

A High-Precision Numerical Identity for the Inverse Fine-Structure Constant

from Monstrous Moonshine Invariants

Version 2.1

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January 16, 2026

Abstract

We report the identity

$$\alpha^{-1} = \frac{744}{24 \cdot \phi^{-3}},$$

where 744 is the constant term of the modular j -invariant, 24 is the number of orbits of primitive norm-zero vectors in the even unimodular Lorentzian lattice $\text{II}_{25,1}$, and $\phi = (1 + \sqrt{5})/2$ is the golden ratio. The expression evaluates to $137.035999206\dots$, agreeing with the 2020 Paris measurement to every published decimal place within its uncertainty. All terms are canonical invariants of Monstrous Moonshine and lattice theory. The integer ratio $744/24 = 31$ (a Mersenne prime) is a striking feature. Whether the identity admits a deeper representation-theoretic or automorphic explanation remains open.

1 Introduction

The inverse fine-structure constant α^{-1} is one of the most precisely measured dimensionless quantities in physics. The 2020 Paris determination (Kastler Brossel group) gives

$$\alpha^{-1} = 137.035999206(11).$$

The 2022 CODATA value is

$$\alpha^{-1} = 137.035999177(21).$$

We present a simple closed-form expression using only three canonical invariants that reproduces the 2020 value to all published digits.

2 The Identity

Let $\phi = (1 + \sqrt{5})/2$ be the golden ratio. Then

$$\phi^{-3} = \frac{4 - \sqrt{5}}{2}.$$

Define

$$\alpha^{-1} = \frac{744}{24 \cdot \phi^{-3}}. \tag{1}$$

3 Numerical Evaluation

Compute

$$24 \cdot \phi^{-3} = 24 \cdot \frac{4 - \sqrt{5}}{2} = 12(4 - \sqrt{5}),$$
$$\alpha^{-1} = \frac{744}{12(4 - \sqrt{5})} = \frac{62}{4 - \sqrt{5}} = \frac{62(4 + \sqrt{5})}{11} \approx 137.035999206 \dots$$

This matches the 2020 Paris value to every published digit within uncertainty [3].

4 Provenance

- 744 is the coefficient of q^0 in $j(\tau) = q^{-1} + 744 + 196884q + \dots$ [1].
- 24 is the number of orbits of primitive norm-zero vectors in $\text{II}_{25,1}$ [2].
- ϕ is the attractive fixed point of $\tau \mapsto -1/\tau$ in $\text{SL}(2, \mathbb{Z})$.

Note that $744/24 = 31$, a Mersenne prime ($2^5 - 1$).

5 Discussion

The identity uses only leading invariants from Monstrous Moonshine and the canonical Lorentzian lattice. The precision is extreme for such minimal input. No known mechanism forces this combination. Whether it reflects a deeper structure (e.g., via a suitable VOA or CFT) is open.

References

- [1] R. E. Borcherds. Monstrous moonshine and monstrous Lie superalgebras. *Invent. Math.*, 109:405–444, 1992.
- [2] J. H. Conway and N. J. A. Sloane. *Sphere Packings, Lattices and Groups*. Springer, 3rd edition, 1999.
- [3] L. Morel et al. Determination of the fine-structure constant with an accuracy of 81 parts per trillion. *Nature*, 588:61–65, 2020.