STATISTICAL BIASES OF SPECTRAL ANALYSIS WITH THE ON-OFF LIKELIHOOD STATISTIC

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ON-OFF spectral Analysis

- Typical source observation covers wide range of observational parameters (e.g. efficiency, zenith, offset)

- Strong IRFs (Instrument Response Function) dependence with the observational parameters and energy ➔ IRFs are calculated for each observation.

- Strong Background event distribution dependence with the observational parameters:
  - ON, OFF (reflected background) ➔ background spectrum for each observation

- But Low counts ➔ Maximum Likelihood (M.L.) method taking into account the Poissonian statistic

- Background: no satisfactory background model so far ➔ marginalize the background ("profile likelihood"):
  - *interest parameters*: spectral model parameters
  - *nuisance parameters*: Background (from ON and OFF)

Motivation

- In order to keep all the information related to the observation conditions:
  - ➔ processes all the observations individually for the likelihood maximization
  - ➔estimators of the number of events in the ON and OFF regions for each reconstructed energy bin.
  - ➔Issue: Where there is as many nuisance parameters as observations: **estimators can be inconsistent** (i.e. do not converge to the right value even for an infinite number of observations)
- Observation grouping require to limit the number of nuisance parameters. How much grouping is needed to have negligible biases?
- *To test the impact of observation grouping on the estimators biases ➔ MC (Monte Carlo) tool that simulate ON & OFF spectra*

MCs tool in the H.E.S.S. analysis framework

- For each observation (given set of observational parameters: zenith angle Z, offset θ and optical efficiency ε): simulation of the ON and OFF data.

For each run j and enrgy bin i:

Intrinsic source spectrum

- **Signal:** $$\overline{S_{i,j}} = \Delta T \int_{E_{m_i}}^{E_{m_{i+1}}} dE_m \int_{E=0}^{\infty} \phi(E) A(E, Z_j, \theta_j, \epsilon_j) R(E, E_m, Z_j, \theta_j, \epsilon_j) dE$$

IRFs

$$\Longrightarrow \quad S_{i,j} = P(\overline{S_{i,j}})$$

MCs tool in the H.E.S.S. analysis framework

- For each run (given set of observational parameters: zenith angle Z, offset θ and optical efficiency ε): simulation of the ON and OFF data.

  - **Signal:** $$\overline{S_{i,j}} = \Delta T \int_{E_{m_i}}^{E_{m_{i+1}}} dE_m \int_{E=0}^{\infty} \phi(E) A(E, Z_j, \theta_j, \epsilon_j) R(E, E_m, Z_j, \theta_j, \epsilon_j) dE$$

  - **Background:**

$$\overline{B_{i,j}}_{E<1\,TeV} = N_{OFF\,i,j}$$

$$\overline{B_{i,j}}_{E>1\,TeV} = \int_{E_i}^{E_{i+1}} \phi_j \times E^{-2.7} dE$$

