CS3014 Tutorial Sheet 5: Handling uncertainty - Solutions

1. Sketch the following fuzzy sets

None:	(0, 0, 0, 0,)
Small:	(2, 4, 2, 3)
Medium:	(7, 10, 2, 1)
Large:	(11, 15, 1, 5)

   See diagrams at the end of this solutions sheet.

   On your diagram (or on a new version of the diagram) highlight the following:

   (a) Small AND Medium

   (b) None OR Small

   (c) Medium OR Large

   (d) NOT Medium

2. The following is a verbal description for the fuzzy variable 'Age':

   "A person is definitely your from the time they are born until they are 20 years old. No-one over the age of forty should be considered young, though some may consider that a person is sort of young up to that age. People may be considered middle-aged between the ages of 35 and 65, and certainly between the ages of 50 – 60. Old age arrives gradually, starting at age 60 and reaching its fullness by age 70, beyond which anyone is definitely old."

   From this description

   (a) Sketch the fuzzy sets associated with Age.
   See diagrams at the end of this solutions sheet.

   (b) Identify the 4-tuples for these sets.
   Young: (0, 20, 0, 20)
   Middle-age: (50, 60, 15, 5)
   Old: (70, ..., 10, 0)

   (c) Calculate the degree of membership in each of these sets for people aged: 20, 37, 55, 63 and 85.
   age = 20
   $\mu(young) = 1$
   $\mu(middle - aged) = 0$
   $\mu(old) = 0$
   age = 37
   $\mu(young) = (20 + 20 - 37)/20 = 3/20 = 0.15$
   $\mu(middle - aged) = (37 - 50 + 15)/15 = 2/15 = 0.13$
   $\mu(old) = 0$
   age = 55
   $\mu(young) = 0$

$\mu(middle - aged) = 1$
$\mu(old) = 0$
age = 68
$\mu(young_{=}0$
$\mu(middle - aged) = (60 + 5 - 63)/5 = 2/5 = 0.4$
$\mu(old) = (63 - 70 + 10)/10 = 3/10 = 0.3$
age = 85
$\mu(young) = 0$
$\mu(middle - aged) = 0$
$\mu(old) = 1$

(d) Using these age values identify the degree of the membership from the connectives:

i. young AND middle-aged
All values are zero except the following:
age = 37: min(0.15, 0.13) = 0.13

ii. middle-aged AND old
All values are 0 except the following:
age = 63: min(0.4, 0.3) = 0.3

iii. young OR middle-aged
All values are 0 except the following:
age = 20: max(1, 0) = 1
age = 37: max(0.15, 0.13) = 0.15
age = 55: max(0, 1) = 1
age = 63: max(0, 0.4) = 0.4

iv. middle-aged OR old
All values are zero except the following:
age = 37: max(0.13, 0) = 0.13
age = 55: max(1, 0) = 1
age = 63: max(0.4, 0.3) = 0.3
age = 85: max(0, 1) = 1

v. NOT young.
All values are 1 except the following:
age = 20: $\mu(notyoung) = 1 - 1 = 0$
age = 37: $\mu(notyoung) = (37 - 40 + 20)/20 = 17/20 = 0.85$

3. A fuzzy rule base for an oven temperature control system contains the following rules:

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Rule1: If temperature is high or current is high then reduce current
Rule2: If temperature is medium then no change to current
Rule3: If temperature is low and current is high then no change to current
Rule4: If temperature is low and current is low then increase current
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The fuzzy sets representing the temperature (in $^{\circ}C$), current (in Amps) and change in current (as a %) are given in 4-tuple form as:

Current	low	(0, 0, 0, 10)
	medium	(10, 10, 10, 10)
	high	(20, 20, 10, 0)
Temperature	low	(0, 100, 0, 100)
	medium	(200, 200, 150, 150)
	high	(400, 450, 150, 0)
Change in current	reduce	(-50, -50, 50, 50)
	no change	(0, 0, 50, 50)
	increase	(50, 50, 50, 50)

Suppose that the temperature is $300°C$ and the current is 17A.

(a) Sketch the sets for current temperature and change in current.
See sheets

(b) Calculate the percentage change in current resulting from these measurements
To calculate this requires that we go through the process of fuzzification. applying the rules, and de-fuzzification.
First fuzzification:
Current = 17A
$\mu(low) = 0$
$\mu(medium) = (10 + 10 - 17)/10 = 0.3$
$\mu(high) = (17 - 20 + 10)/10 = 0.7$
Temperature = $300°C$
$\mu(low) = 0$
$\mu(medium) = (200 + 150 - 300)/150 = 0.33$
$\mu(high) = (300 - 400 + 150)/150 = 0.33$
Next we examine the firing of the rules:
Rule 1: max(0.33, 0.7) is the 'reduce current' value Rule 2: 0.33 is the 'no change to current' value Rule 3: min(0, 0.7) is the 'no change to current' value Rule 4: min(0, 0) is the 'increase current' value
Combine contributions from rules using the OR relation:
So,
reduce signal is 0.7,
'no change' signal is max(0.33, 0) = 0.33
'increase' signal is 0.
Finally we must de-fuzzify these signals:
The area of no-change is: 0.5*1*100 = 50
The area of the missing part of no change is 0.5*0.67*66.67 = 22.33
Hence the area of 0.33 'no change is: 50 - 22.33 = 27.67.
The area of 'reduce' is: 0.5*1*100 = 50
The area of the missing part of reduce is 0.5*0.3*30 = 4.5
Hence the area of 0.7 'reduce' is: 50 - 4.7 = 45.3.

