Council for Technical Education and Vocational Training

Office of the Controller of Examinations

Sanothimi, Bhaktapur

Regular -2079, Ashad

Program:	**Diploma in Engineering All**	**Full Marks: 80**
Year/Part:	**I/I (2021 New Course)**	**Pass Marks: 32**
Subject:	**Engineering Mathematics I**	**Time: 3 hrs**

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Group 'A'

Attempt All questions. [7x(2+2)=28]

1. a) If $A = \{2,3,4,5,6,7\}, B = \{4,5,6,7,8\}$ and $c = \{1,2,3,4,5\}$, find i) $(A \cup B) \cap C$ ii) $(A \cap B) \cup C$

   b) Rewrite $|2x - 1| \leq 5$ without using absolute value sign.

2. a) Prove that : $\sin(2sin^{-1}x) = 2x\sqrt{1-x^2}$.

   b) In any ΔABC, show that. $c(a\ cosB - b\ cosA) = a^2 - b^2$

3. a) If $\frac{CosA}{a} = \frac{CosB}{b}$, prove that the triangle is an isosceles.

   b) Evaluate : $\lim_{x\to 2} \frac{x^2-5x+6}{x^2-x-2}$.

4. a) Find $\frac{dy}{dx}$; when $y = \frac{1}{\sqrt{ax^2+bx+c}}$.

   b) Find $\frac{dy}{dx}$ when $y = \cos(sin\sqrt{3x+5})$

5. a) Integrate : $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx$.

   b) The sum of an infinite G.S. is 15 and the first term is 3. Find the common ratio.

6. a) In how many ways can the letters of the word "MATHEMATICS" be arranged?

   b) Find the seventh term in the expansion of $\left(3x^2 - \frac{1}{2x}\right)^{12}$.

7. a) Find the distance between the parallel lines $3x + 4y - 5 = 0$ and $6x + 8y + 17 = 0$.

b) Find the angle betⁿ two lines represented by $x^2 - 2xy\,cot\theta - y^2 = 0$.

Group 'B'

Attempt All questions. [13x4=52]

8. If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$ prove that $x^x y^y z^z = 1$.

OR

Let $f: R \to R, g: R \to R$ which are defined by $f(x) = x^3 + 1$ and $g(x) = x^5$ respectively then find

a) $fog(x)$ b) $gof(x)$ c) $f^{-1}(x)$

9. Solve : $tan^2 x = secx + 1$.

OR

Solve : $sin^{-1}\frac{2a}{1+a^2} - cos^{-1}\frac{1-b^2}{1+b^2} = 2tan^{-1}x$.

10. If $a^4 + b^4 + c^4 = 2a^2(b^2 + c^2)$ prove that $A = 45°$ or $135°$.

OR

Solve the ΔABC, if $b = \sqrt{3}, c = 1$ and $A = 30°$.

11. Evaluate : $\lim_{x\to\theta} \frac{x\tan\theta - \theta\tan x}{x-\theta}$.

OR

A function $f(x)$ is defined as follows.

$$f(x) = \begin{cases} 2x+1 & for \quad x < 1 \\ 2 & for \quad x = 1 \\ 3x & for \quad x > 1 \end{cases}$$

Is the function continuous at $x = 1$? If not, how can it be made continuous at $x = 1$?

12. Find from first principle, the derivatives of $\sqrt{tanx}$ or $\frac{1}{\sqrt{4-5x}}$.

13. Integrate : (any one)

a) $\int \frac{dx}{x^2\sqrt{9-x^2}}$ b) $\int sec^3 x\,dx$

Cont.....

14. Prove that the AM, GM and HM between any two unequal positive numbers satisfy the relation.

i) $(GM)^2 = AM \times HM$ ii) $AM > GM > HM$

OR

Find the sum to infinity $1 - 3x + 5x^2 - 7x^3 + \cdots\cdots (|x| < 1)$.

15. From 6 gentleman and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done as to include at most two ladies?

16. Prove that : $\frac{1.2}{1!} + \frac{2.3}{2!} + \frac{3.4}{3!} + \cdots \infty = 3e$

17. Find the equations of the bisectors of the angles between the lines $4x - 3y + 1 = 0$ *and* $12x - 5y + 7 = 0$. Also show that bisectors are at right angle.

18. Find the separate equations represented by $2x^2 + xy - 3y^2 + 9x + 26y - 35 = 0$. Also find the angle between them.

OR

Prove that the straight lines joining the origin to the point of intersection of the line $\frac{x}{a} + \frac{y}{b} = 1$ and the curve $x^2 + y^2 = c^2$ are at right angles if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$.

19. Find the equation of circle passing through the points (3, -2) and (-2, 0) whose centre lies on the line $2x - y = 3$.

20. Find $\frac{dy}{dx}$ (any one)

i) $x^2y^2 = tanxy$ ii) $x^y . y^x = a$

Good Luck!