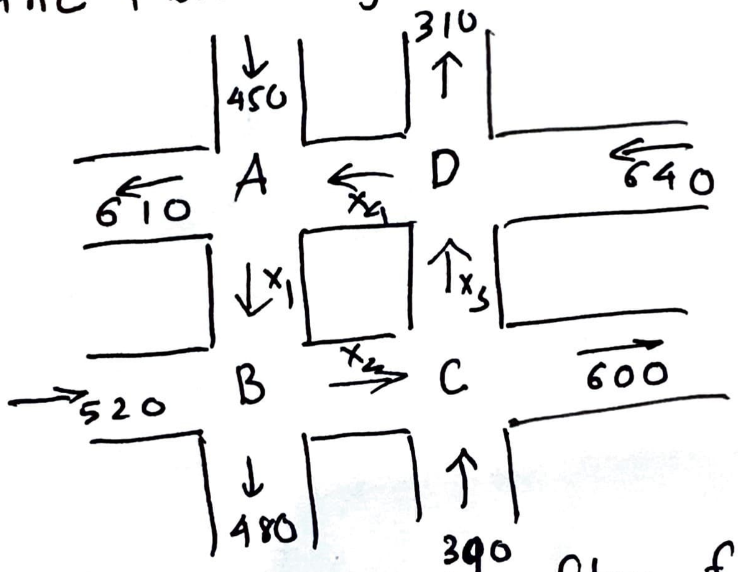
1.2 (cont)

Applications

① Traffic flow : In a down town of some city, the traffic flow is given in the following diagram.



We want to determine the amount of traffic between each intersection

Let x_1 be the traffic flow from A → B

x_2 ____________ B → C

x_3 ____________ C → D

x_4 ____________ D → A

total inflow = total outflow:

At A : $450 + x_4 = 610 + x_1$

$x_1 - x_4 = -160$

At B : $520 + x_1 = 480 + x_2$

$x_1 - x_2 = -40$

At C : $390 + x_2 = 600 + x_3$

$x_2 - x_3 = \cancel{300}\ 210$

$A+D: \quad 640 + x_3 = 310 + x_4.$

$$x_3 - x_4 = -330.$$

Our linear system:

$$\begin{cases} x_1 - x_4 = -160 \\ x_1 - x_2 = -40 \\ x_2 - x_3 = 210 \\ x_3 - x_4 = -330 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -160 \\ 1 & -1 & 0 & 0 & -40 \\ 0 & 1 & -1 & 0 & 210 \\ 0 & 0 & 1 & -1 & -330 \end{array}\right) \xrightarrow{R_2 \to R_2 - R_1} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -160 \\ 0 & -1 & 0 & 1 & 120 \\ 0 & 1 & -1 & 0 & 210 \\ 0 & 0 & 1 & -1 & -330 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 + R_2} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -160 \\ 0 & -1 & 0 & 1 & 120 \\ 0 & 0 & -1 & 1 & 330 \\ 0 & 0 & 1 & -1 & -330 \end{array}\right) \xrightarrow{R_4 \to R_4 + R_3} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -160 \\ 0 & -1 & 0 & 1 & 120 \\ 0 & 0 & -1 & 1 & 330 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \end{array}$$

x_1, x_2, x_3 : leading variables

x_4 : free variable. $\to$ set $x_4 = t$.

$R_3: \quad -x_3 + x_4 = 330 \to x_3 = t - 330.$

$R_2: \quad -x_2 + x_4 = 120 \to x_2 = t - 120$

$R_1: \quad x_1 - x_4 = -160 \to x_1 = t - 160.$

$x = (t-160, t-120, t-330, t)$

$t \geq 330$, t is ~~intger~~ integer.

Chemical Reaction:

In the process of photosynthesis, plants use radiant energy from sunlight to convert carbon dioxide (CO_2) and water (H_2O) into glucose ($C_6H_{12}O_6$) and oxygen (O_2). The equation for this chemical reaction:

$$x_1\, CO_2 + x_2\, H_2O \longrightarrow x_3\, C_6H_{12}O_6 + x_4\, O_2$$

For C: $x_1 = 6x_3 \rightarrow x_1 - 6x_3 = 0.$

For O: $2x_1 + x_2 = 6x_3 + 2x_4 \rightarrow 2x_1 + x_2 - 6x_3 - 2x_4 = 0.$

For H: $2x_2 = 12x_3 \rightarrow 2x_2 - 12x_3 = 0.$

$$\left(\begin{array}{cccc|c} 1 & 0 & -6 & 0 & 0 \\ 2 & 1 & -6 & -2 & 0 \\ 0 & 2 & -12 & 0 & 0 \end{array}\right) \xrightarrow{R_2 \to R_2 - 2R_1} \left(\begin{array}{cccc|c} 1 & 0 & -6 & 0 & 0 \\ 0 & 1 & 6 & -2 & 0 \\ 0 & 2 & -12 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 - 2R_2} \left(\begin{array}{cccc|c} 1 & 0 & -6 & 0 & 0 \\ 0 & 1 & 6 & -2 & 0 \\ 0 & 0 & -24 & 4 & 0 \end{array}\right) \xrightarrow{R_3 \to \frac{R_3}{-24}} \left(\begin{array}{cccc|c} 1 & 0 & -6 & 0 & 0 \\ 0 & 1 & 6 & -2 & 0 \\ 0 & 0 & 1 & -\frac{1}{6} & 0 \end{array}\right)$$

row-echelon.

$$\xrightarrow[R_1 \to R_1 + 6R_3]{R_2 \to R_2 - 6R_3} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{6} & 0 \end{array}\right)$$

$\underbrace{\qquad\qquad}_{\text{reduced}}$

<u>free variable</u>: $x_4 = t$.

R_1: $x_1 - x_4 = 0 \to x_1 = t$

R_2: $x_2 = t$

R_3: $x_3 = \frac{1}{6} t$.