Homework Assignment 5

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Problem 3.23. Show that:

a.
$$\mathscr{L}\left\{t\cos ate^{-bt}\right\} = \frac{(s+b)^2 - a^2}{\left[(s+b)^2 + a^2\right]^2}.$$

Solution. a. Let $f(t) = t \cos at$ and suppose that $\bar{f}(s) = \mathcal{L}\{f(t)\}.$

As shown previously, we know that

$$\bar{f}(s) = \mathcal{L}\{f(t)\} = \mathcal{L}\{t\cos at\} = \frac{s^2 - a^2}{(s^2 + a^2)^2}.$$

Therefore, by Heaviside's First Shifting Theorem,

$$\mathscr{L}\left\{t\cos ate^{-bt}\right\} = \mathscr{L}\left\{f(t)e^{-bt}\right\} = \bar{f}(s+b) = \frac{(s+b)^2 - a^2}{\left[(s+b)^2 + a^2\right]^2},$$

and we are done.

Problem 3.24.

Problem 3.27.

Problem 3.28.

Problem 3.29.

Problem 3.32.

Problem 3.34.

Problem 4.1.