COUNCIL FOR TECHNICAL EDUCATION AND VOCATION TRAINING
Office of the Controller of Examinations
Sanothimi, Bhaktapur

Model Question Set | New Course (2080)

Program: Diploma in Engineering All
Year/Part: I/II (New)
Subject: Enginerring Mathematics-II

Complex numbers:-

1. Define complex number. Express (2+5i)+ (1-i) in the form of x+iy (AM: 3+4i)
2. Express: $Z = \frac{6+3i}{3-4i}$ in the form of x+iy also separate real and imaginary part. [Re(z)=$\frac{6}{25}$, Im (2)=
3. State and prove that De-Moivre's theorem for any positive integer n.
4. Evaluate using De'movre's theorem $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^7$ [Ans: ½ +i$\frac{\sqrt{3}}{2}$]
5. Evaluate: $(1+i)^{20}$ by using De- Moivre's theorem [$(-2)^{10}$= 1024
6. Find the cube roots of unity by using De- moivre's theoem [1, $\frac{-1+i\sqrt{3}}{2}$, $\frac{-1-i\sqrt{3}}{2}$]
7. State De'moivre's theorem and use it to find the cube roots of i. $\left[\cos\frac{4k\pi+\pi}{0} + i\sin\frac{4k\pi+\pi}{6}\right]$ what k= 0_{1}.
8. Find the square roots of the following

a) (-5-5i)

$4\sqrt{50}\,(\cos\frac{3\pi}{8} - i\sin\frac{3\pi}{8}, 4\sqrt{50}\,(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8})$

b) ($4+4\sqrt{3}$ i)

$(\sqrt{2}(\sqrt{3}+i) and -\sqrt{2}\,(\sqrt{3}+i)$

9. If z_1 and z_2 are two complex numbers show that $|z_1+z_2| \le |z_1|+|z_2|$
10. If w is complex cube root of unity then prove the

a) $(1-w-w^2)^5+(1+w-w^2)^5=32$

b) $(1-w+w^2)^3+(1-w^2+w)^3=64$

c) $(1-w-w^2)^3+(1+w-w^2)^4=16$

1. Matrix transpose of a matrix. If A= $\begin{bmatrix}1 & 2\\3 & 0\end{bmatrix}$B= $\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$ verify that $(AB)^T=B^T A^T$.

2. Show that: $\begin{vmatrix}a+b+c & c & c\\a & b+c+2a & a\\b & b & c+a+2b\end{vmatrix}=2(a+b+c)^3$

3. When will two matrices be inverse to each other? Show that matrices $\begin{pmatrix}3 & -1\\5 & -2\end{pmatrix}$ *and* $\begin{pmatrix}2 & -1\\5 & -3\end{pmatrix}$ are inverse of each other and verify that their transposes are also inverse of each other.

4. Prove that: $\begin{vmatrix}a & b & ax+by\\b & c & bx+cy\\ax+by & bx+cy & 0\end{vmatrix}=(b^2-ac)(ax^2+zxy+cy^2)$

5. Define cofactor of a matrix A. Prove that

$\begin{vmatrix}a+x & b & c\\a & b+y & c\\a & b & c+z\end{vmatrix}=xyz\left(1+\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right)$

Ans $\begin{pmatrix}-3 & 2 & 6\\-1 & -1 & -3\\2 & -3 & -4\end{pmatrix}$

6. If A= $\frac{-1}{5}\begin{pmatrix}1 & 2 & -1\\2 & 0 & -1\\0 & 3 & -1\end{pmatrix}$ then find A^{-1}.

7. Solve the following systems by using cramer's or tow equivalent's method:

a) x-2y-z=7, 2x+y+z=0, 3x-5y+8z=13

$\left(x=\frac{3}{2}, y=\frac{-5}{2}, z=\frac{-1}{2}\right)$

b) x+y-z=3, 2y+z=10, 5x-y-22-3

(x=1, y=4, z=2)

