MTH 2775 Quiz 3:

① Solve
$$x_1 - 3x_2 - 10x_3 + 5x_4 = 0$$
$$x_1 + 4x_2 + 11x_3 - 2x_4 = 0$$
$$x_1 + 3x_2 + 8x_3 - x_4 = 0$$

Set S be the set of all solutions. Find a basis of S and find its dimension

$$\left(\begin{array}{cccc|c} 1 & -3 & -10 & 5 & 0 \\ 1 & 4 & 11 & -2 & 0 \\ 1 & 3 & 8 & -1 & 0 \end{array}\right) \xrightarrow[R_3 - R_1]{R_2 - R_1} \left(\begin{array}{cccc|c} 1 & -3 & -10 & 5 & 0 \\ 0 & 7 & 21 & -7 & 0 \\ 0 & 6 & 18 & -6 & 0 \end{array}\right)$$

$$\xrightarrow{R_2/7} \left(\begin{array}{cccc|c} 1 & -3 & -10 & 5 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 6 & 18 & -6 & 0 \end{array}\right) \xrightarrow{R_3 - 6R_2} \left(\begin{array}{cccc|c} 1 & -3 & -10 & 5 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{R_1 + 3R_2} \left(\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 3 & -1 & 0 \end{array}\right)$$

$x_3 = t$, $x_4 = s$
$x_2 = -3t + s$, $x_1 = t - 2s$

$$\longrightarrow S = \left\{ \begin{pmatrix} t-2s \\ -3t+s \\ t \\ s \end{pmatrix} \right\} = \left\{ t \begin{pmatrix} 1 \\ -3 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} -2 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\} = \mathrm{span} \left\{ \begin{pmatrix} 1 \\ -3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$\rightarrow \text{A basis of } S = \begin{pmatrix} 1 \\ -3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$\dim S = 2$

2. (6 pts) Determine whether the following vectors are linearly independent in $\mathbb{R}^{2\times 2}$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad C = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

Set $c_1 A + c_2 B + c_3 C = 0$

$$\begin{pmatrix} c_1 + c_2 & c_2 \\ c_2 + c_3 & c_1 \end{pmatrix} = 0 \rightarrow \begin{matrix} c_1 + c_2 = 0 \\ c_2 = 0 \\ c_2 + c_3 = 0 \\ c_1 = 0 \end{matrix} \quad \begin{matrix} \rightarrow c_1 = 0 \\ \rightarrow c_3 = 0 \end{matrix}$$

So $c_1 = c_2 = c_3 = 0$.

A, B, C are L. I.

3. (6 pts) Find a basis of $S = \{ax^2 - 3bx + 2a - b \mid a, b \in \mathbb{R}\}$, which is a subspace of P_3 and then find its dimension.

$S = \{a(x^2+2) - b(3x+1) \mid a, b \in \mathbb{R}\}$

$= \text{span}\{(x^2+2), 3x+1\}$.

$x^2+2, 3x+1$ are L. I. as if:

$\rightarrow a(x^2+2) - b(3x+1) = 0$

$\rightarrow ax^2 - 3bx + 2a - b = 0$

$\rightarrow a = 0, b = 0$.

Thus a basis of S is $x^2+2, 3x+1$ and $\dim S = 2$.