RYB Symplectic Verlet

$$\mathcal{H}_m = \frac{B_R^2}{2} + \frac{B_\theta^2}{2R^2} + \frac{B_\phi^2}{2R^2\sin^2\theta} - \sum_k \eta_k \frac{1}{\sqrt{R^2 + R_k^2 - 2RR_k\left(\cos\theta\cos\theta_k + \sin\theta\sin\theta_k\cos(\varphi-\varphi_k)\right)}}$$

$$q_{\frac{1}{2}} = q_0 + \frac{h}{2}\frac{\partial}{\partial \mathbf{p}} H(p_0, q_{\frac{1}{2}})$$

$$p_1 = p_0 - \frac{h}{2}\left[\frac{\partial}{\partial \mathbf{q}} H(p_0, q_{\frac{1}{2}}) + \frac{\partial}{\partial \mathbf{q}} H(p_1, q_{\frac{1}{2}})\right]$$

$$q_1 = q_{\frac{1}{2}} + \frac{h}{2}\frac{\partial}{\partial \mathbf{p}} H(p_1, q_{\frac{1}{2}})$$

$$q_{\frac{1}{2}} = q_0 + \frac{h}{2}\frac{\partial}{\partial \mathbf{p}} H(p_0, q_{\frac{1}{2}})$$

$$R_{\frac{1}{2}} = R_0 + \frac{h}{2}\frac{\partial H}{\partial B_r}(\mathbf{B_0}, \mathbf{q}_{\frac{1}{2}})$$

$$\Leftrightarrow \boxed{R_{\frac{1}{2}} = R_0 + \frac{h}{2} B_{R,0}} \quad (1)$$

$$\theta_{\frac{1}{2}} = \theta_0 + \frac{h}{2}\frac{\partial H}{\partial B_\theta}(\mathbf{B_0}, \mathbf{q}_{\frac{1}{2}})$$

$$\Leftrightarrow \boxed{\theta_{\frac{1}{2}} = \theta_0 + \frac{h}{2}\frac{B_{\theta,0}}{R_{\frac{1}{2}}^2}} \quad (2)$$

$$\phi_{\frac{1}{2}} = \phi_0 + \frac{h}{2}\frac{\partial H}{\partial B_\phi}(\mathbf{B_0}, \mathbf{q}_{\frac{1}{2}})$$

$$\Leftrightarrow \boxed{\phi_{\frac{1}{2}} = \phi_0 + \frac{h}{2}\frac{B_{\phi,0}}{R_{\frac{1}{2}}^2\sin^2(\theta_{\frac{1}{2}})}} \quad (3)$$

$$p_1 = p_0 - \frac{h}{2}\left[\frac{\partial}{\partial \mathbf{q}} H(p_0, q_{\frac{1}{2}}) + \frac{\partial}{\partial \mathbf{q}} H(p_1, q_{\frac{1}{2}})\right]$$

$$B_{R,1} = B_{R,0} - \frac{h}{2}\left[-\frac{B_{\theta,0}^2}{R_{\frac{1}{2}}^3} - \frac{B_{\phi,0}^2}{R_{\frac{1}{2}}^3\sin^2(\theta_{\frac{1}{2}})} - \frac{B_{\theta,1}^2}{R_{\frac{1}{2}}^3} - \frac{B_{\phi,1}^2}{R_{\frac{1}{2}}^3\sin^2(\theta_{\frac{1}{2}})} - 2\cdot\sum_k \frac{\eta_k}{-2}\frac{2R_{\frac{1}{2}} - 2R_{k,\frac{1}{2}}\left(\cos\theta_{\frac{1}{2}}\cos\theta_{k,\frac{1}{2}} + \sin\theta_{\frac{1}{2}}\sin\theta_{k,\frac{1}{2}}\cos(\varphi_{\frac{1}{2}}-\varphi_{k,\frac{1}{2}})\right)}{\sqrt{R_{\frac{1}{2}}^2 + R_{k,\frac{1}{2}}^2 - 2R_{\frac{1}{2}}R_{k,\frac{1}{2}}\left(\cos\theta_{\frac{1}{2}}\cos\theta_{k,\frac{1}{2}} + \sin\theta_{\frac{1}{2}}\sin\theta_{k,\frac{1}{2}}\cos(\varphi_{\frac{1}{2}}-\varphi_{k,\frac{1}{2}})\right)}}\right]$$