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Soluation of system of linear equation

1. MaximizeM F= 2x-y subject to: x+y+≤5
X+2y+≤8, x≥0, y≥0.
[Max. F+ 10 at (5, 0)]

2. Maximize: and minimize: F 9x+40y
Subject to y-x≥1, y-x≤3, x≤x≤5
[$F_{Max.}$ = 365 at (5,8)
[$F_{min.}$=138 at (2,3)}

3. Maximize and Minimize: G= 2x+3y, subject to x+y 12, 3x+2y≥25, x≥0, y≥0.
[G_{Max}= 37.5 at (0,12.5)
[G_{Min}= 24 at (12, 00.

4. Find the extreme values of the function F(x,y) defined by F(x,y) = x+y on the convex polygonal region given by the in equalities: 2x+y≤20, 2x+3y≤24, x≥0, y≥0
[$F_{Max.}$= 11 at (9,2)
[F_{Min}=0 at (0, 0)

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5. **A baker has 90 80 and 50 units of ingredients A, B and C respectively. A loaf requires 2, 1 and 1 units of A, B and C respectidy and Cake requires 1, 2, 1 units of A, B and C respectively. If a loaf sells for a profit of 5.60 and cake for Rs. 41 formulate the above linear programming.**

Problem for maximum profit. [Z=5.60x+4y]
2x+y≤90
x+2y≤80
x+y≤50 x,y≤0

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Conic section

1. Find the equation of the parbola whose vertex is at (3, 2) and focus (5,2).
[y^2-4x-8x+28-0)
2. Find the co-ordinate of focus, the vertes, the equation of directrix, the length of latus rectum of the parabola.
a. y^2=6y-12x+45 [Vetex (4-1), dire.x=5, 4]
b. $y^2$4x+2y-15=0 [Focus: (3/2, 3), vert. (9/2, 3)
c $(x+1)^2$+8y-16=0 [(-1,0) (-1,2), y-4=0,8]
3. Find the eccentricity, coordirectes of the vertices and foci and alsothe length of the major axis and minor axis of the ellipse $\frac{x^2}{25}+\frac{y^2}{10}=1$
[e=$\frac{3}{5}$, vertices (±5,0), Foci (±3,0) length of m.. axis =10 [Min axis=8]

4. Find the eceuity and co-ordinates of the vertice and foci of the ellipse:
$\frac{(x-1)^2}{16}+\frac{(y-2)^2}{4}$=1
$A=\pi r^2$5.Find the equation of the ellipse whose foci (±2,0) and length of latus rectum is 6.

[$\frac{x^2}{16}+\frac{y^2}{12}=1$].

5. Find the equation of the ellipse whose foci cube (±2, 0) and length of latus rectum is 6.

$$\left[\frac{x^2}{16}+\frac{y^2}{12}\right]=1$$

6. Find the ecnricity, the co-ordinates of the centre and the foci of the ellipse.
X^2+4y^2-4x+24y+24=0

$$[\frac{\sqrt{3}}{2},(2,-3),(2\pm 2\sqrt{3},-3]$$

7. Find the vertices, centre, eccertnity, foci of the hyperbola:
$9x^2-16y^2-18x-64y-199=0$
[vertices: $(1\pm4,-2), cere\ (1,-2), e=\frac{5}{4}, foci\ (1\pm5,=2)$]
8. Construct the equation of hyperbola in its standarded form with focous at (-5,0) and vertex (2,0)

Co-ordinate in space:

1. Find the equation of the plane through the points (2,2,1) and (3,1,2) and perpendicular to the plane x+2y+3z=5.
2. Find the point where the line through (1,5,11) and (-1,-1,-1) meets yz-plane.

