APM 2663 Quiz 1

Fall 2024

Instructor: Eddie Cheng
Time Limit: 45 minutes

Important:

- Each question is worth 1 mark.
- Recall that the word *if* in a definition means *if and only if*.
- You may use a calculator.
- Recall that $\mathbb{N}$ is the set of natural numbers, that is, the set of positive integers.
- Recall that $\mathbb{Z}$ is the set of integers.
- Recall that $\mathbb{Q}$ is the set of rational numbers.
- Recall that $\mathbb{R}$ is the set of real numbers.
- Recall that $\varnothing$ is the empty set.
- Cheating is a serious academic misconduct. Oakland University policy requires that all suspected instances of cheating be reported to the Academic Conduct Committee for adjudication. I have forwarded cases to the Office of the Dean of Students/Academic Conduct Committee before and I will not hesitate to do this again if I suspect academic misconduct has occurred. Anyone found responsible of cheating in this exam will receive a course grade of F, in addition to any penalty assigned by the Academic Conduct Committee.
- Discussion with anyone about this quiz prior to the release of the answers to the questions on this quiz by me will be considered as academic misconduct.
- Write down your answers on the provided answer sheet.

(1) Write down your name. By writing down your name, you acknowledge that you have read the instructions and you will adhere to them.

(2) Consider the following statements: (1) For every $x \in \mathbb{R} - \{0\}$, there exists a $y \in \mathbb{R}$ such that $\pi y/x = 2024$; (2) There exists an $x \in \mathbb{R} - \{0\}$ such that for every $y \in \mathbb{R}$, $\pi y/x = 2024$; (3) There exist an $x \in \mathbb{Z}$ and a $y \in \mathbb{Z}$ such that $x \neq y$ and $y/x = 2024$. Which of the following is correct?
   (a) Only (1) is correct.
   (b) Only (2) is correct.
   (c) Only (3) is correct.
   (d) Only (1) and (2) are correct.
   (e) Only (1) and (3) are correct.

(3) Consider the following statements: (1) For every $x \in \mathbb{N}$, there exists a $y \in \mathbb{N}$ such that $x + y = 2024$; (2) There exists an $x \in \mathbb{N}$ such that for every $y \in \mathbb{Z}$, $x + y = 2024$; (3) For every $x \in \mathbb{Z}$, there exists a $y \in \mathbb{Z}$ such that $x + y = 2024$. Which of the following is correct?
   (a) Only (1) is correct.
   (b) Only (2) is correct.
   (c) Only (3) is correct.
   (d) Only (1) and (2) are correct.
   (e) Only (1) and (3) are correct.

(4) Let $A = \{\varnothing\}$, $B = \varnothing$ and $C = \{\☺\}$. Consider the following statements: (1) $A \cup C = B \cup C$; (2) $B \in A$; (3) $B \subseteq A$; (4) $A \subseteq C$; (5) $A \cap C = B \cap C$. Which of the following is correct?
   (a) Only (1) and (2) are not correct.
   (b) Only (1) and (3) are not correct.
   (c) Only (1) and (4) are not correct.
   (d) Only (2) and (4) are not correct.
   (e) Only (2) and (5) are not correct.

(5) Let $A = \{\☺, \☹, \longrightarrow\}$ and $B = \{1, 2, 3\}$. Consider the following statements: (1) $A \times B$ has 9 elements; (2) $A \cap B$ has 1 element; (3) $A \cup B$ has 5 elements; (4) The power set

of A is the same as the power set of B; (5) The symmetric difference of A and B is $A \cup B$. Which of the following is correct?

(a) Only (1) and (5) are correct.
(b) Only (1) and (4) are correct.
(c) Only (1) and (3) are correct.
(d) Only (1) and (2) are correct.
(e) Only (1) is correct.

(6) Let $A = \{\ddot{\smile}, \ddot{\frown}, 1, 2, 3, 4, 5\}$ and $B = \{\square, \heartsuit, x, y, z\}$. Consider the following statements: (1) There exists a surjective function from A to B; (2) There exists an injective function from A to B; (3) There exists no bijective functions from A to B. Which of the following is correct?

(a) Only (1) is correct.
(b) Only (2) is correct.
(c) Only (3) is correct.
(d) Only (1) and (3) are correct.
(e) Only (2) and (3) are correct.

(7) Let A, B and C be sets in some universal set. Which of following is not correct.

(a) $\overline{A \cup B} = \overline{A} \cup \overline{B}$.
(b) $A \cap \overline{A} = B \cap \overline{B}$.
(c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
(d) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.
(e) $A \cup \overline{A} = B \cup \overline{B}$.

(8) Let $A \neq \varnothing$ be a set. Consider the following statements: (1) $\varnothing$ is a symmetric binary relation on A; (2) $\varnothing$ is an anti-symmetric binary relation on A; (3) $\varnothing$ is a transitive binary relation on A; Which of the following is correct?

(a) Only (1) and (3) are correct.
(b) Only (1) and (2) are correct.
(c) Only (2) and (3) are correct.
(d) None is correct.
(e) All are correct.

(9) Consider the following statements: (1) If 99 is prime, then $\int_0^2 x^2 \, dx = 5$; (2) If 99 is composite, then $1 + 1 = 2$; (3) If 99 is prime, then $1 + 1 = 3$. Which of the following is correct?

(a) Only (1) and (3) are correct.

(b) Only (1) and (2) are correct.
(c) Only (2) and (3) are correct.
(d) None is correct.
(e) All are correct.

(10) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = 2663x^{12} + 2024$. Which of the following is correct?
(a) f is not a function.
(b) f is a function but is neither injective nor surjective.
(c) f is injective but not surjective.
(d) f is surjective but not injective.
(e) f is injective and surjective.