Homework Assignment 12

Matthew Tiger

December 6, 2015

Problem 1. Use the methods of this section to approximate the solution to

$$y'' + y = 3x^2, \quad y(0) = 0, y(2) = 3.5$$

For basis functions, take $n = 2$ and $\phi_1(x) = x(x-2), \phi_2(x) = x^2(x-2)$.

Solution. Note that $u(x) = \frac{7}{4}x$ satisfies the boundary conditions of the problem, i.e. $u(0) = 0$ and $u(2) = 3.5$. Therefore our approximation to the differential equation is given by a linear combination of the basis functions $\phi_1(x), \phi_2(x)$ and $u(x)$. So the approximation is given by

$$y_2 = u(x) + a_1\phi_1(x) + a_2\phi_2(x) \tag{1}$$

to the solution of the original differential equation which also satisfies the boundary conditions.

We wish to find values of the coefficients a_1, a_2 such that

$$\int_0^2 \left(y_2'' + y_2 - 3x^2\right)\phi_i(x)dx = 0 \quad \text{for } i = 1, 2. \tag{2}$$

Using our definition of the approximation found in (1), we carry out the computations in eqrefsystem with MATLAB to arrive at the following system of equations

$$\begin{bmatrix} 8/5 & 8/5 \\ 8/5 & 64/21 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 37/15 \\ 18/5 \end{bmatrix}.$$

The solution to this system yields that the coefficients are given by $a_1 = 173/228$ and $a_2 = 119/152$.

Therefore, the approximation to the solution to the original differential equation is given by

$$y_2(x) = \frac{7}{4}x + \frac{173}{228}x(x-2) + \frac{119}{152}x^2(x-2).$$

□

Problem 3. Use the methods of this section to approximate the solution to

$$y'' + y^2 = x, \quad y(0) = y(1) = 0.$$

Let $n = 1$ and for a basis function take $\phi_1(x) = x\sin(\pi x)$.

Solution. Note that $\phi_1(x) = x\sin(\pi x)$ satisfies the boundary conditions, i.e. $\phi_1(0) = \phi_1(1) = 0$. Therefore our approximation to the differential equation is given by a linear combination of the basis function $\phi_1(x)$. Thus, the approximation is

$$y_1 = a_1 x \sin(\pi x).$$

We wish to find the value of a_1 such that

$$\int_0^1 \left(y_1'' + y_1^2 - x\right) x\sin(\pi x)dx = 0.$$

Using MATLAB, we see that the above equation reduces to the following quadratic equation

$$\left(\frac{2\pi^2}{3\pi^3} - \frac{40}{9\pi^3}\right)a_1^2 + \left(\frac{-\pi^2}{6} - \frac{1}{4}\right)a_1 + \frac{4-\pi^2}{\pi^3} = 0.$$

Choosing the solution to the equation $a_1 = -0.099539826159056$ give us that the approximate solution is

$$y_1 = -0.099539826159056\sin(\pi x).$$

$\square$

Problem 5. The solution to

$$((x+1)y')' - (x+2)y = \frac{x+2}{e-1}, \quad y(0) = 0, y(1) = 1.$$

is $y = \frac{e^x - 1}{e-1}$. Use the methods of this section to compute approximate solutions $y_5(x)$, $y_{10}(x)$, and $y_{15}(x)$ and compare these approximations to the actual solution.

Solution. $\square$