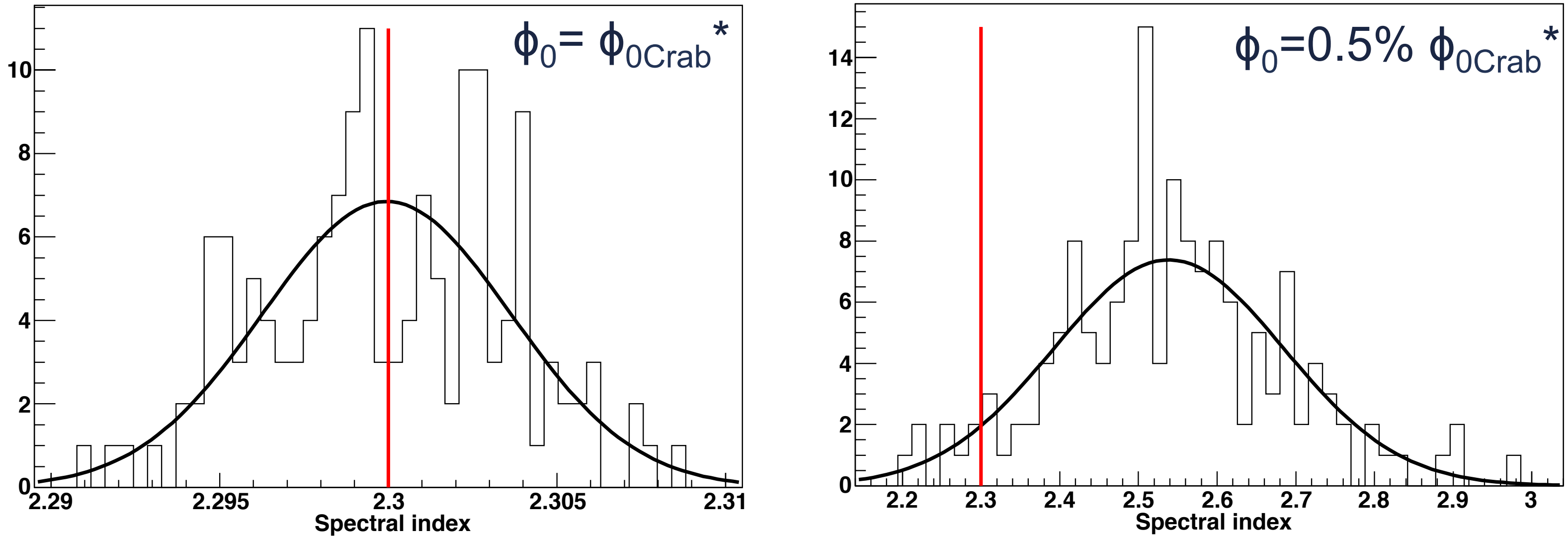
$$N_{OFF\,i,j} = \overline{B_{i,j}} \qquad (E < 1TeV)$$

$$N_{OFF\,i,j} = P(\overline{B_{i,j}}) \quad (E > 1TeV)$$

MC simulation for bright and faint sources

- 250h of observation with a large variation of the observational parameters
- We generate150 simulated spectra and apply each time the fitting procedure to these individual fake data

Distribution of the spectral index Γ fitted on 150 simulated spectra



Processes all the observations individually for the likelihood maximization

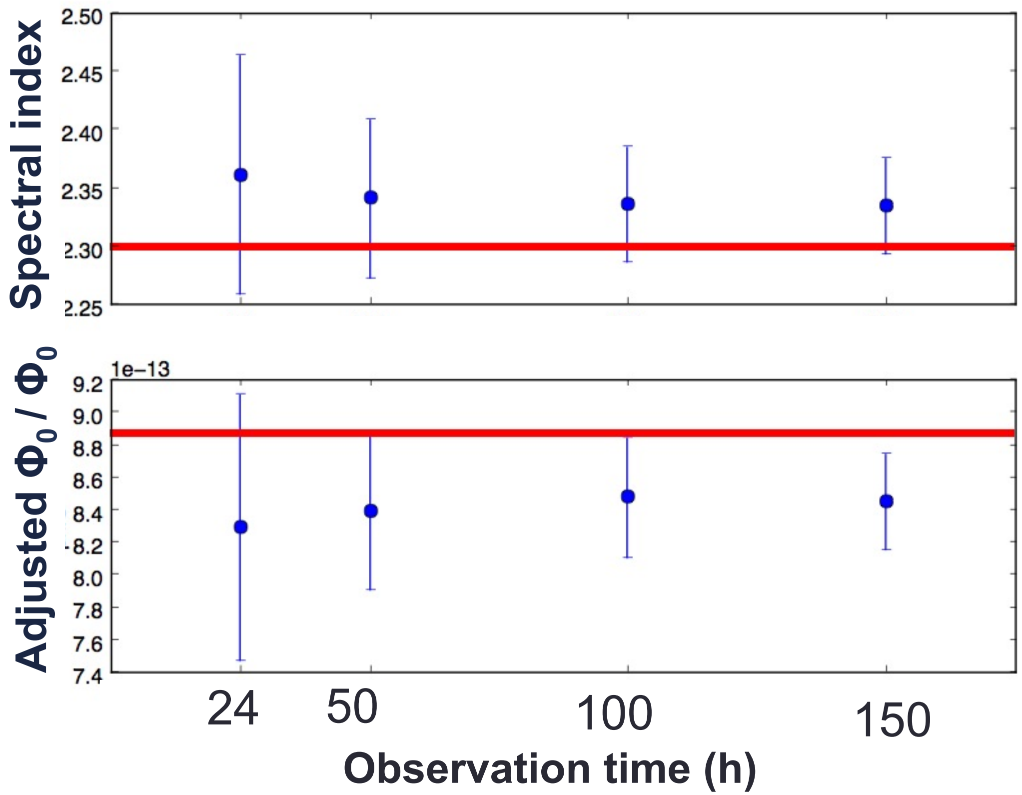
- Bright source: estimators consistent and not biased
- For faint sources: Necessity of spectral grouping to avoid inconsistency

