

Homework Assignment 7

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Problem 1. Write at least two necessary conditions and at least two sufficient conditions for a function f to be concave.

Solution.

□

Problem 2. Let $S \subset \mathbb{R}^n$ be a convex set and let $\mathbf{x}^* \in S$. Prove that a vector $\mathbf{d} \in \mathbb{R}^n$ is a feasible direction at \mathbf{x}^* (relative to S) if and only there exists $t_0 > 0$ such that $\mathbf{x}^* + t_0\mathbf{d} \in S$ with $\mathbf{d} \neq \mathbf{0}$.

Solution.

□

Problem 3. Recall that

$$\max\{\alpha, \beta\} := \begin{cases} \alpha & \text{if } \alpha \geq \beta \\ \beta & \text{if } \alpha < \beta \end{cases}.$$

Given two convex functions $f_1 : \mathbb{R}^n \rightarrow \mathbb{R} \cup \{+\infty\}$ and $f_2 : \mathbb{R}^n \rightarrow \mathbb{R} \cup \{+\infty\}$, prove that for $\mathbf{x} \in \mathbb{R}^n$, the function

$$f(\mathbf{x}) := \max\{f_1(\mathbf{x}), f_2(\mathbf{x})\}$$

is convex.

Solution.

□

Problem 4. Consider the pair of linear programming problems in asymmetric duality:

$$\begin{array}{ll} (P_a) & \text{minimize } f(\mathbf{x}) = \mathbf{c}^\top \mathbf{x} \\ & \text{subject to } A\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{array} \quad \begin{array}{ll} (D_a) & \text{maximize } F(\boldsymbol{\lambda}) = \boldsymbol{\lambda}^\top \mathbf{b} \\ & \text{subject to } \boldsymbol{\lambda}^\top A \leq \mathbf{c}^\top \end{array}$$

- Prove that (D_a) is a convex programming problem.
- Write the KKT conditions for (D_a) .
- Suppose that \mathbf{x}^* is feasible for (P_a) and $\boldsymbol{\lambda}^*$ is feasible for (D_a) . Use the KKT conditions to prove that if $(\mathbf{c}^\top - \boldsymbol{\lambda}^{*\top} A)\mathbf{x}^* = 0$, then $\boldsymbol{\lambda}^*$ is optimal for (D_a) .

Solution.

□