

Алгоритмы

Заметки для Профессионалов

Chapter 15: Applications of Dynamic Programming

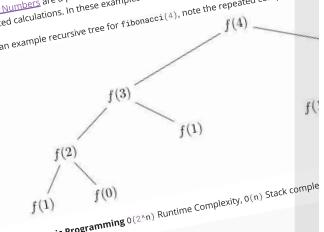
The basic idea behind dynamic programming is breaking a complex problem down to several problems that are repeated. If you can identify a simple subproblem that is repeatedly calculated in a dynamic programming approach to the problem.

As this topic is titled Applications of Dynamic Programming, it will focus more on applications of creating dynamic programming algorithms.

Section 15.1: Fibonacci Numbers

Fibonacci Numbers are a prime subject for dynamic programming as the traditional recursive solution has a lot of repeated calculations. In these examples I will be using the base case of $f(0) = f(1) = 1$.

Here is an example recursive tree for $f_{\text{fibonacci}}(4)$, note the repeated computations:



Non-Dynamic Programming $O(2^n)$ Runtime Complexity, $O(n)$ Stack complexity

```
def fibonaccin():
    if n < 2:
        return 1
    return fibonaccin(n-1) + fibonaccin(n-2)
```

This is the most intuitive way to write the problem. At most the stack space will be $O(2^n)$. Runtime complexity proof that can be seen here: [Complexity](#)

The $O(2^n)$ runtime complexity proof that can be seen here: [Complexity](#). The main point to note is that the runtime is exponential, which means the subsequent term, $f_{\text{fibonacci}}(15)$ will take twice as long as $f_{\text{fibonacci}}(14)$.

Memoized $O(n)$ Runtime Complexity, $O(n)$ Space complexity, $O(n)$ Stack complexity

```
memo = []
memo.append(0) # f(0) = 0
memo.append(1) # f(1) = 1
memo.append(1) # f(2) = 1

def fibonaccin():
    if len(memo) > n:
        return memo[n]

    if fibonaccin() == None:
        memo.append(fibonaccin(n-1) + fibonaccin(n-2))

    return memo[n]
```

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Chapter 6: Check if a tree is BST or not

Section 6.1: Algorithm to check if a given binary tree is BST

A binary tree is BST if it satisfies any one of the following condition:

1. It is empty.
2. It has no subtrees.
3. For every node x in the tree all the keys (if any) in the left sub tree must be less than $\text{key}(x)$ and all the keys (if any) in the right sub tree must be greater than $\text{key}(x)$.

So a straightforward recursive algorithm would be:

```
is_BST(root):
    if root == NULL:
        return true

    // Check values in left subtree
    if root->left != NULL:
        max_key_in_left = find_max_key(root->left)
        if max_key_in_left > root->key:
            return false

    // Check values in right subtree
    if root->right != NULL:
        min_key_in_right = find_min_key(root->right)
        if min_key_in_right < root->key:
            return false

    return is_BST(root->left) && is_BST(root->right)
```

The above recursive algorithm is correct but inefficient, because it traverses each node multiple times.

Another approach to minimize the multiple visits of each node is to remember the min and max possible values of the keys in the subtree we are visiting. Let the minimum possible value of any key be K_{MIN} and maximum value be K_{MAX} . When we start from the root of the tree, the range of values in the tree is $[K_{\text{MIN}}, K_{\text{MAX}}]$. Let the key of root node be x . Then the range of values in left subtree is $[K_{\text{MIN}}, x]$ and the range of values in right subtree is $(x, K_{\text{MAX}}]$. We will use this idea to develop a more efficient algorithm.

```
is_BST(root, min, max):
    if root == NULL:
        return true

    // Is the current node key out of range?
    if root->key < min || root->key > max:
        return false

    // check if left and right subtree is BST
    return is_BST(root->left, min, root->key) && is_BST(root->right, root->key+1, max)
```

It will be initially called as:

```
is_BST(my_tree.root, KEY_MIN, KEY_MAX)
```

Another approach will be to do inorder traversal of the Binary tree. If the inorder traversal produces a sorted sequence of keys then the given tree is a BST. To check if the inorder sequence is sorted remember the value

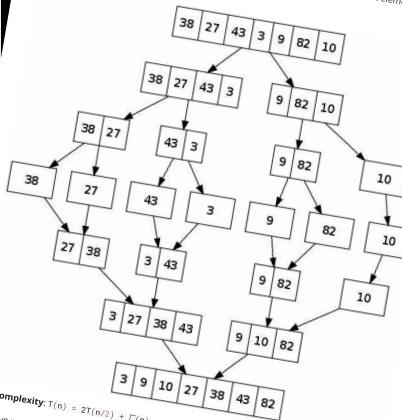
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Chapter 30: Merge Sort

Section 30.1: Merge Sort Basics

Merge Sort is a divide-and-conquer algorithm. It divides the input list of length n in half successively until there are n lists of size 1. Then, pairs of lists are merged together with the smaller first element among the pair of lists being added in each step. Through successive merging and through comparison of first elements, the sorted list is built.

An example:



Time Complexity: $T(n) = 2T(n/2) + C(n)$

The above recurrence can be solved either using Recurrence Tree method or Master method. It falls in case II of Master Method and solution of the recurrence is $C(n \log n)$. Time complexity of Merge Sort is $C(n \log n)$ in all 3 cases (worst, average and best as merge sort always divides the array in two halves and take linear time to merge two halves).

Auxiliary Space: $O(n)$

Algorithmic Paradigm: Divide and Conquer

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