MTH 2775 **Sample Exam I** (8 problems, 105 points) Name: ____________________

1. (3 points for each) Answer whether the following statements are true or false. Explain shortly your answers.

   (a) Let A be an $n \times n$ matrix and b be $n \times 1$ column vector. System $Ax = b$ has a solution $x = A^{-1}b$.

   (b) There exists a nonsigular 2×2 matrix A with $A^{-1} = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$.

   (c) The following system

   $$\left[\begin{array}{ccc|c} 1 & -3 & 0 & 3 \\ 0 & 1 & 2 & -1 \end{array}\right]$$

   has infinitely many solutions.

   (d) This set $S = \{x \in \mathbb{R}^3 | \; x_1 - 2x_3 = 3x_4\}$ is a subspace of $\mathbb{R}^3$.

   (e) Let A and B be two 3×3 matrices. Then $\det(A + B) = \det A + \det B$.

2. (5 for each) Let $A := \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$, $B := \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$. Find each of the following items. If an item does not exist, say "DNE".

(a) AB and BA.

(b) $\det(A)$ and $\det(B)$.

(c) How many solutions to the linear system $Ax = 0$ are there? How many solutions to the linear system $By = 0$ are there? Explain in details your answer for full credit.

3. Given $A = \begin{bmatrix} 1 & 3 & 2 & 1 \\ 1 & 4 & 3 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(a) (7.5 pts) Find $\det(A)$.

(b) (7.5 pts) Find A^{-1}.

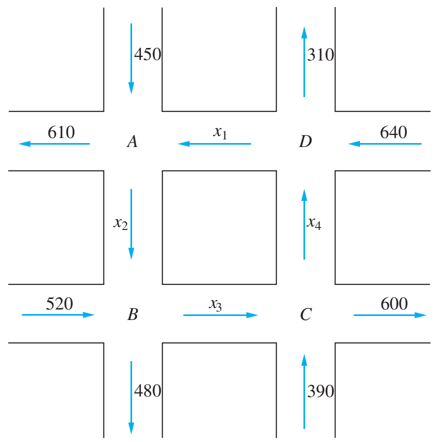
(c) (5 pts) Using (b) above, solve the system $Ax = b$ with $b = (1, 2, 3, 4)^T$.

4. Consider the linear system: $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 1 & 4 & 2 & 4 \\ 0 & 6 & a^2 & a \end{array}\right]$

(a) (5 pts) Find all values of a such that the above system a unique solution.

(b) (5 pts) Find all values of a such that the above system has infinitely many solutions. **Find all these solutions in this case.**

5. (10 pts) In the downtown section of a certain city, two sets of one-way street intersect as shown below. The average hourly volume of traffic entering and leaving this section during rush hour is also given in the below diagram. Determine the amount of traffic between each of the four intersections.



6. Let A be an $n \times n$ matrix with $A^2 = 4I$.

(a) (5 pts) Verify that A is nonsingular and $A^{-1} = \frac{1}{4}A$.

(b) (5 pts) Show that $A - 2I$ is nonsingular and $(A - I)^{-1} = \frac{1}{3}(A + I)$.

7. (7.5 pts for each) Determine whether the following sets are subspaces:

(a) $X = \{f \in C^1(0,1) |\ f(x) - 2f'(x) = 0\}$, where $C^1(0,1)$ is the set of all continuously differentiable functions in $(0,1)$.

(b) $Y = \{A \in \mathbb{R}^{3\times 3} |\ \det(A) = 0\}$.

8. (10 points) Let A be an $m \times n$ matrix. Explain why $A^T A$ and AA^T are possible.

1. (a) F (b) F (c) T (d) T (e) F.

2. (a) AB DNE and $BA = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$.

(b) $\det A = -1$ and $\det B$ DNE.

(c) $Ax = 0$ has a unique solution. $By = 0$ has infinitely many solutions.

3. $\det A = 1$ and $A^{-1} = \begin{pmatrix} 3 & -2 & -1 & 1 \\ -1 & 1 & 0 & -1 \\ 1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 0 \end{pmatrix}$.

4. (a) $a \neq 2, -2$ (b) $a = 2$.

5. $(x_1, x_2, x_3, x_4) = (t + 330, t + 170, t + 210, t)$.

7. (b) Verify $(A - I)(A + I) = 3I$.

8. (a) Yes. (b) No