UNIT –6

Objective:

- To explore dictionaries.

Syllabus:

Hashing:Basic concepts

Hashing Functions: (Division Method, Multiplication Method),

Collision resolution techniques:-

Open Hashing-Examples,

Closed Hashing:-Linear Probing,QuadraticProbing,Double Hashing Examples

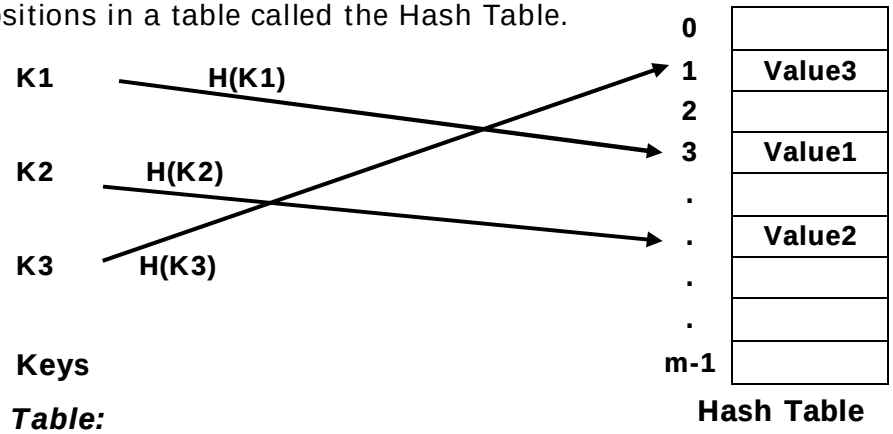
Learning Outcomes:

Student will be able to:

1. Describe Hashing and Various Hash Functions.
2. Insert elements into the Hash Table using various Hash Functions
3. Apply various Collision Resolution Techniques to resolve collisions in Hashing.
4. Implement Hash Table Restructuring when the Hash Table is nearly full.

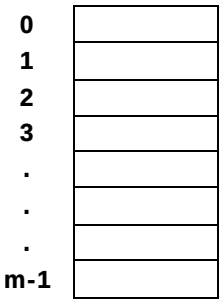
LEARNING MATERIAL

- ***Hashing:***
  - The main goal of Hashing/Hashed search is to find the data with only one test.
  - Hashing is a process of computing the address where an element is to be inserted or found in a hash table by applying a hash function to a given key.
  - Hashing is a Key-to-index transformation in which the keys map to index in an array called as Hash Table.
  - This method used a Hash function (Hashing algorithm) to map keys into positions in a table called the Hash Table.



- ***Hash Table:***
  - *Definition:*A Hash table is a data structure that stores elements and allows insertions, lookups, and deletions to be performed in O(1) time.
  - A hash table consists of an array in which data is accessed via a special index called a Key.
  - Insertion of data in the hash table is based on the key value. Hence every entry in the hash table is associated with some key.

- For example, for storing an employee record in the hash table the employee ID will work as a key.

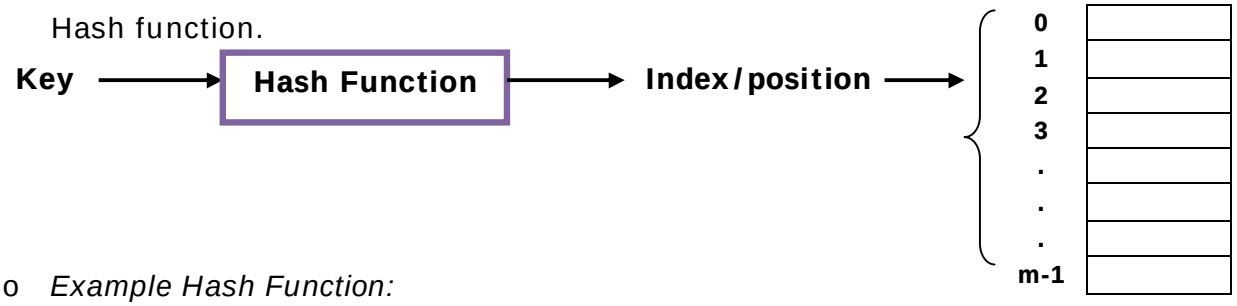


The positions in the hash table are indexed 0 through m-1

Hash table of Size m

- Load factor (α) = (no. of elements) / (no. of table slots)

- ***Hash Function:***
  - Hash functions are primarily used in hash tables, to quickly locate a data record given its search key
  - *Definition:* Hash function is used to map the search key to an index; the index gives the place in the hash table where the corresponding record should be stored.
  - A Key is mapped into one of the m slots of the Hash table using Hash function.



