Classical analysis in VHE γ-ray astronomy

Spectrum and morphology

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Outline

Introduction

Spectral analysis

Morphology analysis

Introduction

- A reminder of standard high level VHE data analysis techniques
  - Instrument Response Functions
  - Background estimation for spectra and maps
  - Forward folding methods
  - Statistics : maximum likelihood & profile likelihood

- Using `sherpa` for spectral and morphology fitting of HESS data
  - Requires converters for end products of HESS analysis chains to fits files (spectra, IRFs and maps)
  - Provide a common set of high level tools for all HESS analysis chains
  - Easy to interface with other packages e.g. for physical model fitting

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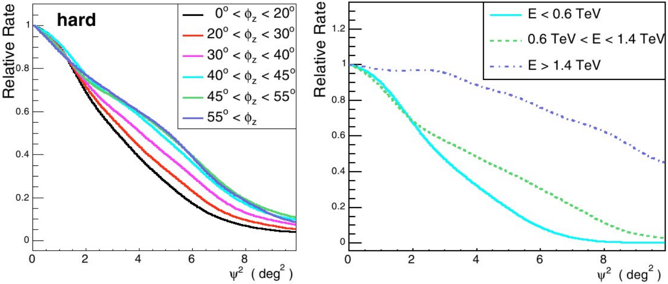
Hadron acceptance

- Dominant background due to γ-like hadrons passing selection cuts
- Total count-rate and morphology in the FoV strongly depend on :
  - observing conditions : zenith angle, optical efficiency, sky position (NSB).
  - measured event energy



from Berge et al. (2007)

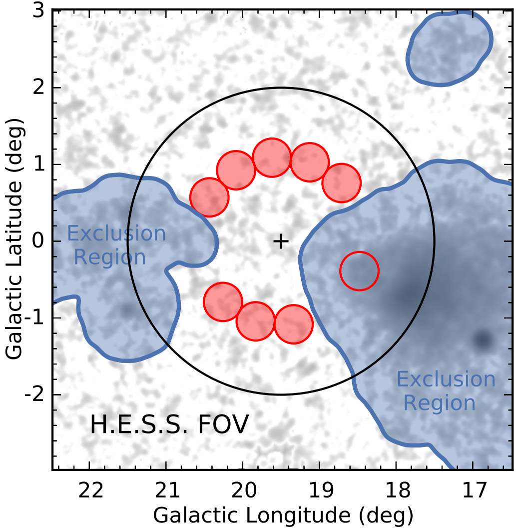
Background subtraction techniques

- Need 2 datasets N_{ON} (signal + bkg) and N_{OFF} (bkg)
- Excess events $N_{ex} = N_{ON} - \alpha N_{OFF}$ with α, ratio of hadron acceptance for ON and OFF datasets.

$$\alpha = \frac{\int_{ON} Acc}{\int_{OFF} Acc}$$

- Background usually estimated from the same observation :
  - Reflected background (spectral analysis)
  - Ring background (images)
- In order to avoid contamination by γ-ray events in the data, define exclusion regions around regions with significant excess

Extracting ON & OFF spectra : reflected background



from HGPS paper, HESS collab. in prep

Instrument Response Functions (IRF)

- Consider a source located at position (x_0, y_0) in the FoV
- For a spectrum Φ, the expected number of events measured at position (x_r, y_r) and energy E_r during time t is written :

$$
\begin{aligned}
\bar{N}(x_r, y_r, E_r | x_0, y_0, F) = \int dE_0 \; \Phi(E_0) \times t \\
\times A_{eff}(x_0, y_0, E_0) \\
\times PSF(x_r, y_r | x_0, y_0, E_0) \\
\times ED(E_r | x_0, y_0, E_0)
\end{aligned}
$$

- In practice, IRFs are strongly dependent on observing conditions, esp. zenith angle, optical efficiency etc.

