Homework Assignment 8

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Problem 7.2.2. If $D : [0, 1) \to [0, 1)$ is the doubling map $D(x) = 2x \mod 1$ and $f : S^1 \to S^1$ is the angle doubling map, $f(z) = z^2$, show that f is a factor of D.

Solution. Recall that a dynamical system $f : S^1 \to S^1$ is a factor of the dynamical system $D : [0, 1) \to [0, 1)$ if there exists a continuous, onto function $h : [0, 1) \to S^1$ such that $h \circ D = f \circ h$.

Define $h : [0, 1) \to S^1$ by $h(x) = e^{2\pi ix}$. Then it is easy to see that h is continuous. To show that it is onto, let $z \in S^1$ be given. Then $z = e^{it}$ for some $t \in [0, 2\pi)$. Choose $x \in [0, 1)$ such that $t = 2\pi x$. Then it is clear that $h(x) = e^{2\pi ix} = e^{it} = z$ and h is onto.

Now, we see that

$$f \circ h(x) = f(e^{2\pi ix}) = e^{2\pi ix}$$

and

$$\begin{aligned} h \circ D(x) &= \begin{cases} h(2x) & \text{if } x \in [0, 1/2) \\ h(2x - 1) & \text{if } x \in [1/2, 1) \end{cases} \\ &= \begin{cases} e^{4\pi ix} & \text{if } x \in [0, 1/2) \\ e^{4\pi ix - 2\pi i} & \text{if } x \in [1/2, 1) \end{cases}. \end{aligned}$$

However, $e^{4\pi ix - 2\pi i} = e^{-2\pi i} e^{4\pi ix} = e^{4\pi ix}$ so in either case $h \circ D(x) = e^{4\pi ix} = f \circ h(x)$ and f is a factor of D.

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Problem 7.2.3.

Solution. □

Problem 7.2.4.

Solution. □

Problem 7.3.2.

Solution. □

Problem 7.3.4.

Solution. □

Problem 7.3.5.

Solution. □