A Numerical Solution to a Second Order Ordinary Differential Equation

Hunter, Marcus   Tiger, Matthew   Wigfield, Jacob

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1. Introduction

The authors were tasked by a client with finding the solution to the following family of differential equations

$$-u''(x) + cu(x) = f(x), \quad a \leq x \leq b, \quad u(a) = \epsilon \text{ and } u(b) = \delta.$$

Assumptions were placed on this family so that $c > 0$ and $f \in C^k([a, b])$ for sufficiently large k so that f is relatively well-behaved on the defined interval. The client has also requested to be provided with a means of plotting the solution once obtained.

In this report we will detail the analytical solution to this family of differential equations and explain why this solution is not amenable to practical use. We therefore provide a numerical scheme to approximate the solution to the family of differential equations and examine the convergence, consistency and stability of the numerical scheme. Using the solution provided by the numerical scheme, we then explore the different options for plotting the solution.

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