Forward folding

- Compare observed number of excess events to expected number for a given model of signal
- For spectral analysis, assume source spectrum follows $\Phi(E_0)$ depending on few numerical parameters
- In reconstructed energy bin i, we expect :

$$m_i = m\left(E_i < E_{rec} < E_{i+1}\right) = \int_{E_i}^{E_{i+1}} dE_{rec} \int dE_0 \; t \; \Phi(E_0) \\ \times A_{eff}(E_0) \times ED(E_{rec}|E_0)$$

- If we have a background model b_i and have observed ON_i, we can use a ML fit with Cash statistics :

$$\log \mathscr{L} = \sum_i ON_i \log\left(m_i + b_i\right) - \left(m_i + b_i\right)$$

Profile likelihood

- We have ON_i and OFF_i events per bin i. We have want to maximize :

$$\log \mathscr{L} = \sum_i ON_i \log(m_i + b_i) - (m_i + b_i) + OFF_i \log\left(\frac{b_i}{\alpha}\right) - \frac{b_i}{\alpha}$$

- We have no model for background, so we want to marginalize over b_i

$$\frac{\partial \log \mathscr{L}}{\partial b_i} = \frac{ON_i}{m_i + b_i} + \frac{OFF_i}{b_i} - \left(1 + \frac{1}{\alpha}\right) = 0$$

- Hence

$$b_i = f(m_i) = \frac{1}{2}\left[\frac{ON_i + OFF_i}{(1 + \frac{1}{\alpha})} - m_i\right] + \frac{1}{2}\sqrt{\left[m_i - \frac{ON_i + OFF_i}{(1 + \frac{1}{\alpha})}\right]^2 + 4\frac{m_i\, OFF_i}{(1 + \frac{1}{\alpha})}}$$

Profile likelihood

- Finally :

$$\log \mathscr{L} = \sum_i ON_i \log\left(m_i + f(m_i)\right) - \left(m_i + f(m_i)\right) + OFF_i \log\left(\frac{f(m_i)}{\alpha}\right) - \frac{f(m_i)}{\alpha}$$

- Marginalization has reduced the number of fitted parameters to the few describing the signal
- Yet the actual number of parameters is as large as the number of energy bins times spectra
  - Might lead to ill-posed problems
  - Requires careful grouping of observations to limit statistical biases

  see L. Jouvin's talk
- Profile likelihood is called `wstat` in XSpec & sherpa

Spectral fitting with `sherpa` : `hspec`

- Spectral fitting for VHE is very similar to X-rays.
  - dataset split into many different observations
  - low signal to noise ratio
- Using `sherpa` is interesting for VHE high level analysis :
  - A python package that can be used with other packages (e.g. astropy, naima...)
  - A set of well tested tools for 1D and 2D fitting
  - Easy arithmetics of models and parameters
  - A common set of tools for all HESS analysis chains
- Create converters for H.E.S.S. spectra and IRFs to OGIP compliant fits files
  - format defined in memo OGIP 92-007 K. Arnaud et al.
  - spectra stored in `pha` files, IRFs in `arf` and `rmf` files
  - converters implemented using the START C++ package (from J. Lefaucheur, V. Marandon)

The OGIP format : `arf` and `rmf`

- we rewrite the expected number of signal counts in bin i :

$$m\,(E_i < E_{rec} < E_{i+1}) = \int dE_0\; t_{obs}\; \Phi(E_0) \times A_{eff}(E_0) \times \int_{E_i}^{E_{i+1}} ED(E_{rec}|E_0)$$
$$= \int dE_0\; t_{obs}\; \Phi(E_0) \times \mathtt{arf}(E_0) \times \mathtt{rmf}(E_0, i)$$

- `arf` ancillary response function
  - 3 columns fits table (ENER_LO, ENER_HI, SPECRESP)
- `rmf` redistribution matrix function
  - MATRIX extension with 6 columns
  - for each true energy (ENER_LO, ENER_HI), redistribution matrix in contiguous groups (N_GRP, F_CHAN, N_CHAN, MATRIX)
  - EBOUNDS extension : 3 col table (CHANNEL, E_MIN, E_MAX) : bin number and rec. energy

Combining observations

- Most of the time it is required to combine spectra to improve statistics
- For run i and energy bin j :

$$ON_{tot,j} = \sum_i ON_{i,j} \text{ and } OFF_{tot,j} = \sum_i OFF_{i,j}$$

- In order to recover correct expected number of signal events :

$$t_{tot} = \sum_i t_i$$

$$\mathtt{arf}_{tot} = \frac{1}{t_{tot}} \sum_i t_i \, \mathtt{arf}_i$$

$$\mathtt{rmf}_{tot} = \frac{1}{\sum_i t_i \, \mathtt{arf}_i} \sum_i t_i \, \mathtt{arf}_i \, \mathtt{rmf}_i$$

