

Directions: Show all work for full credit. Turn off all electronic devices. Using calculator to make matrix operations is prohibited.

1. Let $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & -1 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & -2 \\ 2 & 2 \end{bmatrix}$.

(a) (3 pts) Without calculating, circle all the operations below that do not exist: (explanation is not needed)

(1) $A \cdot B$ (2) $B + C$ (3) $C + A$ (4) AB (5) BC (6) CA

(b) (4 pts) Compute $AC - 2B$.

AC DNE

so $AC - 2B$ DNE

2. (6 pts) Let $A = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$. Find A^2 and A^n with $n \geq 3$.

$$A^2 = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$A^n = A^2 \cdot A^{n-2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} A^{n-2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \text{ for any } n \geq 3$$

3. (7 pts) Given $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 2 & 1 & 1 & 2 \\ 2 & 3 & 4 & 3 \end{pmatrix}$. Find A^{-1} .

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 2 & 2 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 2 & 0 & 0 & 1 & 0 \\ 2 & 3 & 4 & 3 & 0 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\substack{R_2 - R_1 \\ R_3 - 2R_1 \\ R_4 - 2R_1}} \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & -2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & -2 & 0 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{\substack{R_3 + R_2 \\ R_4 - R_2}} \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -3 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 1 \end{array} \right) \xrightarrow{R_3 \leftrightarrow R_4} \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -3 & 1 & 1 & 0 \end{array} \right)$$

$$\xrightarrow{\substack{R_2 - R_4 \\ R_1 - R_4}} \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 0 & 4 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -3 & 1 & 1 & 0 \end{array} \right) \xrightarrow{\substack{R_1 - R_2 \\ R_2 - R_3}} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 3 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -3 & 1 & 1 & 0 \end{array} \right)$$

A^{-1}

$$\text{So } A^{-1} = \begin{pmatrix} 2 & -1 & 0 & 0 \\ 3 & 1 & -1 & -1 \\ -1 & -1 & 0 & 1 \\ -3 & 1 & 1 & 0 \end{pmatrix}$$