We will take the centres with reference to the lhs of the graph. So the centre of reduce is 50 and the centre of no change is 100.

Hence the defuzzified value is (45.3*50 + 27.67*100)/(45.3 + 27.67) = 5032/72.97 = 68.96 from the lhs.

(c) Perform the same calculation by means of a dilatation of the aggregate.

The bulk of the calculation is the same as before, up to the defuzzificaton part. There are several ways to do it, but the simplest is to simply scale the apex to the appropriate level.

In this case:

The area of 'no-change' is: 0.5*0.33*100 = 16.5

The area of 'reduce' is: 0.5*0.7*100 = 35

The centres are as previously.

Hence the defuzzified value is (35*50 + 16.5*100)/(35 + 16.5) = 3400/51.5 = 66.02 from the lhs.

4. An intelligent anaesthetic delivery system uses measurements of two physiological variables: heart rate (HR) and respiration rate (R), in conjunction with a fuzzy rule base to decide how much anaesthetic drug (D) to give.

The sets associated with each fuzzy variable are:

HR: {(very low = <40, 40, 0, 20 >), (low = <60, 60, 20, 10>), (normal = <70, 90, 10, 10>), (high = <100, 100, 10, 20>), (very high = <120, 120, 20, 0>)}
R: (low = <0, 3, 0, 3>), (normal = <6, 8, 3, 2>), (high = <10, 12, 2, 0>)
D: (very small = <0, 0, 0, 2>), (small = <2, 2, 2, 2>), (moderate = <4, 4, 2, 2>), (average = <6, 6, 2, 2>), (high = <8, 8, 2, 2>), (very high = <10, 10, 2, 2>)

and some of the rules in the fuzzy rule base are:

Rule 1: IF HR = normal AND R = normal THEN D = average
Rule 2: IF HR = low AND R = normal THEN D = moderate
Rule 3: IF HR = low AND R = low THEN D = small
Rule 4: IF HR = very low AND R = low THEN D = very small
Rule 5: IF HR = high AND R = high THEN D = large
Rule 6: IF HR = very high AND R = high THEN D = very large.

For this system:

(a) Sketch the sets for each fuzzy variable.

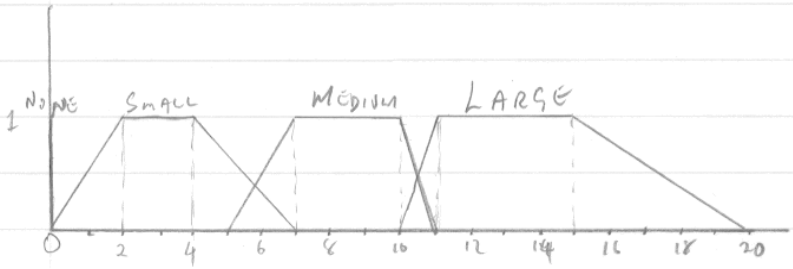
(b) Calculate the amount of anaesthetic drug delivered when HR = 55 and R = 4.

HR = 55

$\mu(VL) = \frac{(40+20-55)}{20} = 0.25$

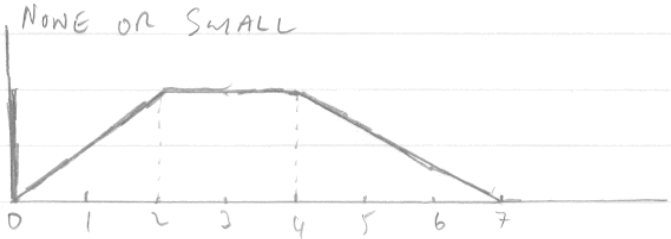
$\mu(L) = \frac{(55-60+20)}{20} = 0.75$
$\mu(N) = \mu(H) = \mu(VH) = 0$
R = 4
$\mu(L) = \frac{(3+3-4)}{3} = 0.667$
$\mu(L) = \frac{(4-6+3)}{3} = 0.33$
$\mu(H) = 0$
R1: min(0, 0) = 0 is the Drug 'average' value
R2: min(0.75, 0.333) = 0.333 is the Drug 'moderate' value
R3: min(0.75, 0.667) = 0.667 is the Drug 'small' value
R4: min(0.25, 0.667) = 0.25 is the Drug 'very small' value
R5: min(0, 0) = 0 is the Drug 'high' value
R6: min(0, 0) = 0 is the Drug 'very high' value
Moderate is 0.333
Area = (0.5*1*4) - (0.5*0.667*4*0.667) = 1.111
Area*centre = 4*1.111 = 4.444
Small is 0.667
Area = (0.5*1*4) - (0.5*0.333*4*0.333) = 1.778
Area*centre = 2*1.778 = 3.556
Very small is 0.25
Area = (0.5*4*1) - (.5*.25*4*.25) = 1.875
Area*centre = 1*1.875 =1.875
Defuzzified value = (4.444+3.556+1.875)/(1.111+1.778+1.875) = 2.073