$^*\phi_{0Crab} = 4.44 \times 10^{-11}\ cm^{-2}\ s^{-1}\ TeV^{-1}$

No grouping: Inconsistent estimators

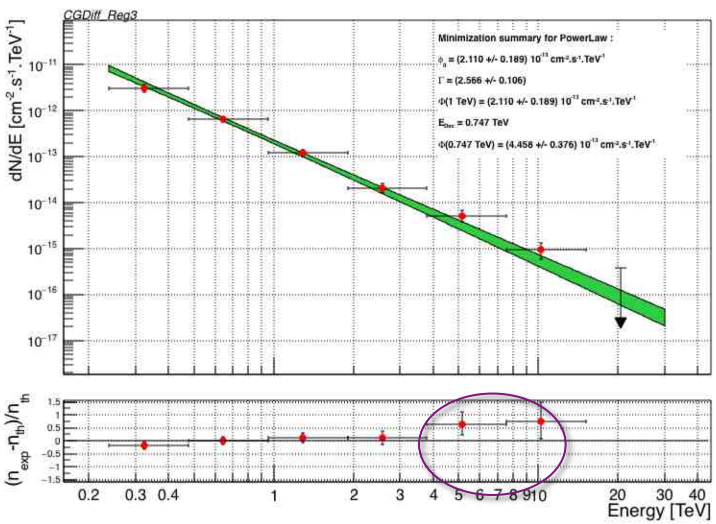
- Several observations time: 24h, 50h, 100h, 150h
- Power-law: Γ=2.3 and ϕ_0= 2% ϕ_{0Crab}*

Mean value of the adjusted parameters. distribution on the 150 simulated spectra.

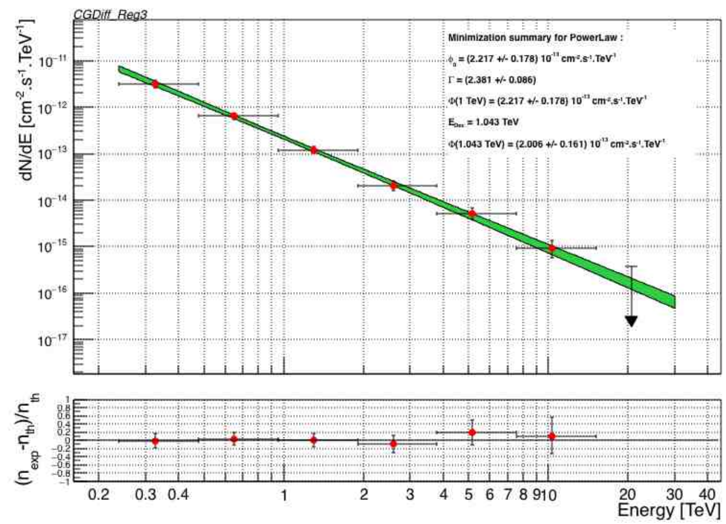


Observations grouping

- The IRFs and OFF event distribution: large variations depending on the observational parameters
- The observation grouping **combines the observations with similar observational conditions** in various bands of zenith, offset and efficiency. `
- *The maximization of the likelihood function is done band by band with this observations grouping option.*



