is even.

3. **(a)** The product of two odd numbers is odd. *Proof*: Let $2n + 1$ and $2m + 1$ be the numbers. Their product is $(2n+1)(2m+1) = 4nm + 2n + 2m + 1 = 2(2nm + n + m) + 1$, which is odd by definition.

(b) The product of two even numbers is even. *Proof*: Let $2n$ and $2m$ be the numbers. Their product is $(2n)(2m) = 2(2nm)$, which is even by definition.

(c) The product of an even number and an odd number is even. *Proof*: Let $2n$ and $2m + 1$ be the numbers. Their product is $(2n)(2m + 1) = 2(2nm + n)$, which is even by definition.

5. **(a)** Let $x = 6n$ be the given number. Then $x = (3 \cdot 2)n = 3(2n)$, so x is a multiple of 3.

(b) This is false. Look at 9, for example; it is a multiple of 3 but not a multiple of 6.

(c) We give an indirect proof, by proving the equivalent proposition, "If x is a multiple of 6, then x is a multiple of 2." Let $x = 6n$ be the given number. Then $x = (2 \cdot 3)n = 2(3n)$, so x is a multiple of 2.