Hash Table

  - *Example Hash Function:*

    H(K)= K%m

    Here K is the Key, H is the Hash Function & m is the size of the Hash Table
  - A Hash function may map several keys into the same position (index) of the hash table.
  - Each position of the hash table is often called as a **'Bucket'**. Bucket consists of number of slots for holding elements.

- Position h(K) is the **'Home Bucket'** for the element whose key is K.

- ***Example:***

  Consider inserting the keys 80, 40, 65, 24, 44, 58 into a hash table of size m=11 using hash function h(k)=k mod m

  1. K=80
     h(80) = 80 mod 11 = 3
  2. K=40
     h(40) = 40 mod 11 = 7
  3. K=65
     h(65) = 65 mod 11 = 10
  4. K=24
     h(24) = 24 mod 11 = 2
  5. K=44
     h(44) = 44 mod 11 = 0
  6. K= 58
     h(58) = 58 mod 11 = 3 (already occupied by 80 -> Collision has occurred)

0	44
1	
2	24
3	80 *
4	
5	
6	
7	40
8	
9	
10	65

Hash Table

- A **Collision** occurs whenever two or more different keys have the same Home Bucket. (in our example keys 80 & 58 have same Home bucket 3)
- *"The situation in which the hash function returns the same index for more than one key is called as collision"*.
- An **Overflow**occurs when there is no room in the home bucket for the new element.
- ***Applications of Hash Tables:***
  - Used to implement Associative Arrays
  - Database Management Systems – Telephone book database, Library books Catalogue, Employee details
  - Compiler - Symbol Table
  - Operating System - Page Mapping Tables, Cache Memory
- ***Analysis of Hash tables:***Time complexity

Operation	Average case	Worst case
Search	O(1)	O(n)
Insert	O(1)	O(n)
Delete	O(1)	O(n)

➢ HASH FUNCTIONS/ HASHING METHODS:

- Hash function is used to map the search key to an index; the index gives the place in the hash table where the corresponding record should be stored.
- A Key is mapped into one of the m slots of the Hash table using Hash function.
- Some Commonly used Hash functions are:
  1. Division Method or Modulo Division
  2. Multiplication Method

1. Division Method:

- This method divides the key by the hash table size and uses the remainder as the index of the hash table.
- In this method, we map a key K into one of the m slots of the hash table by taking remainder of K divided by m.
- The hash function is

  H(K)= K % m

  Here K is the Key, H is the Hash Function & m is the size of the Hash Table
- *Example:*

  Consider inserting keys 26,33,76,86 into a hash table of size m=11 using division method

  h(26) = 26 % 11 = 4

  h(33) = 33 % 11 = 0

  h(76) = 76 % 11 = 10

  h(86) = 86 % 11 = 9

0	33
1	
2	
3	
4	26
5	
6	
7	
8	
9	86
10	76

Hash Table

- Efficiency of this method depends on the value of 'm'
- When using a division method, we usually avoid certain values of m.
  a. m should not be a power of 2

b. A prime number not too close to an exact power of 2 is often a good choice for m. Because it results in fewer collisions than other values for m.

2. Multiplication Method:

- The multiplication method for generating hash functions operates in two steps:
  1) Multiply the key **k** by a constant **A** in the range $0 < A < 1$ and extract the fractional part of **kA**
  2) Multiply this fractional part by **m** and take the floor.
- The hash function is

$$h(k) = \lfloor m(kA \bmod 1) \rfloor$$

Where **kA mod 1 = kA - ⌊kA⌋** i.e the fractional part of KA

A = 0.61804 (Golden value suggested by Knuth)

