# Homework Assignment 3

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February 29, 2016

**Problem 1.** Solve the following linear program using the Simplex Algorithm in conjunction with Bland's rule:

$$\begin{array}{rrl} \text{maximize} & 2x_1 + 5x_2 & \\ \text{subject to} & x_1 & \leq 4 \\ & x_2 & \leq 6 \\ & x_1 + x_2 & \leq 0 \\ & x_1, x_2 & \geq 0. \end{array}$$

*Solution.* □

**Problem 2.** a. Prove that if (ALP) has a feasible solution $(x_1, \dots, x_n; y_1, \dots, y_m)$ with objective function value zero then $y_1 = 0, \dots, y_m = 0$.

b. What do you do if after Phase I (ALP) does not have any optimal feasible solution with objective function value zero?

*Solution.* □

**Problem 3.** Consider the linear program

$$\begin{array}{rrl} \text{maximize} & 2x_1 + x_2 & \\ \text{subject to} & 0 \leq x_1 & \leq 5 \\ & 0 \leq x_2 & \leq 7 \\ & x_1 + x_2 & \leq 9. \end{array}$$

Convert the problem to standard form and solve it using the simplex method.

*Solution.* $\square$

**Problem 4.** Solve the following linear programs using the revised simplex method:

a.

$$\begin{array}{lrl} \text{maximize} & -4x_1 - 3x_2 & \\ \text{subject to} & 5x_1 + x_2 & \geq 11 \\ & -2x_1 - x_2 & \leq -8 \\ & x_1 + 2x_2 & \geq 7 \\ & x_1, x_2 & \geq 0. \end{array}$$

b.

$$\begin{array}{lrl} \text{maximize} & 6x_1 + 4x_2 + 7x_3 + 5x_4 & \\ \text{subject to} & x_1 + 2x_2 + x_3 + 2x_4 & \leq 20 \\ & 6x_1 + 5x_2 + 3x_3 + 2x_4 & \leq 100 \\ & 3x_1 + 4x_2 + 9x_3 + 12x_4 & \leq 75 \\ & x_1, x_2, x_3, x_4 & \geq 0. \end{array}$$

*Solution.* $\square$

**Problem 5.** Suppose that we apply the simplex method to a given linear programming problem and obtain the following canonical tableau:

$$\begin{array}{ccccc} 0 & \beta & 0 & 1 & 4 \\ 1 & \gamma & 0 & 0 & 5 \\ 0 & -3 & 1 & 0 & 6 \\ 0 & 2-\alpha & 0 & 0 & \delta \end{array}$$

For each of the following conditions, find the set of all parameter values $\alpha, \beta, \gamma, \delta$ that satisfy the condition.

a. The problem has no solution because the objective function values are unbounded.

b. The current basic feasible solution is optimal, and the corresponding objective function value is 7.

c. The current basic feasible solution is not optimal, and the objective function value strictly decreases if we remove the first column of $A$ from the basis.

*Solution.* □