Homework Assignment 6

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Problem 1. Prove that if A and $A + \delta A$ are invertible, then

$$\|\delta x\| < \|x\| \to \frac{\|\delta x\|}{\|x\|} \leq 2\text{cond}(A)\frac{\|\delta A\|}{\|A\|}$$

Solution. Note that we have already proved that if A and $A + \delta A$ are invertible, then

$$\|\delta x\| < \|x\| \to \frac{\|\delta x\|}{\|x + \delta x\|} \leq \text{cond}(A)\frac{\|\delta A\|}{\|A\|}. \tag{1}$$

Since $\|x + \delta x\| \leq \|x\| + \|\delta x\|$, we know that

$$\frac{1}{\|x\| + \|\delta x\|} \leq \frac{1}{\|x + \delta x\|}. \tag{2}$$

Using our assumption that $\|\delta x\| < \|x\|$ we see that $\|x\| + \|\delta x\| < \|x\| + \|x\|$ or

$$\frac{1}{\|x\| + \|x\|} < \frac{1}{\|x\| + \|\delta x\|}. \tag{3}$$

Combining (2) and (3) and multiplying by $\|\delta x\|$, we can see that

$$\frac{\|\delta x\|}{2\|x\|} = \frac{\|\delta x\|}{\|x\| + \|x\|} < \frac{\|\delta x\|}{\|x\| + \|\delta x\|} \leq \frac{\|\delta x\|}{\|x + \delta x\|}$$

Using the above inequality and (1), it is clear that if A and $A + \delta A$ are invertible and $\|\delta x\| < \|x\|$, then

$$\frac{\|\delta x\|}{2\|x\|} \leq \frac{\|\delta x\|}{\|x + \delta x\|} \leq \text{cond}(A)\frac{\|\delta A\|}{\|A\|}$$

and we are done. □