Homework Assignment 9

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Problem 8.2.2. If $Sf(x)$ is the Schwarzian derivative of $f(x)$ with $f \in C^3$ and $F(x) = \dfrac{f''(x)}{f'(x)}$, show that $Sf(x) = F'(x) - (F(x))^2/2$.

Solution. Recall that the Schwarzian derivative of f is given by

$$Sf(x) = \frac{f'''(x)}{f'(x)} - \frac{3}{2}\left[\frac{f''(x)}{f'(x)}\right]^2.$$

We readily see from the definition of $F(x)$ that

$$\begin{aligned} F'(x) - \frac{1}{2}\left[F(x)\right]^2 &= \frac{f'(x)f'''(x) - f''(x)^2}{f'(x)^2} - \frac{1}{2}\left[\frac{f''(x)}{f'(x)}\right]^2 \\ &= \frac{f'''(x)}{f'(x)} - \frac{3}{2}\left[\frac{f''(x)}{f'(x)}\right]^2 \\ &= Sf(x). \end{aligned}$$

□

Problem 8.2.5.

Solution. □

Problem 8.2.6.

Solution. □

Problem 8.2.10.

Solution. □

Problem 10.3.4.

Solution. □

Problem 10.3.6.

Solution. □

Problem 10.3.7.

Solution. □