Aiding Television Media Planning Through Bayesian Inference and Forecasting

Matthew Tiger

Towson University

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Problem

TV Advertising Buying and Selling

Motivating Example

Baseball example

Formal Statement of Problem

Bayesian Inference

Baseball example

Types of Data

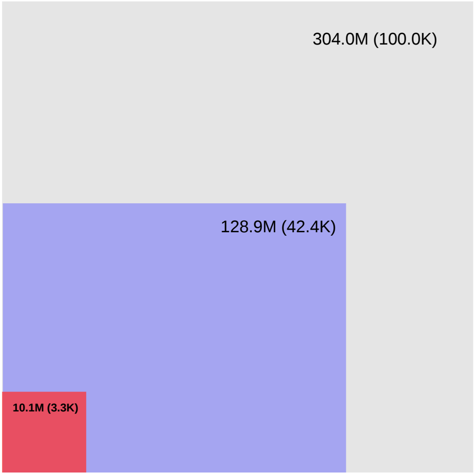
Programming Schedule

Forecasted Impressions

Audience Measurement

Training Data

Audience Size Comparison



Units of Observation and Analysis

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- Derived from Audience Measurement Data
  - **Genre**
  - **Live-program**
  - **First-run**

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- A sequence of random variable is exchangeable if the "joint probability density $p(y_1, \ldots, y_k)$ is invariant to permutations of the indexes."
- This allows us to model the data as independently and identically distributed given the covariates and unknown parameters.

Model Description

Define model $\mathcal{M}$ to be

$$
\begin{aligned}
y_i | X_i, n_i, \pi_i, \omega_i, \kappa_i &\sim \text{Bin}(n_i, \pi_i) \\
\pi_i | \omega_i, \kappa_i &\sim \text{Beta}\left(\omega_i \kappa_i + 1, (1 - \omega_i)\kappa_i + 1\right) \\
\omega_i &= \text{logit}^{-1}\left(\beta_0 + \sum_{j=1}^{m-1} \beta_j X_{ij}\right), \quad \beta_j \sim t_4(0, \sigma_j^2) \\
&\text{for } 0 \leq j \leq m \\
\kappa_i | X_{im} &\sim \text{Exp}(\lambda_p X_{im}), \quad \text{for } p = 0, 1,
\end{aligned}
$$

where $\text{logit}^{-1}(\alpha) = \frac{\exp \alpha}{1 + \exp \alpha}$.

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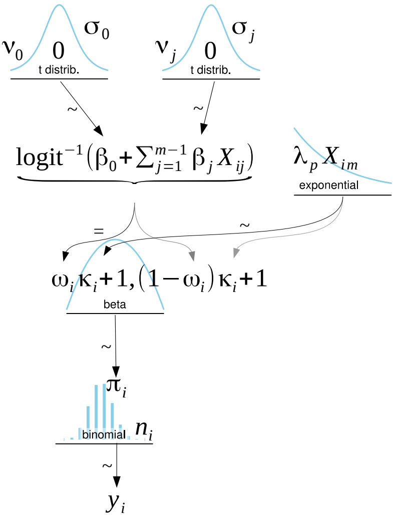

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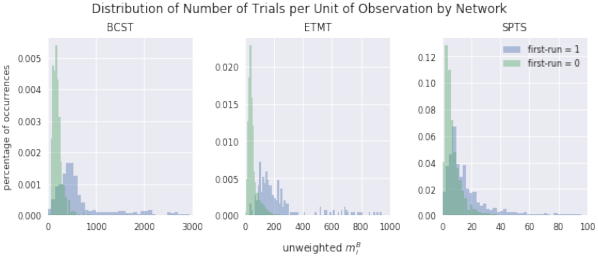

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- Parameters used for sampling:
  - `target_accept`: 0.95
  - tuned samples: 3000
  - drawn samples: 500
  - number of chains: 4

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- For each network model, we have that $0.99 \leq \hat{R} \leq 1.01$ and $\hat{n_{\text{eff}}} > 400$ for all model parameters.

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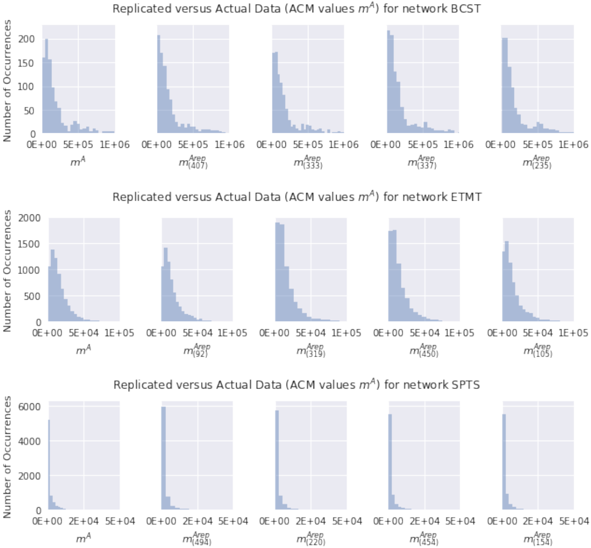
Let y be the observed data and θ be the vector of model parameters. Define y^{rep} to be the replicated data that could have been generated given θ, i.e.

$$p(y^{\text{rep}}|y) = \int p(y^{\text{rep}}|\theta)p(\theta|y)d\theta. \quad (1)$$

