Homework Assignment 4

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Problem 4.56. Suppose that on each play of the game a gambler either wins 1 with probability p or loses 1 with probability $1 - p$. The gambler continues betting until she or he is either up n or down m. What is the probability that the gambler quits a winner?

Solution. $\square$

Problem 4.59. For the gambler's ruin problem of Section 4.5.1, let M_i denote the mean number of games that must be played until the gambler either goes broke or reaches a fortune of N, given that he starts with i for $i = 0, 1, \ldots, N$. Show that M_i satisfies

$$M_0 = M_N = 0; \quad M_i = 1 + pM_{i+1} + qM_{i-1}, \quad i = 1, \ldots, N-1.$$

Solve these equations to obtain

$$M_i = \begin{cases} i(N-i) & \text{if } p = 1/2 \\ \dfrac{i}{q-p} - \dfrac{N}{q-p}\dfrac{1-(q/p)^i}{1-(q/p)^N} & \text{if } p \neq 1/2 \end{cases}.$$

Solution. □

Problem 4.63. For the Markov chain with states 1, 2, 3, 4 whose transition probability matrix $\mathbf{P}$ is as listed below find f_{i3} and s_{i3} for $i = 1, 2, 3$.

$$\mathbf{P} = \begin{bmatrix} 0.4 & 0.2 & 0.1 & 0.3 \\ 0.1 & 0.5 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.2 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution. □

Problem 4.64. Consider a branching process having $\mu < 1$. Show that if $X_0 = 1$, then the expected number of individuals that ever exist in this population is given by $1/(1-\mu)$. What if $X_0 = n$?

Solution. □

Problem 4.66. For a branching process, calculate π_0 when

i. $P_0 = \frac{1}{4}$,$\mathrm{P}_2 = \frac{3}{4}$.

ii. $P_0 = \frac{1}{4}$,$\mathrm{P}_1 = \frac{1}{2}$, $P_2 = \frac{1}{4}$.

iii. $P_0 = \frac{1}{6}$,$\mathrm{P}_1 = \frac{1}{2}$, $P_2 = \frac{1}{3}$.

Solution. □