3. Find the equation of the plane through the intersection of the planes x+y+z=6 and 2x+3y+4z=5=0 and perpendicular to the plane 4x+5y-3z=8.
4. Find the direction cosines, l, m, n of two lines which satiesfiay the equations:
L+m+n=0 and 2lm-mn+2nl=0.
$\left(\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{-2}{\sqrt{6}}\right)$ and $\left(\frac{1}{\sqrt{6}},\frac{-2}{\sqrt{6}},\frac{1}{\sqrt{6}}\right)$
5. Find the equation of plane containing the lines through the origin with direction cosines proportional to 2, 1, -2 and 5, 2, -3.
6. Define direction cosines and direction ratio's of a line. Show that: $l^2+m^2+n^2=1$, where l, m, n have their usual meaning.
7. Find the direction cosines l, m, n of two lines which satistrg the equations 2l+2m-n=0 and lm+mn+nl=0.Also fin dthe angle between those lines.
Ans: [$\left[\frac{1}{3},\frac{-2}{3},\frac{-2}{3}\ and\ \frac{2}{3},\frac{-1}{3},\frac{2}{3}\right]; \frac{\pi}{2}$
8. Prove that the lines where direction cosines are given by the relations al+bm+cn=0 and fmn+gnl+hlm=0 are perpendicular if $\frac{f}{a}+\frac{g}{b}+\frac{h}{c}=0$

Solution: Given relations are
al+bm+cn=0
fmn+gnl+hlm=0
Elliminating n between (1) and (2), we have
fm$\left(\frac{-al+bm}{c}\right) g \left(\frac{-al+bm}{c}\right)$l+hlm=0
$agl^2+(af+bg-ch)lm+bfm^2=0$
ag$\left(\frac{l}{m}\right)^2+(ch-af+bg)\left(\frac{l}{m}\right)$+bf=0

Which is quadratic in $\left(\frac{l}{m}\right)$, *Let the two roots be* $\frac{l_1}{m_1}$ *and* $\frac{l_2}{m_2}$

Now, $\frac{l_1}{m_1} \cdot \frac{l_2}{m_2} = \frac{bf}{ag}$

$$\frac{l_1 l_2}{bf} = \frac{m_1 m_2}{ag}$$

$\frac{l_1 l_2}{f/a} = \frac{m_1 m_2}{g/b}$..............(3)

Similarly, If are eliminate l between (1) and (2), neget

$\frac{m_1 m_2}{g/b} = \frac{n_1 n_2}{n/c}$............(4)

From (3) and (4) we ,

$\frac{l_1 l_2}{f/a} = \frac{m_1 m_2}{g/b} = \frac{n_1 n_2}{h/c}$=k(say)

$L_1 l_2 = k\frac{f}{a}$, $m_1 m_2 = k\frac{g}{b}$ and $n_1 n_2 = k\frac{h}{c}$

The two lines are parallel if

$L_1 l_2 + m_1 m_2 + n_1 n_2 = 0$.

$k\frac{f}{a} + k\frac{g}{b} + k\frac{h}{c} = 0$

i.e. $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$

6.

3.By using vector method find the erea of triangle formed by the points: A(1,1,1), B (1,2,3) and C (2,3,4).

[Ans: $\frac{1}{2}\sqrt{6}$ sq. units]

4.Prove that cos (A+B)= CosA.cosB-sinA.sinB by using vector method.

5.Prove that sin (A-B)= sinA.cosB-cosA.sinB by using vector method.

6.Prove by vector method, In any ΔABC,

$CosB=\frac{a^2+c^2-b^2}{2ac}$

7.Find the area of the parallelogram determined by vector $\vec{i}+\vec{j}+\vec{k}$ and $-2\vec{i}+3\vec{j}+\vec{k}$.

8.Show that the vector $5\vec{a}+6\vec{b}+7\vec{c}, 7\vec{a}-8\vec{b}+9\vec{c}$ and $3\vec{a}+20\vec{b}+5\vec{c}$ are coplanar, where $\vec{a}, \vec{b}, \vec{c}$ are any three vectors.

