Homework Assignment 5

Matthew Tiger

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Problem 1. a. Explain in a specific example why, when A and $\boldsymbol{b}$ have integer components, a general integer programming problem

$$\begin{array}{llrl} (GILP) & \text{Minimize (Maximize)} & f(\boldsymbol{x}) = \boldsymbol{c}^\top \boldsymbol{x} & \\ & \text{subject to} & A\boldsymbol{x} & \leq (\geq, =)\boldsymbol{b} \\ & & \boldsymbol{x} \geq (\leq)\boldsymbol{0}, \boldsymbol{x} \in \mathbb{Z}^n & \end{array}$$

can be reduced (or is equivalent) to a standard integer programming problem

$$\begin{array}{llrl} (ILP) & \text{Minimize (Maximize)} & f(\boldsymbol{X}) = \boldsymbol{C}^\top \boldsymbol{X} & \\ & \text{subject to} & \mathscr{A}\boldsymbol{X} & = \boldsymbol{B} \\ & & \boldsymbol{X} \geq \boldsymbol{0}, \boldsymbol{X} \in \mathbb{Z}^n & \end{array}$$

by adding variables or any of the transformations discussed in class that change $\boldsymbol{x}$ into $\boldsymbol{X}$.

More precisely, explain why (GILP) has a solution $\boldsymbol{x} \in \mathbb{Z}^n$ if and only if (ILP) has a solution $\boldsymbol{X} \in \mathbb{Z}^n$.

b. How do we solve (GILP) when A or $\boldsymbol{b}$ do not have integer components?

Solution. □

Problem 2. Solve the shipping problem studied in MATH 111 with the replaced constraints over integers using the Gomory Cutting-Plane Method.

More precisely, solve:

$$\begin{array}{lll} \text{Maximize} & 9x_1 + 13x_2 & \\ \text{subject to} & 4x_1 + 3x_2 & \leq 300 \\ & x_1 + 2x_2 & \leq 625/6 \\ & -2x_1 + x_2 & \leq 0 \\ & x_1, x_2 \geq 0 & \\ & x_1, x_2 \in \mathbb{Z} & \end{array}.$$

Solution. $\square$

Problem 3. Let $f : \mathbb{R}^n \to \mathbb{R}^m$ and $\Omega \subset \mathbb{R}^n$ be an open subset. Explain the meaning of $f \in C^1(\Omega)$. More precisely, give all the definitions needed and present some examples and results concerning $C^1(\Omega)$ functions.

Solution. $\square$