Homework Assignment 3

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Problem 1.   a. Give an example of an asymptotic relation $f(x) \sim g(x)$ $(x \to \infty)$ that cannot be exponentiated; that is $e^{f(x)} \sim e^{g(x)}$ $(x \to \infty)$ is false.

b. Show that if $f(x) - g(x) \ll 1$ $(x \to \infty)$, then $e^{f(x)} \sim e^{g(x)}$ $(x \to \infty)$.

Solution.   □

Problem 2. Find and classify all the singular points (including the point at ∞) of the equations:

$$x(1-x)y'' + [2-(a+b)x]y' - aby = 0, \qquad (x^2+1)y'' - xy = 0.$$

Here, $a, b \in \mathbb{R}$.

Solution. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\square$

Problem 3. Find the Taylor series solution of the IVP

$$(1 - x^3)y''' + 2xy' = 0, \quad y(0) = 3, y'(0) = 3, y''(0) = 0.$$

Solution.   □

Problem 4. Find two linearly independent solutions to $x(1 - x)y'' - 3xy' - y = 0$.

Solution.   □

Problem 5. Find two linearly independent solutions to $x^2y'' + 3xy' + (1 - 2x)y = 0$.

Solution. □

Problem 6. Find the leading behavior of both solutions of $x^5 y'' - y = 0$ near $x = 0$.

Solution.   □

Problem 7. Find the first four terms in the asymptotic series for the solutions of $y'' = e^{-2/x}y$ as $x \to +\infty$.

    Hint: When you are performing the asymptotic analysis to extract the leading behavior of the solution as $x \to +\infty$, you may (and probably want) to replace $e^{-2/x}$ with a reasonable simpler approximation.

Solution.   □