Homework Assignment 1

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Problem 1.4.1. Find the fixed points and determine their stability for the function

$$f(x) = \frac{6}{x} - 1.$$

Solution. $\square$

Problem 1.4.2. Let $f : \mathbb{R} \to \mathbb{R}$. If $f'(x)$ exists with $f'(x) \neq 1$ for all $x \in \mathbb{R}$, prove that f has at most one fixed point. (Hint: Use the Mean Value Theorem).

Solution. $\square$

Problem 1.4.4. Let $S_\mu(x) = \mu \sin(x)$, $0 \le x \le 2\pi$, $0 < \mu \le \pi$ and $C_\mu(x) = \mu \cos(x)$, $\pi \le x \le \pi$ and $\pi \le \mu \le \pi$, $\mu \ne 0$.

i. Show that S_μ has a super-attracting fixed point at $x = \pi/2$, when $\mu = \pi/2$.

ii. Find the corresponding values for C_μ having a super-attracting fixed point.

Solution. $\square$

Problem 1.4.7. Let N_f be the Newton function of the map $f(x) = x^2 + 1$. Clearly there are no fixed points of the Newton function as there are no zeros of f. Show that there are points c where $N_f^2(c) = c$ (called *period 2-points* of N_f).

Solution. $\square$

Problem 1.4.8. i. Suppose that $f(c) = f'(c) = 0$ and $f''(c) \neq 0$. If $f''(x)$ is continuous at $x = c$, show that the Newton function $N_f(x)$ has a removable discontinuity at $x = c$. (Hint: Apply LHopitals rule to N_f at $x = c$.)

ii. If in addition, $f'''(x)$ is continuous at $x = c$ with $f'''(c) \neq 0$, show that $N_f'(c) = 1/2$, so that $x = c$ is not a super-attracting fixed point in this case.

iii. Check the above for the function $f(x) = x^3 x^2$ with $c = 0$.

Solution. □