- *Example:*

Consider m=32 and key k=100

(i). kA = 100 * 0.61804

= 61.804

Taking the fractional part of the result

kA mod 1 = 61.804 mod 1

= 0.804

(ii). m* (kA mod 1) = 32 * 0.804

= 25.728

By taking the floor of the result the index we got is 25

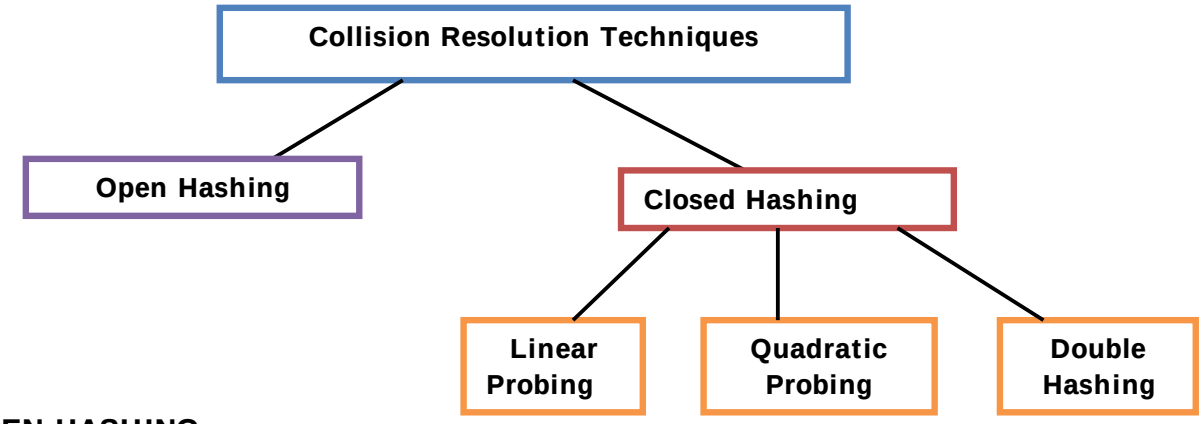
h(32) = 25

- The advantage of this method is that the value of m is not critical. Typically it is chosen to be a power of 2.

➢ COLLISION RESOLUTION TECHNIQUES:

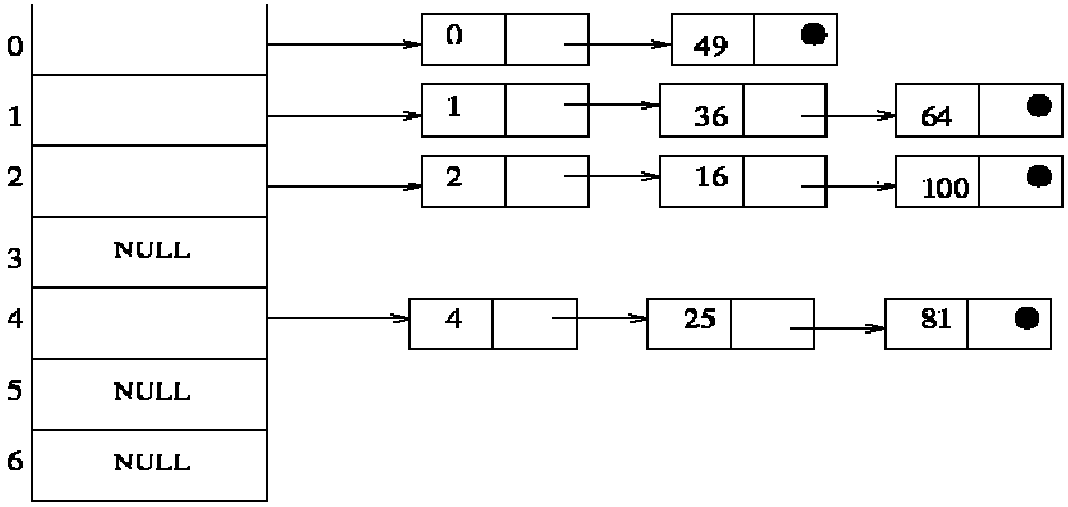
- ***Definition:*** The situation in which a hash function maps two or more keys to same index (Home bucket) in hash table is called as Collision.
- When two keys hash to the same position in a hash table they collide.

