Homework Assignment 5

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Problem 2.7. Show, using the geometric series $1/(1-x) = \sum_{j=0}^{\infty} x^j$ for $|x| < 1$, that $1/(1-\phi z) = -\sum_{j=1}^{\infty} \phi^{-j} z^{-j}$ for $|\phi| > 1$ and $|z| \geq 1$.

Solution. If $|\phi| > 1$ and $|z| \geq 1$, then $|\phi z| = |\phi||z| > 1$ and $1/|\phi z| = 1/(|\phi||z|) < 1$. Now,

$$\sum_{j=0}^{\infty} \phi^{-j} z^{-j} = \sum_{j=0}^{\infty} (\phi z)^{-j} = \sum_{j=0}^{\infty} \left(\frac{1}{\phi z}\right)^j.$$

Note that this is a geometric series where $|\frac{1}{\phi z}| < 1$. So the series converges and

$$\sum_{j=0}^{\infty} \left(\frac{1}{\phi z}\right)^j = \frac{1}{1 - \frac{1}{\phi z}} = \frac{\phi z}{\phi z - 1}.$$

This implies that

$$\frac{\phi z}{\phi z - 1} = \sum_{j=0}^{\infty} \left(\frac{1}{\phi z}\right)^j = 1 + \sum_{j=1}^{\infty} \left(\frac{1}{\phi z}\right)^j$$

so that

$$\sum_{j=1}^{\infty} \left(\frac{1}{\phi z}\right)^j = \frac{\phi z}{\phi z - 1} - 1 = \frac{\phi z - (\phi z - 1)}{\phi z - 1} = \frac{1}{\phi z - 1}.$$

From this identity it is clear that

$$-\sum_{j=1}^{\infty} \phi^{-j} z^{-j} = -\sum_{j=1}^{\infty} \left(\frac{1}{\phi z}\right)^j = \frac{1}{-(\phi z - 1)} = \frac{1}{1 - \phi z}$$

and we are done. □