2024 Putnam Competition and Putnam Seminar (MTH 3001)

If you like solving problems, you might be interested in participating the William Lowell Putnam Mathematical Competition. This event takes place the first Saturday in December (December 7 this year) in two sessions (from 10 a.m. to 1 p.m. and from 3 p.m. to 6 p.m.). The competition is available to regularly enrolled undergraduates in colleges and universities in the United States and Canada who have not yet received a college degree. The Department of Mathematics and Statistics offers a 0- or 2-credit problem-solving seminar (MTH 3001) to help prepare students for the competition. If you wish to participate in the competition and/or take (or just attend) the seminar, contact Professor László Lipták, 346 MSC, liptak@oakland.edu. The seminar is scheduled for Wednesdays, 4–5:17 p.m. in 307 SFH. If you would like to take the seminar or to attend the seminar or to just attend sometimes or to participate in the Putnam Competition, you will be put on the Moodle page for the seminar, which will provide a considerable amount of preparation material.

The following is a description of the competition: "The competition will be constructed to test originality as well as technical competence. It is expected that the contestant will be familiar with the formal theories embodied in undergraduate mathematics. It is assumed that such training, designed for mathematics and physical science majors, will include more sophisticated mathematical concepts than is the case in minimal courses.... Questions will be included that cut across the bounds of various disciplines, and self-contained questions that do not fit into any of the usual categories may be included."

Although most of the problems that have appeared in past competitions have been difficult, there are usually a few problems that are easier and can be solved by an average students. Below are a few problems that have appeared in past competitions. Hints are given on the back side of this announcement.

1. How many primes among the positive integers, written in base 10, are such that their digits are alternating 1's and 0's beginning and ending with 1?

2. For a chess board, we say that two cells are adjacent if they share a side or a corner. For any numbering of the cells in a chess board from 1 to 64, let d be the maximum difference of entries in adjacent cells. Among all of the numberings what is the smallest possible value for d?

3. Can a countably infinite set have an uncountable collection of non-empty subsets where the intersection of any two of them is finite?

4. a) If every point of the plane is painted one of three colors, do there necessarily exist two points of the same color that are exactly one inch apart? b) What if "three" is replaced by 9?

5. Let R be the region consisting of points (x, y) in the Cartesian plane satisfying both $|x|-|y| \leq 1$ and $|y| \leq 1$. Sketch the region R and find its area.

Hints: These hints make sense if you have gotten far enough on the problem.

1. Use a geometric sum.

2. Start with the square where 1 is placed. Go to any square adjacent to it. Then go to any square adjacent to that one. How many steps are required to go from the starting square to any other square?

3. Take the rational numbers, and for any real number x, consider the range of a sequence of rational numbers that converges to x.

4. Look at an equilateral triangle of side 1, and look at a rhombus with side 1 and one interior angle of 60 degrees.

5. This one is really easy.