

Homework Assignment 1

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Problem 1.1.2. Use Example 1.1.3 for affine maps to find the solutions to the difference equations:

i. $x_{n+1} - \frac{x_n}{3} = 2, x_0 = 2.$

ii. $x_{n+1} + 3x_n = 4, x_0 = -1.$

Solution.

□

Problem 1.1.3. A *logistic difference equation* is one of the form $x_{n+1} = \mu x_n(1 - x_n)$ for some fixed $\mu \in \mathbb{R}$. Find exact (closed form) solutions to the following logistic difference equations:

- i. $x_{n+1} = 2x_n(1 - x_n)$. Hint: Use the substitution $x_n = (1 - y_n)/2$ to transform the equation into a simpler equation that is easily solved.
- ii. $x_{n+1} = 4x_n(1 - x_n)$. Hint: Set $x_n = \sin^2(\theta_n)$ and simplify to get an equation that is easily solved.

Solution.

□

Problem 1.1.4. You borrow $\$P$ at $r\%$ per annum and pay off $\$M$ at the end of each subsequent month. Write down a difference equation for the amount owing $A(n)$ at the end of each month (so $A(0) = P$). Solve the equation to find a closed form for $A(n)$. If $P = 100,000$, $M = 1,000$, and $r = 4$, after how long will the loan be paid off?

Solution.

□

Problem 1.1.7. Let $f(x) = x^2 + bx + c$. Give conditions on b and c for $f : [0, 1] \rightarrow [0, 1]$ to be a dynamical system. Hint: Recall that the maximum and minimum values of a continuous function defined on a closed interval $[a, b]$ occur either at the end points, or at the critical points of the function.

Solution.

□

Problem 1.2.1. Give conditions on b and c for the map $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^2 + bx + c$ to have a fixed point. Use these conditions to show that $f_c(x) = x^2 + c$ has a fixed point provided $c \leq 1/4$.

Solution.

□

Problem 1.2.6. Consider the eventual fixed points of the logistic map $L_\mu : [0, 1] \rightarrow [0, 1]$, $L_\mu(x) = \mu x(1 - x)$ for $0 < \mu < 4$.

- i. Show that there are no eventual fixed points associated with the fixed point $x = 0$, other than $x = 1$.
- ii. Show that for $1 < \mu \leq 2$, the only eventual fixed point associated with the fixed point $x = 1 - 1/\mu$ is $x = 1/\mu$.
- iii. Show that there are additional eventual fixed points associated with $x = 1 - 1/\mu$ when $2 < \mu < 3$.
- iv. Investigate the eventual fixed points of the logistic map when $\mu = 5/2$.

Solution.

□