Ch after I: Matrix and Systems of Equations

Tuesday, January 7, 2025 6:08 PM

1.1 Systems of Linear Equations.

- $y = mx + b$ is a linear equation
- Linear system:

$$x - y = 2 \quad (1)$$
$$2x + y = 7 \quad (2)$$

(system of linear equations)

Method of substitution:

- Solve for x from (1): $x = y + 2$
- Substitute it into (2):

$$2(y+2) + y = 7$$
$$\underbrace{2y} + 4 + \underbrace{y} = 7$$
$$3y + 4 = 7$$
$$3y = 3$$
$$\left.\begin{aligned} y &= 1 \\ x &= 1+2 \\ x &= 3 \end{aligned}\right\} \rightarrow (x, y) = (3, 1).$$

(●) Method of elimination:

- Eliminate y by adding (1) to (2):

$$3x = 9$$
$$x = 3$$

- Substitution back to (1):

$$3 - y = 2$$

$$y = 1$$

$\rightarrow (x, y) = (3, 1)$ is the solution of our system.

The system of linear equations has the following format:

$$\left.\underbrace{\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \cdots\cdots & \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}}\right\} \begin{matrix} m \text{ equations} \\ (L) \end{matrix}$$

n variables: $x_1, x_2, \ldots, x_n$.

a_{ij} $(1 \le i \le m, 1 \le j \le n)$ are known coefficients.

The system is called consistent if it has a solution $(x_1, x_2, \ldots, x_n)$ satisfying the system (L).

——— inconsistent ——— does not have a solution.

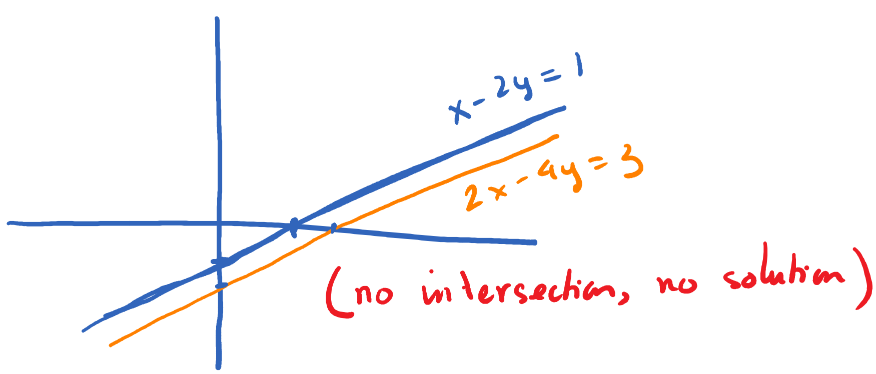
ex: $x - 2y = 1$ ①
$2x - 4y = 3$ ②

$\xrightarrow{\times(-2)}$ $+\begin{cases} -2x + 4y = -2 \\ 2x - 4y = 3 \end{cases}$

$0x + 0y = 1$

$0 = 1$

(not possible, no solution)



③ Solve $x_1 + 2x_2 + x_3 = 3$ ①
$3x_1 - x_2 - 3x_3 = -1$ ②

equivalent

$$3x_1 - x_2 - 3x_3 = -1 \quad (2)$$
$$2x_1 + 3x_2 + x_3 = 4 \quad (3)$$

equivalent

Elimination:

$-3(1) + (2)$:

$$\begin{array}{rrrcl} & -3x_1 & -6x_2 & -3x_3 & = -9 \\ + & 3x_1 & -x_2 & -3x_3 & = -1 \\ \hline & & -7x_2 & -6x_3 & = -10 \end{array} \quad (2_{new})$$

$-2(1) + (3)$:

$$\begin{array}{rrrcl} & -\cancel{2x_1} & -4x_2 & -2x_3 & = -6 \\ + & \cancel{2x_1} & +3x_2 & +x_3 & = 4 \\ \hline & & -x_2 & -x_3 & = -2 \end{array} \quad (3_{new})$$

$(2_{new}) - 7(3_{new})$:

$$\begin{array}{rrrcl} & -7x_2 & -6x_3 & = -10 \\ - & -7x_2 & -7x_3 & = -14 \\ \hline & & x_3 & = 4 \end{array}$$

Back substitution:

- Substitute x_3 into (3_{new}):

$$-x_2 - 4 = -2$$
$$-x_2 = 2$$
$$x_2 = -2$$

- Substitute x_2, x_3 into (1):

$$x_1 + (-4) + 4 = 3$$
$$x_1 = 3$$

$(x_1, x_2, x_3) = (3, -2, 4)$ is the solution.

We have 3 elementary operations for system of linear equations :

① Interchange 2 equations

② Multiply an equation by a nonzero number.

③ Replace an equation by its sum with a multiple of another equation.