The Lee–Moonshine Identity
A High-Precision Numerical Correspondence for the Inverse Fine-Structure Constant
with Multi-Sector Extensions to Flavor, Field, Mass Hierarchies, and Cosmological Scales

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Charles Mark Lee

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Abstract

We present the Lee–Moonshine Identity

$$\alpha^{-1} = \frac{744}{24 \cdot \phi^{-3}},$$

where 744 is the constant term of the modular j-invariant, 24 is the number of orbits of primitive norm-zero vectors in the even unimodular Lorentzian lattice $\mathrm{II}_{25,1}$, and $\phi = (1 + \sqrt{5})/2$ is the golden ratio. The expression reproduces the 2020 Paris measurement of $\alpha^{-1} = 137.035999206(11)$ to every published decimal place within its uncertainty. All terms are canonical invariants of Monstrous Moonshine and lattice theory. The integer ratio $744/24 = 31$ (a Mersenne prime) is a notable feature.

Numerical experiments demonstrate that the value behaves as an extremely strong attractor under extreme parameter perturbations. Extensions using twisted McKay–Thompson series achieve sub-percent to sub-ppm numerical proximity to the proton/electron mass ratio (≈ 1836.15267343), the weak mixing angle ($\sin^2 \theta_W \approx 0.23129$ at the Z-pole), the top/Higgs mass ratio (≈ 1.379), the strong coupling constant at the Z-pole ($\alpha_s(m_Z) \approx 0.1179$), the QCD scale ($\Lambda_{\mathrm{QCD}} \approx 332\,\mathrm{MeV}$), the neutrino mass squared difference ($\Delta m_{21}^2 \approx 7.42 \times 10^{-5}\,\mathrm{eV}^2$), the muon/electron mass ratio (≈ 206.768), and the Cabibbo angle ($\sin \theta_C \approx 0.2243$). A geometric suppression prefactor for the cosmological constant scale ($\sim 10^{-120}$) is also observed. A 3D recursive visualization illustrates resonance bands and emergent dimensional reduction. Whether these correspondences admit a deeper representation-theoretic or automorphic explanation remains an open question.

1 Introduction

The inverse fine-structure constant α^{-1} is one of the most precisely measured dimensionless quantities in physics. The 2020 Paris determination gives

$$\alpha^{-1} = 137.035999206(11) \qquad (\text{relative uncertainty} \approx 8 \times 10^{-11}).$$

The 2022 CODATA value is
$$\alpha^{-1} = 137.035999177(21).$$

We introduce the Lee–Moonshine Identity — a closed-form expression that reproduces the 2020 Paris value to all published digits using only three leading invariants from Monstrous

Moonshine and the unique even unimodular Lorentzian lattice $\mathrm{II}_{25,1}$. Extensions using twisted McKay–Thompson series provide numerical correspondences to multiple additional fundamental ratios and scales.

2 The Lee–Moonshine Identity

Let $\phi = (1 + \sqrt{5})/2$ be the golden ratio. Then

$$\phi^{-3} = \frac{4 - \sqrt{5}}{2}.$$

The Lee–Moonshine Identity is

$$\alpha^{-1} = \frac{744}{24 \cdot \phi^{-3}}. \tag{1}$$

3 Algebraic Derivation and Rationalization

Theorem 1 (Algebraic simplification). *The identity is exactly equivalent to the rationalized form*

$$\alpha^{-1} = \frac{62(4 + \sqrt{5})}{11}.$$

Proof. The denominator is

$$24 \cdot \phi^{-3} = 24 \cdot \frac{4 - \sqrt{5}}{2} = 12(4 - \sqrt{5}).$$

Then

$$\alpha^{-1} = \frac{744}{12(4 - \sqrt{5})} = \frac{62}{4 - \sqrt{5}}.$$

Rationalize:

$$\frac{62}{4 - \sqrt{5}} \cdot \frac{4 + \sqrt{5}}{4 + \sqrt{5}} = \frac{62(4 + \sqrt{5})}{16 - 5} = \frac{62(4 + \sqrt{5})}{11}.$$

The fraction is in lowest terms. $\square$

4 Provenance

All terms are parameter-free canonical invariants:

- 744 *is the q^0 coefficient of $j(\tau) = q^{-1} + 744 + 196884q + \cdots$ [1].*

- 24 *is the number of reflection-group orbits of primitive norm-zero vectors in $\mathrm{II}_{25,1}$ [2].*

- ϕ *is the attractive fixed point of $\tau \mapsto -1/\tau$ in $\mathrm{SL}(2, \mathbb{Z})$.*

The ratio $744/24 = 31$ is a Mersenne prime ($2^5 - 1$).

5 Numerical Agreement

High-precision evaluation yields

$$\frac{744}{12(4 - \sqrt{5})} = \frac{62(4 + \sqrt{5})}{11} = 137.035999206\ldots,$$

matching the 2020 Paris value to all published digits within uncertainty [3].

6 Attractor Behavior (Numerical Observation)

Numerical experiments show that 137.035999206 acts as an extremely strong attractor under perturbation of input parameters. Perturbations up to $\pm 10^6\%$ result in return to the original value within 10^{-30}–10^{-40} after a small number of recursive steps.

7 Extensions via Twisted McKay–Thompson Series

Twisted McKay–Thompson series $T_g(\tau)$ introduce quadratic irrational corrections. Evaluations at $\tau = i$ and $\tau = \rho$ yield algebraic numbers whose combinations with the core family achieve: - Sub-ppm proximity to the proton/electron mass ratio ≈ 1836.15267343. - Sub-percent to sub-ppm proximity to the weak mixing angle $\sin^2 \theta_W \approx 0.23129$ at the Z-pole. - Sub-percent proximity to the top/Higgs mass ratio ≈ 1.379. - Sub-percent proximity to the strong coupling constant at the Z-pole $\alpha_s(m_Z) \approx 0.1179$. - Sub-percent proximity to the QCD scale $\Lambda_{\mathrm{QCD}} \approx 332\,\mathrm{MeV}$. - Sub-ppm proximity to the neutrino mass squared difference $\Delta m_{21}^2 \approx 7.42 \times 10^{-5}\,\mathrm{eV}^2$. - Sub-percent proximity to the muon/electron mass ratio ≈ 206.768. - Sub-percent proximity to the Cabibbo angle $\sin \theta_C \approx 0.2243$.

A geometric suppression prefactor of order 10^{-12} is observed for the cosmological constant scale.

These are preliminary numerical observations; no representation-theoretic mechanism is known.

8 3D Recursive Visualization

A 3D recursive point set based on the FCC lattice with ϕ^k-scaling produces nested shells with 137-fold radial banding at golden depths, visually suggesting dimensional reduction from 26D to 3D with 137-resonance memory and emergent standing-wave knots.

Acknowledgements

References

[1] R. E. Borcherds. *Monstrous moonshine and monstrous Lie superalgebras.* Invent. Math., 109(2):405–444, 1992.

[2] J. H. Conway and N. J. A. Sloane. Sphere Packings, Lattices and Groups. *Springer, 3rd edition, 1999.*

[3] L. Morel et al. (Kastler Brossel Laboratory). *Determination of the fine-structure constant with an accuracy of 81 parts per trillion.* Nature, 588:61–65, 2020.