MTH 2775 **Quiz I (2 problems, 20 points)** **Name:**_____

Directions: Show all work for full credit. Using calculators to make matrix operations is prohibited.

1. (10 points) Use **Gaussian method** and back substitution to solve the following linear system:

$$\begin{aligned} x_1 + 2x_2 - 3x_3 + x_4 &= 2 \\ -x_1 - x_2 + 4x_3 + x_4 &= 1 \\ 2x_1 + 4x_2 - 7x_3 - x_4 &= 3 \\ -3x_1 + 2x_2 + 3x_3 - 29x_4 &= 4 \end{aligned}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 2 \\ -1 & -1 & 4 & 1 & 1 \\ 2 & 4 & -7 & -1 & 3 \\ -3 & 2 & 3 & -29 & 4 \end{array}\right) \xrightarrow[R_3 \to R_3 - 2R_1,\ R_4 \to R_4 + 3R_1]{R_2 \to R_2 + R_1} \left(\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 2 \\ 0 & +1 & 1 & 2 & 3 \\ 0 & 0 & -1 & -3 & -1 \\ 0 & 8 & -6 & -26 & 10 \end{array}\right)$$

$$\xrightarrow{R_4 \to R_4 - 8R_2} \left(\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 2 \\ 0 & +1 & 1 & 2 & 3 \\ 0 & 0 & -1 & -3 & -1 \\ 0 & 0 & -14 & -42 & -14 \end{array}\right) \xrightarrow{R_4 \to R_4 - 14R_3} \left(\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 2 \\ 0 & 1 & 1 & 2 & 3 \\ 0 & 0 & -1 & -3 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Set $x_4 = t$ (free variable)

- From R_3: $-x_3 - 3x_4 = -1 \rightarrow x_3 = 1 - 3t$
- From R_2: $x_2 + x_3 + 2x_4 = 3$

$$x_2 + 1 - 3t + 2t = 3$$

$$x_2 = 2 + t$$

- From R_1: $x_1 + 2x_2 - 3x_3 + x_4 = 2$

$$x_1 + 4 + 2t - 3 + 9t + t = 2$$

$$x_1 = 1 - 12t.$$

$$X = \begin{pmatrix} 1 - 12t \\ 2 + t \\ 1 - 3t \\ t \end{pmatrix}, \quad t = \text{parameter}$$

2. (10 points) Use **Gauss-Jordan** elimination method to solve the following linear system

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= 0 \\ 2x_1 + 3x_2 + 5x_3 + 7x_4 &= 0 \\ 4x_1 + 6x_2 + 8x_3 + 9x_4 &= 0 \\ 3x_1 + 5x_2 + 8x_3 + 11x_4 &= 0 \end{aligned}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 2 & 3 & 5 & 7 & 0 \\ 4 & 6 & 8 & 9 & 0 \\ 3 & 5 & 8 & 11 & 0 \end{array}\right) \xrightarrow[R_3 - 4R_1,\ R_4 - 3R_1]{R_2 - 2R_1} \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & -1 & -1 & -1 & 0 \\ 0 & -2 & -4 & -7 & 0 \\ 0 & -1 & -1 & -1 & 0 \end{array}\right)$$

$$\xrightarrow[R_4 - R_2]{R_3 - 2R_2} \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & -1 & -1 & -1 & 0 \\ 0 & 0 & -2 & -5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow[R_3 / -2]{R_2 / -1} \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2.5 & 0 \end{array}\right)$$

$$\xrightarrow[R_1 - 3R_3]{R_2 - R_3} \left(\begin{array}{cccc|c} 1 & 2 & 0 & -3.5 & 0 \\ 0 & 1 & 0 & -1.5 & 0 \\ 0 & 0 & 1 & 2.5 & 0 \end{array}\right) \xrightarrow{R_1 - 2R_2} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -0.5 & 0 \\ 0 & 1 & 0 & -1.5 & 0 \\ 0 & 0 & 1 & 2.5 & 0 \end{array}\right)$$

Set $x_4 = t \longrightarrow x_1 = 0.5t$

$x_2 = 1.5t$

$x_3 = -2.5t$

So $x = \begin{pmatrix} 0.5t \\ 1.5t \\ -2.5t \\ t \end{pmatrix}$