- If x1 and x2 are two different keys, it is possible that **h(x1) = h(x2)**, then collision is said to occur between keys x1 and x2.
- Whatever may be the hash function used in hashing, complete removal of collisions is almost impossible. So collisions in hashing cannot be ignored.
- Two techniques used to resolve collisions are
  1. Open hashing or Separate chaining
  2. Closed hashing or Open addressing



1) OPEN HASHING:

- It is also known as Separate chaining
- In this technique, hash table is implemented as an array of Linked lists.
- In this technique*, "hash table is an array of pointers and each pointer will point to one linked list".*
- Whenever a collision occurs then a linked list (chain) is maintained at that home bucket.
- All the elements that hash to the same value are placed in the same home bucket in a Linked list.
- Hash table that used open hashing is called as Open Hash table(or chained hash table).
- **Example:**
  Consider the keys 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100. Let the hash function be:h(x) = x % 7



- ***Advantages:***
  1. Simple and effective approach to collision resolution.
  2. Hash table size need not be a prime number
- ***Disadvantages:***
  1. Additional Data structure (linked list) needs to be used to accommodate collision data.
  2. Wastage of memory space for storing pointers (linked lists).
  3. More number of collisions leads to unevenly distributed keys resulting in
     a. Long length lists
     b. Increased search time
     c. Many empty spaces in the hash table

2) CLOSED HASHING:

- It is also known as Open Addressing.
- In Open Addressing, all the elements are stored in the hash table itself. That is, each table entry contains either an element or NULL.
- It avoids pointers
- Instead of following pointers as in case of Open hashing, we compute the sequence of slots to be examined.
- The process of examining the locations in the hash table is called ***Probing***.
- To perform insertion using open addressing, we successively examine, or probe, the hash table until we find an empty slot in which to put the key.
- Collisions are handled by generating a sequence of rehash values

h : U × {0,1,2,..... } -->{0,1,2,.....,m-1}

Universe of Keys | Probe number | Hash Table

- Given a key x, it has a hash value h(x,0) and a set of rehash values h(x,1), h(x,2), h(x,3),.....,h(x,m-1)
- Closed Hashing Techniques:
  1. Linear Probing
  2. Quadratic Probing
  3. Double Hashing

1. ***Linear Probing:***

- Linear probing is a scheme for resolving collisions of values by *sequentially searching the hash table for a free location*.
- Suppose the hash table size is m, and key value is mapped to index i, with a hash function. If collision has occurred at i^{th} index. then follow the following sequence of locations in hash table and do sequential / linear search.

  i+1, i+2,.....,m,0,1,2,.....i
- The search will continue until any one of the following cases occur
  1. The key value is already present in the table
  2. The unoccupied location(empty slot) is encountered.
  3. It reaches to the location where search was started.
- Here hash table is considered as circular, so that when the last location is reached, the search proceeds to the starting location of the table.
- Hash function in linear probing is defined as **$h^1(k,i) = (h(k) + i) \bmod m$** fori=0,1,2,3,....,m-1 and h(k) is the primary hash function
- **Example:**Insert the following keys 15,11,25,16,9,8,12,8,23,47into hash table of size m=11 using linear probing, consider the primary hash function as h(k)=k mod m

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Initially hashtable is empty

0	
1	
2	
3	
4	15
5	
6	
7	
8	
9	
10	

Inserting 15

0	11
1	
2	
3	
4	15
5	
6	
7	
8	
9	
10	

Inserting 11

0	11
1	
2	
3	25
4	15
5	
6	
7	
8	
9	
10	

Inserting 25

0	11
1	
2	
3	25
4	15
5	16
6	
7	
8	
9	
10	

Inserting 16

0	11
1	
2	
3	25
4	15
5	16
6	
7	
8	
9	9
10	

Inserting 9

0	11
1	
2	
3	25
4	15
5	16
6	
7	
8	8
9	9
10	

Inserting 8

0	11
1	12
2	
3	25
4	15
5	16
6	
7	
8	8
9	9
10	

Inserting 12

0	11
1	12
2	
3	25
4	15
5	16
6	
7	
8	8
9	9
10	

Inserting 8

Here already key 8 is presented in hash table. So does not insert once again.

