Homework Assignment 12

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Problem 1. Use the methods of this section to approximate the solution to

$$y'' + y = 3x^2, \quad y(0) = 0, y(2) = 3.5$$

For basis functions, take $n = 2$ and $\phi_1(x) = x(x-2), \phi_2(x) = x^2(x-2)$.

Solution. Note that $u(x) = \frac{7}{4}x$ satisfies the boundary conditions of the problem, i.e. $u(0) = 0$ and $u(2) = 3.5$. So we choose the approximation

$$y_2 = u(x) + a_1\phi_1(x) + a_2\phi_2(x) \tag{1}$$

to the solution of the original differential equation which also satisfies the boundary conditions.

We wish to find values of the coefficients a_1, a_2 such that

$$\int_0^2 \left(y_2'' + y_2 - 3x^2\right)\phi_i(x)dx = 0 \quad \text{for } i = 1, 2. \tag{2}$$

Using our definition of the approximation found in (1), we carry out the computations in eqrefsystem with MATLAB to arrive at the following system of equations

$$\begin{bmatrix} 8/5 & 8/5 \\ 8/5 & 64/21 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 37/15 \\ 18/5 \end{bmatrix}.$$

The solution to this system yields that the coefficients are given by $a_1 = 173/228$ and $a_2 = 119/152$.

Therefore, the approximation to the solution to the original differential equation is given by

$$y_2(x) = \frac{7}{4}x + \frac{173}{228}x(x-2) + \frac{119}{152}x^2(x-2).$$

□