Homework Assignment 5

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Problem 1. Use the method of stationary phase to find the leading behavior of the following integral as $x \to +\infty$:

$$\int_0^1 e^{ixt^2} \cosh t^2 dt.$$

Solution. $\square$

Problem 2. Use second-order perturbation theory to find approximations to the roots of the following equation:

$$x^3 + \varepsilon x^2 - x = 0.$$

Solution. □

Problem 3. Analyze in the limit $\varepsilon \to 0$ the roots of the polynomial

$$\varepsilon x^8 - \varepsilon^2 x^6 + x - 2 = 0.$$

Solution. $\square$

Problem 4. Solve perturbatively

$$\begin{cases} y'' = (\sin x)y \\ y(0) = 1 \\ y'(0) = 1 \end{cases}.$$

Is the resulting perturbation series uniformly valid for $0 \leq x \leq \infty$? Why?

Solution. $\square$

Problem 5. Find leading-order uniform asymptotic approximations to the solution of the following equation in the limit $\varepsilon \to 0^+$:

$$\varepsilon y'' + (x^2 + 1)y' - x^3 y = 0$$
$$y(0) = 1, \; y(1) = 1.$$

Solution. □

Problem 6. Obtain a uniform approximation accurate to order ε^2 as $\varepsilon \to 0^+$ for the problem

$$\varepsilon y'' + (1+x)^2 y' + y = 0$$
$$y(0) = 1, \ y(1) = 1.$$

Solution. $\square$

Problem 7. For what real values of the constant α does the singular perturbation problem

$$\begin{aligned} &\varepsilon y''(x) + y'(x) - x^{\alpha} y(x) = 0 \\ &y(0) = 1,\ y(1) = 1. \end{aligned}$$

have a solution with a boundary layer near $x = 0$ as $\varepsilon \to 0^{+}$?

Solution. □