0	11
1	12
2	23
3	25
4	15
5	16
6	
7	
8	8
9	9
10	

Inserting 23

Here collision occurs at 1st index. So next index is 2

0	11
1	12
2	23
3	25
4	15
5	16
6	47
7	
8	8
9	9
10	

Inserting 47

Here collision occurs at 3, 4, 5 indexes. So next index is 6

- **rawbacks of linear probing:**

  a. The major drawback isthat, as half of the hash table is filled, there is a tendency towards clustering. The key values are clustered in large groups and as a result sequential search becomes slower. This kind of clustering known as primary clustering.

  b. Performance degrades as the hash table gets nearly full.

2. Quadratic Probing:

- Quadratic probing is a collision resolution method that eliminates the primary clustering problem in linear probing.
- If there is a collision at index i

  In linear probing , the next indexes to be probe are i+1, i+2, i+3,----------

  But in quadratic probe, the next indexes to be probe are $i+1^2$, $i+2^2$, $i+3^2$,------
- If m is the size of the hash table and h(k) is the hash function , then quadratic probing search the indexes as

  $h^1(k,i) = (h(k)+ i^2) \bmod m$ for i=0,1,2,3,----------
- ***Example:*** Insert the following keys 89, 18, 49, 58, 69 into hash table of size m=10 using quadratic probing, consider the Hash function h(k) = k % m

  Hash function used in quadratic probing is **$h^1(k,i) = (h(k)+ i^2) \bmod m$**

  1. $h^1(89,0)= (h(89) + 0)\%10$

  $h(89) = 89\%10 = 9$

  $h^1(89,0) = 9$ (empty slot)

  2. $h^1(18,0) = (h(18) + 0)\%10$

  $h(18) = 18\%10 = 8$

  $h^1(18,0) = 8$ (empty slot)

  3. **a)** $h^1(49,0) = (h(49) + 0)\%10$

  $h(49) = 49\%10 = 9$

  $h^1(49,0) = 9$ (Collision)

  b) $h^1(49,1) = (h(49) + 1^2)\%10$

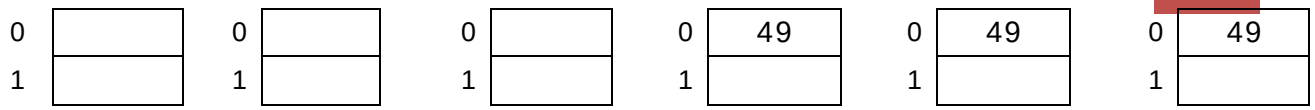
  $= (9+1) \%10$

  $h^1(49,1) = 0$ (empty slot)

  4. **a)** $h^1(58,0) = (h(58) + 0)\%10$

  $h(58) = 58\%10 = 8$

  $h^1(58,0) = 8$ (Collision)



b) $h^1(58,1) = (h(58) + 1^2)\%10$

$=(8+1)\ \%10$

$h^1(58,1) = 9$ (Collision)

c) $h^1(58,2) = (h(58) + 2^2)\%10$

$= (8+4)\ \%10$

$= 2$ (empty slot)

5. a) $h^1(69,0) = (h(69) + 0)\%10$

$h(69) = 69\%10 = 9$

$h^1(69,0) = 9$ (Collision)

