Homework Assignment 11

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Problem 5.1. The sunspot numbers $\{X_t, t = 1, \ldots, 100\}$, filed as `SUNSPOTS.TSM`, have sample autocovariances $\hat{\gamma}(0) = 1382.2$, $\hat{\gamma}(1) = 1114.4$, $\hat{\gamma}(2) = 591.73$, and $\hat{\gamma}(3) = 96.216$. Use these values to find the Yule-Walker estimates of ϕ_1, ϕ_2, and σ^2 in the model

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + Z_t, \quad \{Z_t\} \sim \mathrm{WN}(0, \sigma^2),$$

for the mean-corrected series $Y_t = X_t - 46.93$, $t = 1, \ldots, 100$. Assuming the data really are a realization of an AR(2) process, find 95% confidence intervals for ϕ_1 and ϕ_2.

Solution. We wish to find $\hat{\phi}_1, \hat{\phi}_2$, and $\hat{\sigma}^2$ given $\hat{\gamma}(0)$, $\hat{\gamma}(1)$, and $\hat{\gamma}(2)$. By the Yule-Walker equations for sample autocovariances,

$$\begin{bmatrix} \hat{\gamma}(0) & \hat{\gamma}(1) \\ \hat{\gamma}(1) & \hat{\gamma}(0) \end{bmatrix} \begin{bmatrix} \hat{\phi}_1 \\ \hat{\phi}_2 \end{bmatrix} = \begin{bmatrix} \hat{\gamma}(1) \\ \hat{\gamma}(2) \end{bmatrix}.$$

Solving this system yields $\hat{\phi}_1 = 1.31755$ and $\hat{\phi}_2 = -0.634168$. Using the Yule-Walker equation $\hat{\sigma}^2 = \hat{\gamma}(0) - \hat{\phi}_1\hat{\gamma}(1) - \hat{\phi}_2\hat{\gamma}(2)$, we see that $\hat{\sigma}^2 = 289.2$.

Since our sample size $n = 100$ is large, a 95% confidence interval for the parameter ϕ_j is given by

$$\hat{\phi}_j \pm \Phi_{1-\frac{\alpha}{2}} n^{-1/2} \hat{\nu}_{jj}^{1/2}$$

where $\Phi_{1-\frac{\alpha}{2}} = 1.96$ and $\hat{\nu}_{jj}$ is the j-the element on the diagonal of $\hat{\sigma}^2\Gamma_2^{-1}$ for $j = 1, 2$. Using this formula, we see that $\nu_{jj} = 0.5979$ for $j = 1, 2$ and 95% confidence intervals for the model parameters are given by

$$\begin{aligned} (1.1660, 1.4961) \quad &\text{for } \phi_1 \\ (-0.7858, -0.4827) \quad &\text{for } \phi_2. \end{aligned}$$

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