Homework Assignment 2

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February 13, 2017

Problem 2.10. Solve the Cauchy problem for the Klein-Gordon equation

$$\begin{aligned} u_{tt} - c^2 u_{xx} + a^2 u = 0, \qquad & -\infty < x < \infty, \quad t > 0, \\ u(x,0) = f(x) \qquad & \text{for } -\infty < x < \infty, \\ \left[\frac{\partial u}{\partial t}\right]_{t=0} = g(x) \qquad & \text{for } -\infty < x < \infty. \end{aligned}$$

Solution. □

Problem 2.12. Solve the equation

$$\begin{aligned} u_{tt} + u_{xxxx} = 0, \qquad & -\infty < x < \infty, \quad t > 0 \\ u(x,0) = f(x), \quad u_t(x,0) = 0 \qquad & \text{for } -\infty < x < \infty. \end{aligned}$$

Solution. □

Problem 2.14. Obtain the Fourier cosine transforms of the following functions:

a. xe^{-ax}, $a > 0$.

Solution. □

Problem 2.15. Find the Fourier sine transform of the following functions:

a. xe^{-ax}, $a > 0$.

b. $\dfrac{1}{x}e^{-ax}$, $a > 0$.

Solution. □

Problem 2.20. Apply the Fourier cosine transform to find the solution $u(x, y)$ of the problem

$$\begin{aligned} u_{xx} + u_{yy} &= 0, && 0 < x < \infty, \quad 0 < y < \infty \\ u(x, 0) &= H(a - x), && x < a \\ u_x(0, y) &= 0, && 0 < x, \quad y < \infty. \end{aligned}$$

Solution. □

Problem 2.22. Solve the diffusion equation in the semi-infinite line

$$u_t = \kappa u_x x, \qquad 0 \leq x < \infty, \quad t > 0,$$

with the boundary and initial data

$$\begin{aligned} u(0,t) &= 0 && \text{for } t > 0, \\ u(x,t) &\to 0 && \text{as } x \to \infty \text{ for } t > 0, \\ u(x,0) &= f(x) && \text{for } 0 < x < \infty. \end{aligned}$$

Solution. □