Homework Assignment 1

Matthew Tiger

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Problem 1. To be comprehensive, the second derivative test for two-variable functions $f = f(x, y)$ studied in Calculus III should contain (among others) the cases:

a. $D(a, b) > 0$ and $f_{xx}(a, b) = 0$,

b. $D(a, b) = 0$ and $f_{xx}(a, b) = 0$.

Why aren't these cases considered? Explain.

Solution. $\square$

Problem 2. Recall that

- (a, b) is called an *absolute maximum* of $f = f(x, y)$ on a domain $D \subset \mathbb{R}^2$ if $f(x, y) \leq f(a, b)$ for every $(x, y) \in D$.
- (The Extreme Value) If f is continuous and D is closed and bounded, then f attains both an absolute maximum value and an absolute minimum value.

a. Describe in steps (and in words) how one finds absolute extrema for a two-variable function $f = f(x, y)$ on a closed bounded $D \subset \mathbb{R}^2$.

b. Apply your procedure derived in (a) to find absolute extrema for $f(x, y) = 2x^3 + xy^2 + xy^2 + 5x^2 + y^2$ over the rectangle $D := \{(x, y) \mid -2 \leq x \leq 3, 0 \leq y \leq 2\}$.

Solution. $\square$

Problem 3. Consider the optimization problem:

$$
\begin{array}{lll}
\text{Min (Max)} & f(x_1, x_2, \ldots, x_n) & \\
\text{subject to} & g_1(x_1, x_2, \ldots, x_n) & = k_1 \\
 & g_2(x_1, x_2, \ldots, x_n) & = k_2 \\
 & \vdots & \\
 & g_m(x_1, x_2, \ldots, x_n) & = k_m
\end{array}
$$

a. Formulate the Lagrangean and describe how we should proceed in order to solve such a problem.

b. Find the relative extrema of $f(x, y, z) = x+2y+3z$ subject to $x-y+z = 1, x^2+y^2 = 1$.

Solution. $\square$

Problem 4. Solve the shipping problem studied in MATH 111 if we replace the constraint $x + 2y \leq 100$ by the constraint $x + 2y \leq 625/6$. Use Mathematica to (at least) graph the feasible set.

Solution. $\square$

Problem 5. Suppose that f, f_1, f_2 are convex functions and $a \geq 0$. Prove that af and $f_1 + f_2$ are convex functions.

Solution. $\square$

Problem 6. For $f : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ we define its *epigraph* as the set

$$\text{epi } f = \{(x, \beta) \in \mathbb{R}^n \times \mathbb{R} | f(x) \leq \beta\} \subset \mathbb{R}^{n+1}.$$

Prove that f is convex if and only if epi f is convex.

Solution. $\square$