Replicated versus Actual Data



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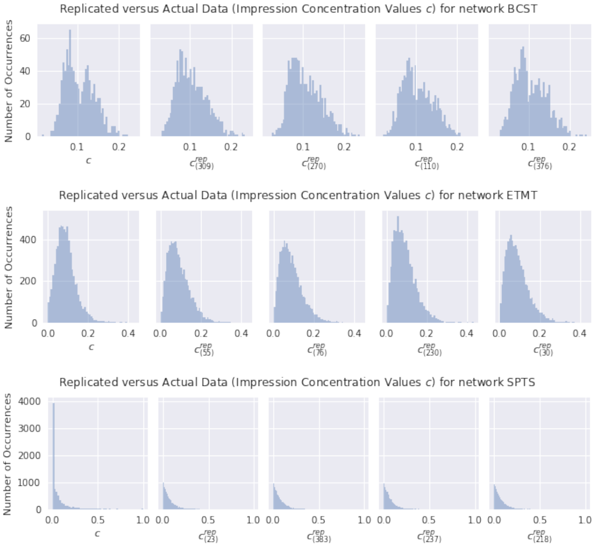

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- Since we use simulated values of the posterior density, we have that the estimated p-value for S simulations is given by:

$$\hat{p_B} = \frac{1}{S} \sum_{i=1}^{S} [T(y_{(i)}^{\text{rep}}, \theta_{(i)}) \geq T(y, \theta_{(i)})]. \tag{2}$$

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- $T_3(y, \theta) := \max(y)$,
- $T_4(y, \theta) := \text{std}(y) = \sqrt{\frac{\sum_{i=1}^{N} (y_i - \overline{y})^2}{N-1}}$.

Test Statistics - Evaluation - BCST network

Test quantity	$T(y,\theta)$	95% int. for $T(y^{\text{rep}},\theta)$	p_B
$T_1(y,\theta)$ (min)	3701	[6245, 14270]	0.99
$T_2(y,\theta)$ (mean)	227457.84	[2266852.49, 236367.09]	0.95
$T_3(y,\theta)$ (max)	4311038	[3443885, 4989241]	0.34
$T_4(y,\theta)$ (std)	334052.86	[325128.37, 364859.10]	0.90

Test Statistics - Evaluation - ETMT network

Test quantity	$T(y, \theta)$	95% int. for $T(y^{\text{rep}}, \theta)$	p_B
$T_1(y, \theta)$ (min)	0	[9, 182]	1.0
$T_2(y, \theta)$ (mean)	16357.80	[16705.39, 17489.11]	1.0
$T_3(y, \theta)$ (max)	452762	[307901, 760822]	0.78
$T_4(y, \theta)$ (std)	17686.89	[20021.24, 23205.09]	1.0

Test Statistics - Evaluation - SPTS network

Test quantity	$T(y,\theta)$	95% int. for $T(y^{\text{rep}},\theta)$	p_B
$T_1(y,\theta)$ (min)	0	[0, 0]	1.0
$T_2(y,\theta)$ (mean)	3972.45	[3714.91, 4559.66]	0.73
$T_3(y,\theta)$ (max)	526816	[607186, 2239365]	0.99
$T_4(y,\theta)$ (std)	22300.18	[20012.59, 39808.44]	0.88

Residual Analysis

- For a model with unknown parameters θ and predictors x_i, the *predicted* value is $\mathrm{E}(y_i|x_i, \theta)$ and the *residual* is $r_i = y_i - \mathrm{E}(y_i|x_i, \theta)$.

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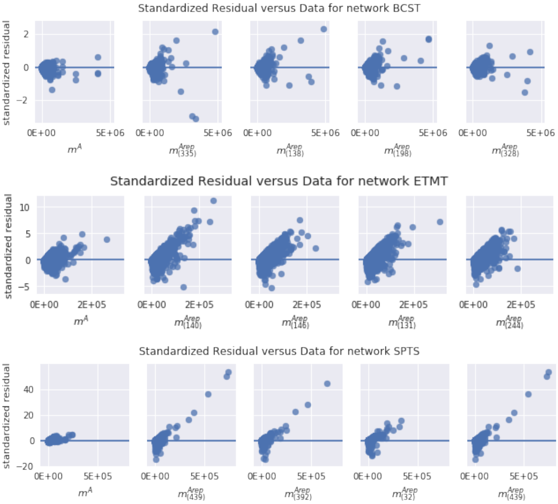
- The *standardized residual* is given by $r_i/\mathrm{std}(y)$.
- Using the simulated posterior density, we can compute $\mathrm{E}(y_i|x_i, \theta)$ to be the mean of the replicated hold-out data itself.

Residual Analysis - Actual versus Replicated



Residual Analysis - Test Statistic Evaluation

We can measure the residual mis-fit through the following test statistic:

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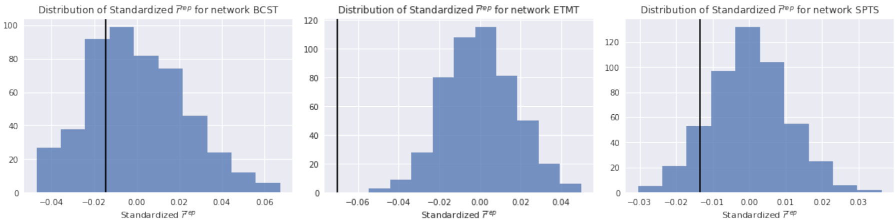

Units of Observation

Quantiled Media Plans

Sample frame title