Homework Assignment 6

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Problem 1. a. Where is the assumption "x^* is regular" essential in the proof of the results of section: Lagrange Multipliers?

b. In the example on page 49 (Example 20.8 in *An Introduction to Optimization*) explain in what way is (P_0) equivalent to (P_1).

c. State the SOSC Theorem on p. 51 (Theorem 20.5 p. 474 in the book) for x^* a local maximizer.

Solution. □

Problem 2. Find local extremizers for the following optimization problem:

$$\begin{aligned} \text{maximize} \quad & x_1 x_2 \\ \text{subject to} \quad & x_1^2 + 4x_2^2 = 1. \end{aligned}$$

Solution. □

Problem 3. Consider the problem

$$\begin{aligned} \text{minimize} \quad & 2x_1 + 3x_2 - 4, \quad x_1, x_2 \in \mathbb{R} \\ \text{subject to} \quad & x_1 x_2 = 6. \end{aligned}$$

a. Use Lagrange's theorem to find all possible local minimizers and maximizers.

b. Use the second-order sufficient conditions to specify which points are strict local minimizers and which are strict local maximizers.

c. Are the points in part b global minimizers or maximizers? Explain.

Solution. □

Problem 4. Consider the problem of minimizing a general quadratic function subject to a linear constraint:

$$\begin{array}{rl} \text{minimize} & \frac{1}{2}\boldsymbol{x}^\top Q\boldsymbol{x} - \boldsymbol{c}^\top\boldsymbol{x} + d \\ \text{subject to} & A\boldsymbol{x} = \boldsymbol{b}, \end{array}$$

where $Q = Q^\top > 0$, $A \in \mathbb{R}^{m\times n}$ with $m < n$, $\operatorname{rank} A = m$ and d a constant. Derive a closed form solution to the problem.

Solution. □

Problem 5. Consider the discrete-time linear system $x_k = 2x_{k-1} + u_k$, $k \geq 1$, with $x_0 = 1$. Find the values of the control inputs u_1 and u_2 to minimize

$$x_2^2 + \frac{1}{2}u_1^2 + \frac{1}{3}u_2^2.$$

Solution. $\square$