Homework Assignment 3

Matthew Tiger

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Problem 4.28. Every time that the team wins a game, it wins its next game with probability 0.8; every time it loses a game, it wins its next game with probability 0.3. If the team wins a game, then it has dinner together with probability 0.7, whereas if the team loses then it has dinner together with probability 0.2. What proportion of games result in a team dinner?

Solution. □

Problem 4.29. An organization has N employees where N is a large number. Each employee has one of three possible job classifications and changes classifications (independently) according to a Markov chain with transition probabilities

$$\begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.4 & 0.5 \end{bmatrix}$$

What percentage of employees are in each classification?

Solution. □

Problem 4.30. Three out of every four trucks on the road are followed by a car, while only one out of every five cars is followed by a truck. What fraction of vehicles on the road are trucks?

Solution. □

Problem 4.33. A professor continually gives exams to her students. She can give three possible types of exams, and her class is graded as either having done well or badly. Let p_i denote the probability that the class does well on a type i exam, and suppose that $p_1 = 0.3$, $p_2 = 0.6$, and $p_3 = 0.9$. If the class does well on an exam, then the next exam is equally likely to be any of the three types. If the class does badly, then the next exam is always type 1. What proportion of exams are type i?

Solution. □

Problem 4.35. Consider a Markov chain with states $0, 1, 2, 3, 4$. Suppose $P_{04} = 1$; and suppose that when the chain is in state i, with $i > 0$, the next state is equally likely to be any of the states $0, 1, \ldots, i-1$. Find the limiting probabilities of this Markov chain.

Solution. □