b) $h^1(69,1) = (h(69) + 1^2)\%\ 10$

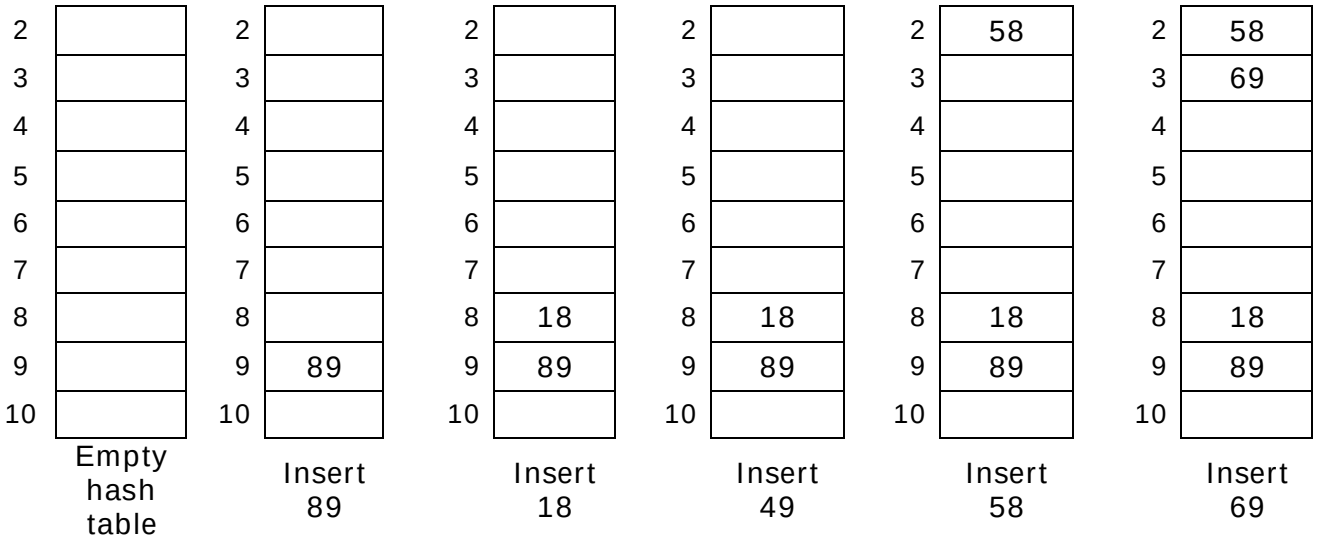
$= (9+1)\%10$

$h^1(69,1) = 0$ (Collision)

c) $h^1(69,2) = (h(69) + 2^2)\%\ 103$

$=(9+4)\%10$

$h^1(69,2) = 3$ (empty slot)



- **Drawbacks of Quadratic probing:**
  a) There is no guarantee of finding an empty cell once more than half of the table gets full or even before that, if the table size is not prime.
  b) It suffers from Secondary Clustering. If two keys have the same initial probe position, then their probe sequences are the same.
  If $h^1(k1,0) = h^1(k2,0)$ then $h^1(k1,i) = h^1(k2,i)$ for i=1,2,3.....

3. *Double Hashing:*

- Double Hashing overcomes the problem of Secondary Clustering.
- This approach uses 2 hash functions. This second hash function results the value of m for a key which is different from the first hash function value.
- Double hashing uses hash function of the form
  h(k,i) = (h1(k) + i h2(k)) mod m for i=0,1,2,3.....
- Here h1 and h2 are the auxiliary hash functions
- There are two important rules to be followed for the second function:
  1) It must never evaluate to Zero
  2) Must make sure that all the slots can be examined
- Generally used hash functions are
  First hash function=>**h1(k)=k mod m**
  Second hash function =>**h2(k)=R-(k mod R)**
- In second hash function, R is a prime number, which is smaller than m.

- ***Disadvantage*** of double hashing is that Complex computation is required. We have to use the second hash function when there is a collision.
- ***Example:***Insert the following keys89, 18, 49, 58, 69, 59into a hash table of size 10 using Double hashing

  h(k,i) = (h1(k) + i h2(k)) mod m for i=0,1,2,3.....

  h1(k) = k%m

  h2(k) = R – k mod r

  m= 10 and consider R = 7 which is prime and less than m

1. h(89,0) = (h1(89)+ 0*h2(89)) mod 10

h1(89) = 89%10 =9

h(89,0) = 9 (empty slot)

2. h(18,0) = (h1(18)+ 0*h2(18)) mod 10

h1(18) = 18%10 =8

h(18,0) = 8 (empty slot)

3. **a)**h(49,0) = (h1(49)+ 0*h2(49)) mod 10

h1(49) = 49%10 = 9

h(49,0) = 9 (Collision)

b)h(49,1) = (h1(49)+ 1*h2(49)) mod 10

h1(49) = 49%10 = 9

h2(49) = 7-(49%7) = 7

h1(49,1) = (9+7) mod 10

= 6 (empty slot)

4. **a)** h(58,0) = (h1(58)+ 0*h2(58)) mod 10

h1(58) = 58%10 = 8

h(58,0) = 8 (Collision)

b)h(58,1) = (h1(58)+ 1*h2(58)) mod 10

h1(58) = 58%10 = 8

h2(58) = 7-(58%7) = 5

h(58,1) = (8+5) mod 10

= 3 (empty slot)

