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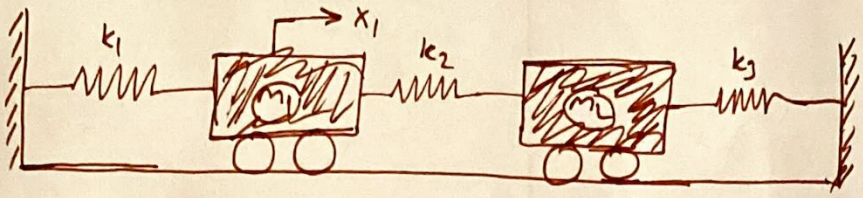
28/03/2022

MECH534: Computer Based Modeling & Simulation

Spring 2022

a)



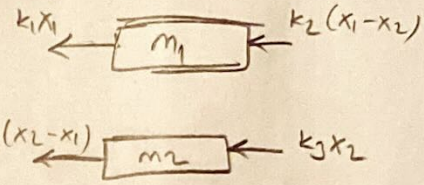
$F = m\ddot{x}$

$kx = m\ddot{x}$

Assume that there is no friction in the system:

$$m_1\ddot{x}_1 = -k_1x_1 + k_2(x_2 - x_1)$$

$$m_2\ddot{x}_2 = -k_2(x_2 - x_1) - k_3x_2$$



b)

$$\ddot{x}_1 = -\frac{k_1}{m_1}x_1 + \frac{k_2}{m_1}x_2 - \frac{k_2}{m_1}x_1$$

$$\ddot{x}_1 = -\left(\frac{k_1+k_2}{m_1}\right)x_1 + \frac{k_2}{m_1}x_2$$

$$\ddot{x}_2 = -\frac{k_2}{m_2}x_2 + \frac{k_2}{m_2}x_1 - \frac{k_3}{m_2}x_2$$

$$\ddot{x}_2 = \frac{k_2}{m_2}x_1 - \left(\frac{k_2+k_3}{m_2}\right)x_2$$

where $\ddot{x} = \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}$

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^t$

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} -\left(\frac{k_1+k_2}{m_1}\right) & \frac{k_2}{m_1} \\ \frac{k_2}{m_2} & -\left(\frac{k_2+k_3}{m_2}\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^t$$

$\ddot{x}$ $\quad$ A $\quad$ x

c) $x = be^{jwt}$

$\ddot{x} = -w^2 be^{jwt}$

$-w^2 be^{jwt} = A\, be^{jwt}$

$-w^2 b = Ab$

If $\lambda = -w^2 \Rightarrow \boxed{\lambda b = Ab}$ //

d) From part b:

$$A = \begin{bmatrix} -\frac{k_1+k_2}{m_1} & \frac{k_2}{m_1} \\ \frac{k_2}{m_2} & -\frac{k_1+k_3}{m_2} \end{bmatrix}$$

$k_1 = k_2 = k_3 = 1$

$m_1 = m_2 = 1$

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$$ //