- Note : Average α will often depend on reconstructed energy bin. This is not possible with regular `backscale` parameter used in `sherpa`

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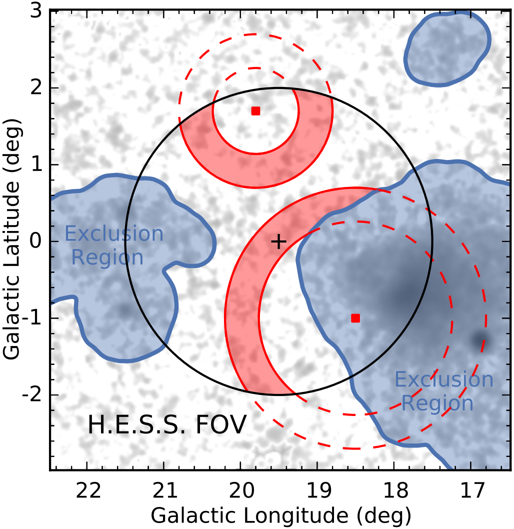
Maps production

- Event (ON) map
- Background exposure map
  - Expected number of bkg events in ON map from blank sky obs. in similar conditions
  - Normalized to observed events outside exclusion regions
- OFF map
  - Local estimation : ring background
- Significance map
  - Correlated (top-hat function) on various scales.
  - Li & Ma significance.
- γ exposure map & flux map
  - Assuming a spectral shape of γ-ray signal Φ

$$\epsilon = \frac{t}{\int dE_0 \Phi(E_0)} \int_{E_{min}}^{\infty} dE_0 \Phi(E_0) \times A_{eff}(x, y, E_0)$$

Extracting background from images

Ex : Adaptive ring background



from HGPS paper, HESS collab. in prep.

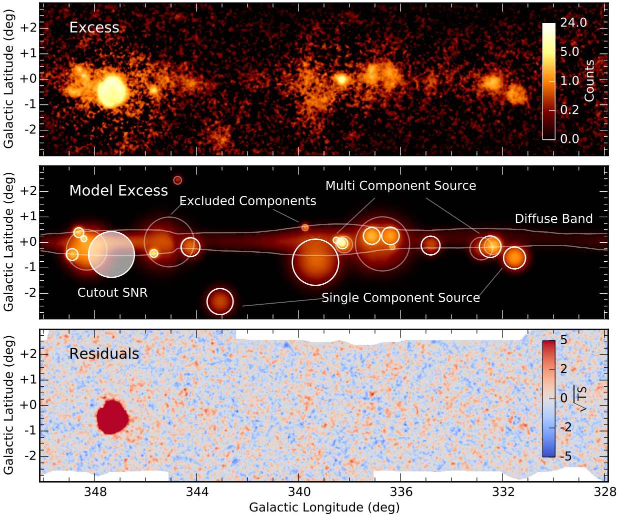
Morphology fitting with `sherpa`

- Export maps to fits dataset with proper astrometry (`wcs`)
  - ON map (event map) N_{ON}
  - OFF map normalized N_{Bkg}
  - exposure map ϵ
  - export PSF (as 3 gaussian parametrization) per ROI
- apply ML fit with Cash statistics to compare N_{ON} to

$$N_{pred} = N_{Bkg} + PSF \otimes \left[\epsilon \times \left(I_{template} + \sum_i I_{Gauss,i} \right) \right] \tag{1}$$

  - Sources represented by gaussian functions : I_{Gauss} .
  - Include extended features $I_{template}$ from fits files using `table_model`
  - Reprojection using NASA/IRSA `montage` package and python wrapper
- Apply iterative source fitting with TS criterion
- Note : No uncertainty on N_{Bkg} is taken into account. `wstat` does not apply straightforwardly because of bin to bin correlation.

Application : HGPS



Deil et al. ICRC 2015, from HGPS paper, HESS collab. in prep.

To conclude

- Spectral and morphology fitting of HESS final high level products with `sherpa`
  - Fits converters for all analysis chains

- Perform high level analysis with python open source tools directly from fits event lists and IRFs

  (*see J. King's talk*)

- Perform 3D analysis of VHE data

  (*see P. Eger talk*)