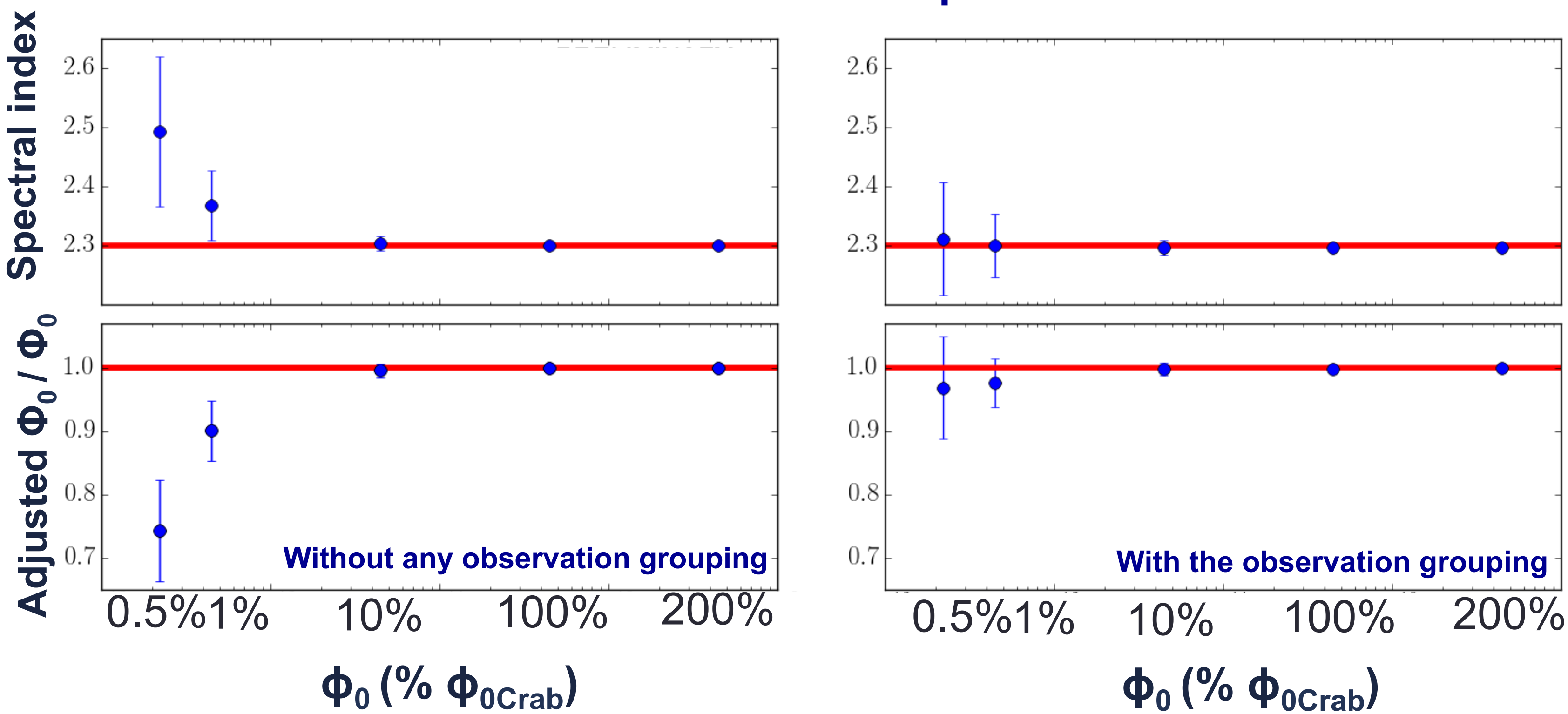
Without any observation grouping



With the observation grouping

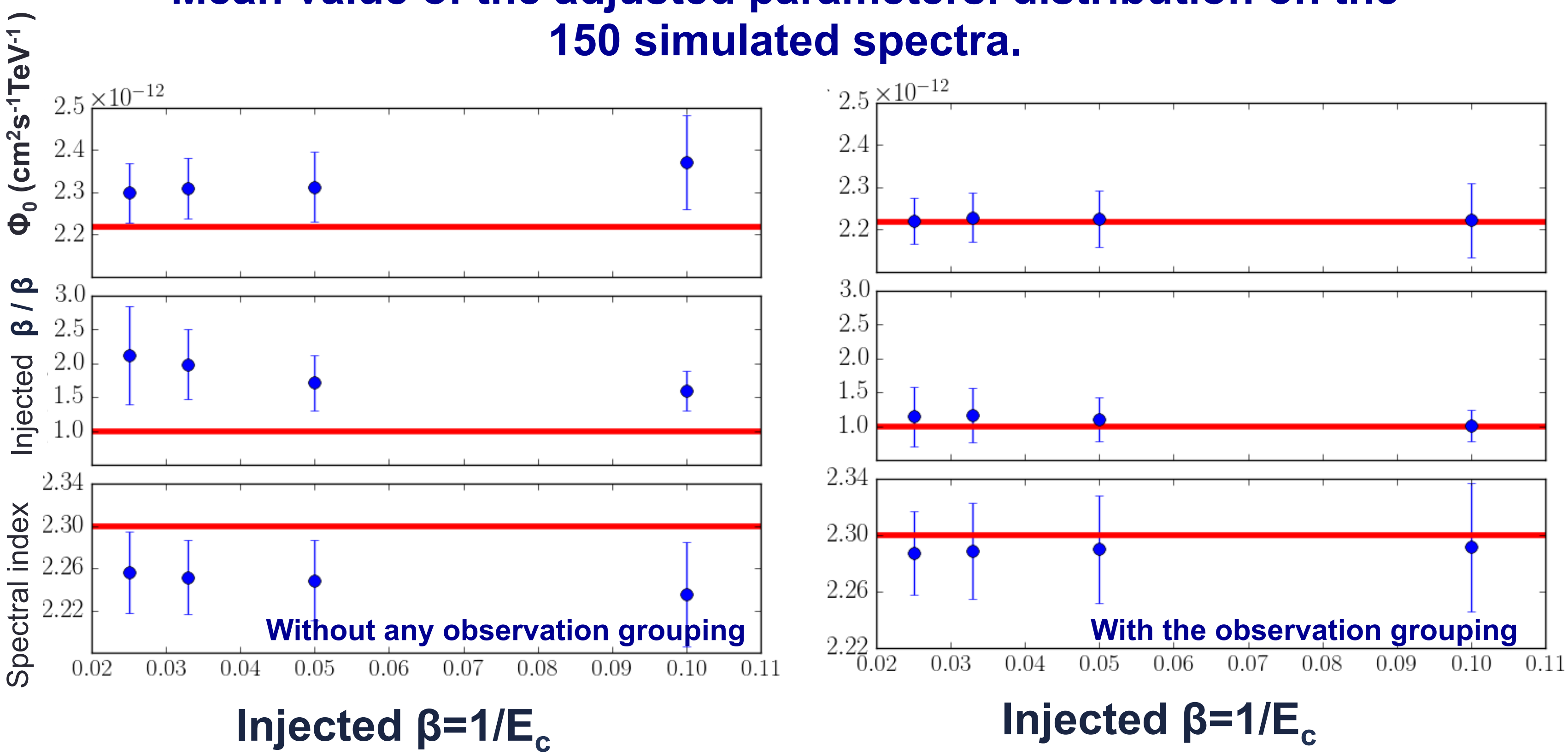
Power-law (Γ=2.3) : impact of the differential flux normalization ϕ_0

Mean value of the adjusted parameters. distribution on the 150 simulated spectra.



Power-law with an exponential cutoff (Γ=2.3 and ϕ_0=5%ϕ_{0Crab}): impact of the cutoff E_c

Mean value of the adjusted parameters. distribution on the 150 simulated spectra.



Conclusion

- **ON-OFF likelihood** can lead to **bias spectral parameters** especially **for faint sources**.
- **Combining** the observations greatly **improves the reconstruction** of the spectral parameters.
- MC simulation is necessary to test the consistency of the fitted spectral parameters.
  - Spectral analysis package must provide the ability to fake a data set
- Using a hadronic background model remove this issue but introduce a systematic effect

Thanks for your attention