Homework Assignment 1

Matthew Tiger

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Problem 3.7. Suppose $p(x, y, z)$, the joint probability mass function of the random variables X, Y, and Z, is given by

$$p(1,1,1) = \tfrac{1}{8}, \quad p(2,1,1) = \tfrac{1}{4},$$

$$p(1,1,2) = \tfrac{1}{8}, \quad p(2,1,2) = \tfrac{3}{16},$$

$$p(1,2,1) = \tfrac{1}{16}, \quad p(2,2,1) = 0,$$

$$p(1,2,2) = 0, \quad p(2,2,2) = \tfrac{1}{4}.$$

What is $E[X|Y = 2]$? What is $E[X|Y = 2, Z = 1]$?

Solution. □

Problem 3.8. An unbiased die is successively rolled. Let X and Y denote, respectively, the number of rolls necessary to obtain a six and a five. Find:

a. $E[X]$,

b. $E[X|Y = 1]$,

c. $E[X]Y = 5]$.

Solution. □

Problem 3.9. Show in the discrete case that if X and Y are independent, then

$$E[X|Y = y] = E[X] \text{ for all y.}$$

Solution. □

Problem 3.10. Suppose X and Y are independent continuous random variables. Show that

$$E[X|Y = y] = E[X] \text{ for all y.}$$

Solution. □

Problem 3.13. Let X be exponential with mean $1/\lambda$; that is,

$$f_X(x) = \lambda e^{-\lambda x}, \quad 0 < x < \infty.$$

Find $E[X|X > 1]$.

Solution. $\square$

Problem 3.14. Let X be uniform over $(0, 1)$. Find $E[X|X < 1/2]$.

Solution. □