Homework Assignment 5

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Problem 2.6.9. i. Use the results of section 2.6 to show that the logistic map $L_4(x) = 4x(1-x)$ cannot have a super-attracting cycle.

ii. Find a point $x_0 \in (0, 1)$ which is not a periodic point for L_4.

Solution. □

Problem 2.8.3. Show that

$$\left\{ \frac{\mu}{1+\mu^3}, \frac{\mu^2}{1+\mu^3}, \frac{\mu^3}{1+\mu^3} \right\}$$

is a 3-cycle for T_μ when $\mu \geq (1+\sqrt{5})/2$.

Solution. □

Problem 3.2.5. Show that the map $f(x) = (x - 1/x)/2$, $x \neq 0$, has no fixed points but it has period 2-points. Find the 2-cycle, and by looking at the graph of $f^3(x)$, check to see whether or not it has a 3-cycle. Why does this not contradict Sharkovskys Theorem?

Solution. □

Problem 3.2.6. A map $f : [1, 7] \to [1, 7]$ is defined so that $f(1) = 4$, $f(2) = 7$, $f(3) = 6$, $f(4) = 5$, $f(5) = 3$, $f(6) = 2$, $f(7) = 1$, and the corresponding points are joined so the map is continuous and piece-wise linear. Show that f has a 7-cycle but no 5-cycle.

Solution. □

Problem 3.2.10. Let $f : \mathbb{R} \to \mathbb{R}$. Write down all the possibilities for a 4-cycle $\{a, b, c, d\}$ with $a < b < c < d$ for f (e.g. $f(a) = c$, $f(c) = d$, $f(d) = b$, and $f(b) = a$). Indicate which are mirror images, and which give rise to a 3-cycle.

Solution. $\square$

Problem 3.2.11. Use Sharkovskys Theorem to prove that if $f : [a, b] \to [a, b]$ is a continuous function and $\lim_n f^n(x)$ exists for every $x \in [a, b]$, then f can have no points of period $n > 1$.

Solution. □