(1)

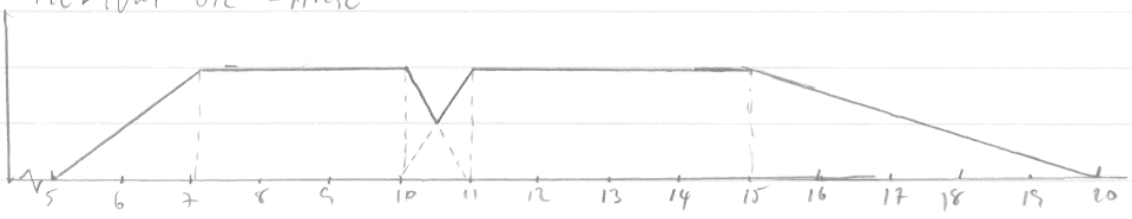


SMALL & MEDIUM

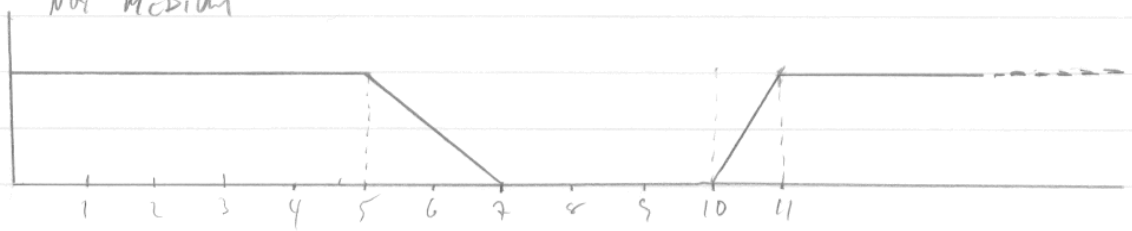
NONE OR SMALL



MEDIUM OR LARGE

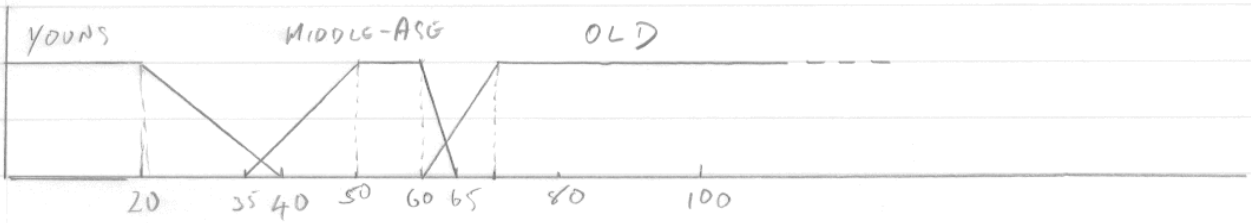


NOT MEDIUM



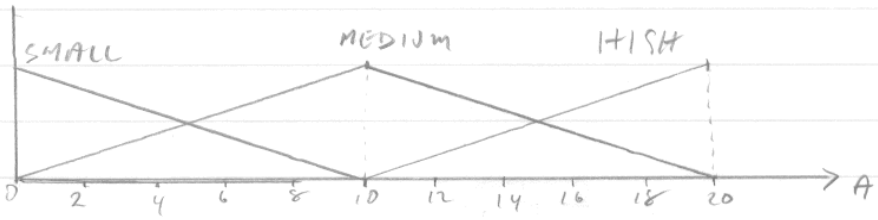
AGE

(2)

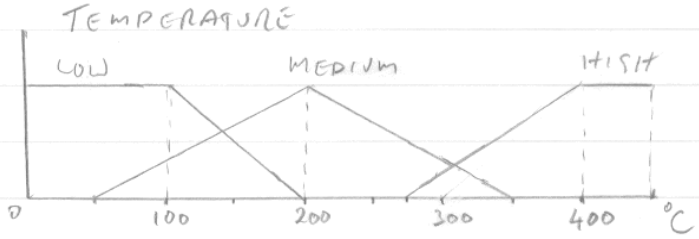


CURRENT

(3)



TEMPERATURE



CHANE IN CURRENT