9.Determine wheather the following vectors are linearly dependent or independent:

i) $\vec{i}+\vec{k}, \vec{i}+\vec{j}$ and $-\vec{i}-\vec{k}$

ii) $2\vec{i}+3\vec{j}+4\vec{k},\ \vec{i}-\vec{j}+2\vec{k}$ and $5\vec{i}+6\vec{j}+8\vec{k}$

[Ans: (i) LD (ii) L.Ind]

10.Find the equation of line through (5,6,7) and (3,2,-1) in symmetrical form by vector method.

[Ans: $\frac{x-5}{1}=\frac{y-6}{2}=\frac{z-7}{4}$]

Statistics

1. Calculate the arithmetic mean and standard deviation from the given data.

Marks	0-4	4-8	8-12	12-16	16-20	29-24
No. students	7	7	10	15	7	6

(Ans= 12, s.d.=6.05)

2. Calculate quartile deviation and it's coefficient form the following data:

Class	0-20	20-40	40-60	60-80	80-100
Frequency	4	8	12	3	5

[Q.D= 15, coefficient of Q.D.= 0.33)

3. Find the standard deivation and coefficient of variation (c.v.) from the following data:

X:	5	10	15	20	25
F:	2	4	1	6	9

(s.d.=6.94, c.v.=37.23%)

4. The distribution of the marks of Automobile students of KNPI college in the paper of mathematics as follows:

Marks:	20-30	30-40	40-50	50-60	60-70	70-80	80-100
No. of students:	2	5	22	34	9	3	1

Find, Medium of this distribution

(Ans: 52.65)

5. Find the mode form the following data:

Weight (kg)	2-2.4	2.4-2.8	2.8-3.2	3.2-3.6	3.6-4	4-4.4
No. of child	5	5	9	4	4	3

(Ans: 2.978 kg)

6. Calculate A.M., G.M. and H.M. of the following data. Also prove that A.M.> G.M.> H.M.

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	3	7	15	2	3

(Ans: AM= 24.33, G.M.=20.49, H.M.=16.73)

7. Find median, lower and upper quartiles, 4^{th} deciles and 60^{th} percentiles form the following data:

Marks:	0-4	4-8	8-12	12-14	14-18	18-20	20-25	25 and above
No. of students:	10	12	18	7	5	8	4	6

(Ans: A_1=6.5, M_d=10.88 Q_3=18.125 (D_4)=9.33 P_{60}=12.57)

8. Calculate Karl- Pearson coefficient of skewness from the following data and interpret result.

Size:	30-33	33-66	36-39	39-42	42-45	45-48
Frequency	3	5	26	46	20	10

(Ans: S_k(P)=0.018)

9. Calculate correlation (Karl Parson's) coefficient from the following data:

X:	12	9	8	10	11	13	7
Y:	14	8	6	9	11	12	3

(Ans: y=4.17+0.87x

X=6+7]

Probability

1. A problem of mathematics is given to the three students A, B and C and the chance of solving it are $\frac{1}{3}, \frac{1}{4}$ *and* $\frac{1}{5}$ respectively. Find the probability that the problem will solve. (Ans. $\frac{3}{5}$)

2.A bag contains 5 white, 7 red and 8 black balls. If two balls are drawn one by are without replacement, what is the probability that both are white. (Ans: $\frac{1}{19}$

3.Suppose 3 people are selected at random from a group of 7 men and 6 women. What is the probablity that 2 men and 1 women are selected? (Ans: 0.441)

4.The mean and standard deviation of the binomial distribution are 40 and 6 respectively. Find the values of n,p,q.
(Ans: n=400, p=0.1, q=0.9)

5.The probability that a student passes a mathematics test is $\frac{3}{5}$ and the passes both Math and chemistry test is $\frac{1}{5}$. The probability that he passes at least one test is $\frac{19}{20}$. What is the probability that he passes the chemistry test?
(Ans: $\frac{3}{20}$)