5. **a)** h(69,0) = (h1(69)+ 0*h2(69)) mod 10

h1(69) = 69%10 = 9

h(69,0) = 9 (Collision)

b)h(69,1) = (h1(69)+ 1*h2(69)) mod 10

h1(69) = 69%10 = 9

h2(69) = 7-(69%7) = 1

h(69,1) = (9+1) mod 10

= 0 (empty slot)

6. a) h(59,0) = (h1(59)+ 0*h2(59)) mod 10

h1(59) = 59%10 = 9

h(59,0) = 9 (Collision)

b)h(59,1) = (h1(59)+ 1*h2(59)) mod 10

h1(59) = 59%10 = 9

h2(59) = 7-(59%7) = 4

h(59,1) = (9+4) mod 10

h(59,1) = 3 (Collision)

c)h(59,2) = (h1(59)+ 2*h2(59)) mod 10

h1(59) = 59%10 = 9

h2(59) = 7-(59%7) = 4

h(59,2) = (9+2*4) mod 10

= (9+8) mod 10

h(59,2) = 7 (empty slot)

	Empty Hash table		**Insert 89**		**Insert 18**		**Insert 49**		**Insert 58**		**Insert 69**		**Insert 59**
0		0		0		0		0		0	69	0	69
1		1		1		1		1		1		1	
2		2		2		2		2		2		2	
3		3		3		3		3	58	3	58	3	58
4		4		4		4		4		4		4	
5		5		5		5		5		5		5	
6		6		6		6	49	6	49	6	49	6	49
7		7		7		7		7		7		7	59
8		8		8	18	8	18	8	18	8	18	8	18
9		9	89	9	89	9	89	9	89	9	89	9	89

UNIT-VI
Assignment-Cum-Tutorial Questions
SECTION-A

Objective Questions

1. The mapping of keys to indices of a hash table is done using _____
2. _____ is the formula used for Multiplication hash function
3. Define Bucket in a hash table
4. Define Home Bucket in a hash table
5. Given a Hash table of size m=17 then its range of indices are _____
6. Load factor (α) = _____
7. In a hash table of length 11, the key value 80 can be placed using division hash function at index _____

   A). 3 B). 4 C). 5 D). 6 **[ ]**
8. _____ occurs when hashing produces same index for two different keys.
9. List different collision resolution techniques
10. The disadvantage of Separate chaining is [ ]

    A). Need to maintain a Separate Data Structure

    B). Need more Memory Space

    C). Both A & B

    D). None of these
11. Closed hashing is also called as _____
12. Primary Clustering occurs in **[ ]**

    A). Quadratic Probing B). Linear Probing

    C). Double hashing D). All of the above
13. In a hash table of size 13, the elements to be inserted are 18, 31, and 44 using Division hash function. With Quadratic probing 44 can be placed in _____ cell.

    A). 6 B). 7 C). 8 D). 9 **[ ]**
14. Primary clustering and secondary clustering are solved by _____
15. _____ is a collision resolution technique that uses linked lists to handle collisions [ ]

    A). Linear probing B). Quadratic probing

    C). Double hashing D). Open Hashing

SECTION-B

Descriptive Questions

1. Calculate the hash table indexes using Division and Multiplication hash functions for the keys: 25, 4, 16, 100, 32, 58 with the size of the hash table as m=11
2. Construct the open hash table using separate chaining for the input: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 140 using the hash function h(k)= k mod 11
3. Show the result of inserting the keys: 12, 44, 13, 88, 23, 94, 11, 39, 16 into a hash table of size m =13 with the primary hash function as h(k) = k % m using Linear Probing
4. Show the result of inserting the keys: 12, 44, 13, 88, 23, 94, 11, 39, 20 into a hash table of size m =11 with the primary hash function as h(k) = k % m using Quadratic Probing
5. Show the result of inserting the keys: 15, 11, 25, 16, 36, 47, 22 into a hash table of size m =11 using Double hashing with h1(k) = k % m and h2(k)=R- (k mod R)where R < m and is prime
6. Consider inserting the keys: 20, 29, 45, 49, 52, 59, 65 into a hash table of size m=10 using the primary hash function as h(k) = k % m. Illustrate the result of inserting these keys using quadratic probing with $h^1(k)=(h(k) + i + 3i^2)$ mod m
7. Consider inserting the keys 10, 22, 31, 4, 15, 28, 17 into a hash table of length m = 11 using the primary hashing function $h(k) = k$ mod m. Illustrate the result of inserting these keys usingDouble hashing with rehashing function $h^1(k) = (h(k) + i(1 + k \text{ mod}(m - 1))$ mod m.
8. Consider inserting the keys 7, 18, 48, 10, 36, 25, 47 into a hash table of size m=10 using linear probing. Apply hash table restructuring and show the resulting new Hash table.

