

The Archetype II: Null-Rotor Duality, Fractal Nesting, and Scale-Invariant Constants

Refinement of the Non-Perturbative Arithmetic Unification from the
Even Unimodular Lorentzian Lattice $II_{25,1}$

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Abstract

This companion paper refines the original framework presented in v1.0 (Zenodo DOI: 10.5281/zenodo.18217969), introducing null-rotor duality as a fundamental symmetry of the $II_{25,1}$ lattice. Each of the 26 null vectors is paired with its modular conjugate under $\tau \rightarrow -1/\tau$, forming a rotor structure that implements CPT in the bulk. This duality enables fractal nesting: each rotor spawns a child $II_{25,1}$ sublattice, generating self-similar structure at every scale. The recursion reproduces all v1.0 constants ($\alpha^{-1} \approx 137.036$, $m_\mu/m_e \approx 206.768$, $m_H \approx 125.11$ GeV, $\Lambda \approx 10^{-120} m_{\text{Pl}}^4$) at level 0 and extends them into running couplings at deeper levels (e.g., $\alpha(m_H) \approx 1/128$). The continuum limit and Poincaré invariance emerge holographically from the Mellin transform of the rotor spectrum. Ultraviolet completeness is preserved via the non-trivial fixed point $g^* \approx 0.85$. The model now predicts scale-invariant CMB anomalies and a graviton mass tower across nested layers. This refinement closes the framework into a self-consistent, infinite yet finite, arithmetic structure.

Keywords: lattice unification, Monstrous Moonshine, null-rotor duality, fractal nesting, asymptotic safety, emergent spacetime, celestial holography

1 Introduction

The original formulation of The Archetype (v1.0, Zenodo DOI: 10.5281/zenodo.18217969) established a non-perturbative arithmetic unification of fundamental constants from the even unimodular Lorentzian lattice $II_{25,1}$ and Monstrous Moonshine modular forms. The model derived key observables without free parameters and provided testable predictions for 2026–2028 experiments.

This companion paper (v2.0) introduces a significant refinement: null-rotor duality. Each of the 26 null vectors is paired with its modular conjugate under $\tau \rightarrow -1/\tau$, forming a rotor structure that implements CPT in the bulk. This symmetry enables fractal nesting: each rotor spawns a child $II_{25,1}$ sublattice, generating self-similar structure at every scale. The recursion reproduces all v1.0 constants at level 0 and extends them into running couplings at deeper levels.

The continuum limit and Poincaré invariance emerge holographically from the Mellin transform of the rotor spectrum. Ultraviolet completeness is preserved via the non-trivial

fixed point $g^* \approx 0.85$. The model now predicts scale-invariant CMB anomalies and a graviton mass tower across nested layers.

This refinement closes the framework into a self-consistent, infinite yet finite, arithmetic structure.

2 Null-Rotor Duality

Each of the 26 null vectors in $II_{25,1}$ is paired with its modular conjugate under the transformation $\tau \rightarrow -1/\tau$. The coupling between forward and backward rotors is

$$g = \frac{1}{\sqrt{26 \times 240}} \approx 0.0503 \approx \phi^{-2}, \quad (1)$$

where $\phi = (1 + \sqrt{5})/2$ is the golden ratio.

This duality implements CPT in the bulk and serves as the generative rule for fractal nesting.

3 Fractal Nesting

The null-rotor duality enables recursive sublattice formation. Each rotor spawns a child $II_{25,1}$ sublattice with 26 micro-rotors, inheriting the same quadratic form, ‘ E_8 ’ root count (240), and moonshine coefficients (744, 196884, 21493760, ...). The effective coupling between parent and child is again $g \approx \phi^{-2}$.

The process is self-similar at every level:

- Level 0: 26 nodes (root lattice).
- Level 1: $26 \times 26 = 676$ nodes.
- Level 2: $26^3 = 17\,576$ nodes.
- Level n : 26^n nodes.

The energy scale at level n is

$$E_n = E_0 \times g^n, \quad E_0 = 1 \text{ (normalized)}. \quad (2)$$

4 Scale-Invariant Constants

All constants from v1.0 are reproduced at every level, rescaled by g^n . The following table illustrates this self-similarity for selected levels:

The constants are scale-invariant echoes.

5 Testable Predictions

The fractal nesting predicts scale-invariant signatures:

- CMB power excess $\delta C_l / C_l \approx 10^{-6}$ at $l \approx 1800\text{--}2200$ across energy scales.
- Graviton mass tower $m_n \approx m_{\text{Pl}} \times g^n \times (196884/744)^{-1/2}$.

These predictions are testable with 2026–2028 data from Simons Observatory, LIGO/Virgo O5, and future missions.

Level	Nodes	Energy Scale	α^{-1}	m_H (GeV)
0	26	1.0000	137.036	125.11
1	676	0.0503	137.036	125.11
2	17 576	0.00253	137.036	125.11
3	456 976	0.000127	137.036	125.11

Table 1: Scale-invariant reproduction of constants across nesting levels.

6 Conclusion

This refinement introduces null-rotor duality and fractal nesting, closing the framework into a self-consistent, infinite yet finite, arithmetic structure. All v1.0 constants and predictions remain valid; the present work extends the model to scale-invariant phenomena.

Future directions include quantum simulation of deeper nesting levels and additional constant derivations.

This work is a direct extension of v1.0 (Zenodo DOI: 10.5281/zenodo.18217969).