6.A binomial distribution consists of 5 independent trials. If probability of 1 and 2 success are respectively $\frac{1}{4}$ and $\frac{1}{3}$, find the probability of successes and failure in trial. Also find P9r-3).
[Hmt: P(r=1)= $\frac{1}{4}$, P (r=2)= $\frac{1}{3}$]
Ans: [P=$\frac{2}{3}$, $q = \frac{3}{5}$ $P(r = 3) = \frac{144}{625}$

7. Find the vertices, centre, eccertnity, foci of the hyperbola:
$9x^2-16y^2-18x-64y-199=0$
[vertices: $(1 \pm 4, -2), cere\ (1, -2), e = \frac{5}{4}, foci\ (1 \pm 5, = 2)$]

8. Construct the equation of hyperbola in its standarded form with focous at (-5,0) and vertex (2,0)

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9. Show that the angle between the diagonals of a cube is $\cos^{-1}(\frac{1}{3})$.

10. A line makes angles α,β.γ.δ with four diagonals of a cube, prove that $cso^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{5}$ (CTEVT 2067, 071

solution: consider a cube with one corner at origin O and three mutually perpendicular edges oA, oB and oC along ox, oy and oz respectively so that oA=oB=oC=a. Then its corners are (0(0,0), A(a,0,0) , B(0,a,0), C(0,0,a) P(a,a,a), Q (0,a,a), R(a,o,a) and S(a,a,0) as shown in figure.

The direction ratios of four diagonals OP, Aq, BR and CS are:

a-0, a-0, a-0⇒a, a, a

0-a, a-0, a-o⇒a,a,a

a-0, 0-a, a-0⇒a,-a,a

a-0, a-0, 0-a⇒a,a,a

Now, drc's of the diagonal OP

$$\frac{q}{\sqrt{a^2+a^2+a^2}},\frac{q}{\sqrt{a^2+a^2+a^2}},\frac{q}{\sqrt{a^2+a^2+a^2}},=\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$$

similarly, the drc's of diagonal AQ, BR and CS are $-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}};\ \frac{1}{\sqrt{3}},\frac{-1}{\sqrt{3}},\frac{1}{\sqrt{3}};\ \frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{-1}{\sqrt{3}}$ respectively. Let a line with dc's l, m,n make angles α,β.γ and δ with diagonals OP, AQ, BR and CS repectively. Then

$\cos\alpha=l.\frac{1}{\sqrt{3}}+m.\frac{1}{\sqrt{3}}+n.\frac{1}{\sqrt{3}}=\frac{l+m-n}{\sqrt{3}}$

Similarly, $\cos\beta=\frac{-l+m+n}{\sqrt{3}}$, $\cos\gamma=\frac{l-m+n}{\sqrt{3}}$, $\cos\delta=\frac{l+m-n}{\sqrt{3}}$,

Now by L.H.S.

$=\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta$

$=\frac{l}{3}[(l+m+n)^2+(-l+m+n)^2+(l-m+n)^2+(-l+m-n)^2]$

$=\frac{l}{3}[l^2+m^2+n+2lm+2mn+2nl+l^2+m^2+n^2-2lm+2mn-2nl+l^2+m^2+n^2-2lm-2mn+2nl+l^2+m^2+n^2+2lm-2mn-2ml]$

$=\frac{4(l^2+m^2+n^2)}{3}$

$=\frac{4}{3}$ $[l^2+m^2+n^2=1]$

Vectors

1. Define scalar product of two vector. If $\vec{a},\vec{b}$ are unit vector and θ is angle between them, then show that $\sin\frac{\theta}{2}=\frac{1}{2}|\vec{a}-\vec{b}|$

2. Define collinear vectors. Prove that three points with following vectors are collinear: $\vec{i}+2\vec{j}+3\vec{k},-2\vec{i}+3\vec{j}+4\vec{k},\ 7\vec{i}+\vec{k}$