Homework Assignment 8

Matthew Tiger

April 29, 2017

Problem 7.1. Show that

a. $\mathscr{H}_0\left\{(a^2 - r^2)H(a-r)\right\} = \dfrac{4a}{\kappa^3}J_1(a\kappa) - \dfrac{2a^2}{\kappa^2}J_0(a\kappa)$.

Solution. a. Let J_n be the integral representation of the Bessel function of order n, i.e.

$$J_n(\kappa r) = \frac{1}{2\pi}\int_{\pi/2-\phi}^{5\pi/2-\phi} \exp\left[i(n\alpha - \kappa r \sin\alpha)\right] d\alpha$$

Then the Hankel transformation of order n of $f(r)$ is defined to be

$$\mathscr{H}_n\left\{f(r)\right\} = \int_0^\infty r J_n(\kappa r) f(r) dr.$$

Using the table of Hankel transforms we see that

$$\mathscr{H}_0\left\{(a^2 - r^2)H(a-r)\right\} = \frac{4a}{\kappa^3}J_1(a\kappa) - \frac{2a^2}{\kappa^2}J_0(a\kappa),$$

and we are done.

□

Problem 7.2.

Solution. □

Problem 7.9.

Solution. $\square$

Problem 7.12.

Solution. $\square$

Problem 7.14.

Solution. $\square$

Problem 7.19.

Solution. □