Homework Assignment 9

Matthew Tiger

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Problem 2.20. In the innovations algorithm, show that for each $n \geq 2$, the innovation $X_n - \hat{X}_n$ is uncorrelated with $X_1, X_2, \ldots, X_{n-1}$. Conclude that $X_n - \hat{X}_n$ is uncorrelated with the innovations $X_1 - \hat{X}_1, X_2 - \hat{X}_2, \ldots, X_{n-1} - \hat{X}_{n-1}$

Solution. □

Problem 2.21. Let X_1, X_2, X_3, X_4, X_5 be observations from the MA(1) model.

$$X_t = Z_t + \theta Z_{t-1}, \{Z_t\} \sim \text{WN}(0, \sigma^2).$$

a. Find the best linear estimate of the missing value X_3 in terms of X_1 and X_2.

b. Find the best linear estimate of the missing value X_3 in terms of X_4 and X_5.

c. Find the best linear estimate of the missing value X_3 in terms of X_1, X_2, X_4, and X_5.

d. Compute the mean squared errors for each of the estimates in (a), (b), and (c).

Solution. □

Problem 2.21. Repeat parts (a)-(d) of Problem 2.21 assuming now that the observations X_1, X_2, X_3, X_4, X_5 are from the causal AR(1) model

$$X_t = \phi X_{t-1} + Z_t, \{Z_t\} \sim \text{WN}(0, \sigma^2)$$

Solution. □