Section C

Questions asked in GATE

1.Which one of the following hash functions on integers will distribute keys most uniformly over 10 buckets numbered 0 to 9 for i ranging from 0 to 2020?**[gate 2002]** **[ ]**

(A) h(i)=i^2 mod10 **(B)** h(i)=i^3 mod10

(C) h(i)=$(11*i^2)$mod10 **(D)** h(i)=(12∗i)mod

2.Given a hash table T with 25 slots that stores 2000 elements, the load factor α for T is _____ **[ ][gate 2005]**

A) 80 B) 0.0125 C) 8000 D) 1.25

3.A hash function h defined h(key)=key mod 7, with linear probing, is used to insert the keys 44, 45, 79, 55, 91, 18, 63 into a table indexed from 0 to 6. What will be the location of key 18?**[gate 2007]** **[ ]**

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6

4.Which of the following statement(s) is TRUE?**[gate 2008]** **[ ]**

1. A hash function takes a message of arbitrary length and generates a fixed length code.
2. A hash function takes a message of fixed length and generates a code of variable length.
3. A hash function may give the same hash value for distinct messages.

(A) I only **(B)** II and III only
(C) I and III only **(D)** II only

5.A hash table of length 10 uses open addressing with hash function h(k)=k mod 10, and linear probing. After inserting 6 values into an empty hash table, the table is as shown below.

0	
1	
2	42
3	23
4	34
5	52
6	46
7	33
8	
9	

Which one of the following choices gives a possible order in which the key values could have been inserted in the table? [ ]

(A) 46, 42, 34, 52, 23, 33 **(B)** 34, 42, 23, 52, 33, 46
(C) 46, 34, 42, 23, 52, 33 **(D)** 42, 46, 33, 23, 34, 52

6.Consider a hash function that distributes keys uniformly. The hash table size is 20. After hashing of how many keys will the probability that any new key hashed collides with an existing one exceed 0.5.**[gate 2009]** **[ ]**

(A) 5 **(B)** 6 **(C)** 7 **(D)** 10

7.A hash table has space for 100 records. What is the propability of collision before the table is 10% full?**[gate 2015]**

A). 0.45 B). 0.5 C). 0.3 D). 0.34 **[ ]**

8.Consider a 13 element hash table for which f(key)=key mod 13 is used with integer keys. Assuming linear probing is used for collision resolution, at which location would the key 103 be inserted, if the keys 661, 182, 24 and 103 are inserted in that order?**[gate 2016]** **[ ]**

(A) 0 **(B)** 1 **(C)** 11 **(D)** 12

9.Consider a hash table with 9 slots. The hash function is $h(k) = k \bmod 9$. The collisions are resolved by chaining. The following 9 keys are inserted in the order: 5, 28, 19, 15, 20, 33, 12, 17, 10. The maximum, minimum, and average chain lengths in the hash table, respectively, are

[Gate 2017] **[ ]**

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(A) 3, 0, and 1 (B) 3, 3, and 3 (C) 4, 0, and 1 D) 3, 0, and 2

10.Consider a hash table of size seven, with starting index zero, and a hash function $(3x+4) \bmod 7$. Assuming the hash table is initially empty, which of the following is the contents of the table when the sequence 1, 3, 8, 10 is inserted into the table using closed hashing? Note that - denotes an empty location in the table.**[gate 2007]** **[ ]**

.A) 8, - , - , - , - , - , 10 B) 1, 8, 10, - , - , - , 3 (C) 1, - , - , - , - , - , 3 (D) 1, 10, 8, - , - , - , 3