$$\Leftrightarrow$$

$$B_{R,1} = B_{R,0} + \frac{h}{2}\left[\frac{B_{\theta,0}^2 + B_{\theta,1}^2}{R_{\frac{1}{2}}^3} + \frac{B_{\phi,0}^2 + B_{\phi,1}^2}{R_{\frac{1}{2}}^3\sin^2(\theta_{\frac{1}{2}})} + 2\sum_k \frac{\eta_k\left[-R_{\frac{1}{2}} + R_{k,\frac{1}{2}}\left(\cos\theta_{\frac{1}{2}}\cos\theta_{k,\frac{1}{2}} + \sin\theta_{\frac{1}{2}}\sin\theta_{k,\frac{1}{2}}\cos(\varphi_{\frac{1}{2}}-\varphi_{k,\frac{1}{2}})\right)\right]}{\sqrt{R_{\frac{1}{2}}^2 + R_{k,\frac{1}{2}}^2 - 2R_{\frac{1}{2}}R_{k,\frac{1}{2}}\left(\cos\theta_{\frac{1}{2}}\cos\theta_{k,\frac{1}{2}} + \sin\theta_{\frac{1}{2}}\sin\theta_{k,\frac{1}{2}}\cos(\varphi_{\frac{1}{2}}-\varphi_{k,\frac{1}{2}})\right)}}\right]$$

$$\boxed{B_{R,1} = B_{R,0} + \frac{h}{2}\left[\frac{B_{\theta,0}^2 + B_{\theta,1}^2}{R_{\frac{1}{2}}^3} + \frac{B_{\phi,0}^2 + B_{\phi,1}^2}{R_{\frac{1}{2}}^3\sin^2(\theta_{\frac{1}{2}})} + 2\frac{\alpha}{\lambda_{\frac{1}{2}}}\right]} \quad (4)$$

$$\mathcal{H}_m = \frac{B_R^2}{2} + \frac{B_\theta^2}{2R^2} + \frac{B_\phi^2}{2R^2\sin^2\theta} - \sum_k \eta_k \frac{1}{\sqrt{R^2 + R_k^2 - 2RR_k\left(\cos\theta\cos\theta_k + \sin\theta\sin\theta_k\cos(\varphi-\varphi_k)\right)}}$$

$$B_{\theta,1} = B_{\theta,0} - \frac{h}{2}\left[-\frac{2\left(B_{\phi,0}^2 + B_{\phi,1}^2\right)}{2R_{\frac{1}{2}}^2\sin^2(\theta_{\frac{1}{2}})\tan(\theta_{\frac{1}{2}})} - 2\cdot\sum_k \frac{\eta_k}{-2}\frac{-2R_{\frac{1}{2}}R_{k,\frac{1}{2}}\left(-\sin\theta_{\frac{1}{2}}\cos\theta_{k,\frac{1}{2}} + \cos\theta_{\frac{1}{2}}\sin\theta_{k,\frac{1}{2}}\cos(\varphi_{\frac{1}{2}}-\varphi_{k,\frac{1}{2}})\right)}{\sqrt{R_{\frac{1}{2}}^2 + R_{k,\frac{1}{2}}^2 - 2R_{\frac{1}{2}}R_{k,\frac{1}{2}}\left(\cos\theta_{\frac{1}{2}}\cos\theta_{k,\frac{1}{2}} + \sin\theta_{\frac{1}{2}}\sin\theta_{k,\frac{1}{2}}\cos(\varphi_{\frac{1}{2}}-\varphi_{k,\frac{1}{2}})\right)}}\right]$$

$\Leftrightarrow$

$$B_{\theta,1} = B_{\theta,0} + \frac{h}{2}\left[\frac{B_{\phi,0}^2 + B_{\phi,1}^2}{R_{\frac{1}{2}}^2\sin^2(\theta_{\frac{1}{2}})\tan(\theta_{\frac{1}{2}})} + 2\cdot\sum_k \frac{\eta_k\left[R_{\frac{1}{2}}R_{k,\frac{1}{2}}\left(-\sin\theta_{\frac{1}{2}}\cos\theta_{k,\frac{1}{2}} + \cos\theta_{\frac{1}{2}}\sin\theta_{k,\frac{1}{2}}\cos(\varphi_{\frac{1}{2}}-\varphi_{k,\frac{1}{2}})\right)\right]}{\sqrt{R_{\frac{1}{2}}^2 + R_{k,\frac{1}{2}}^2 - 2R_{\frac{1}{2}}R_{k,\frac{1}{2}}\left(\cos\theta_{\frac{1}{2}}\cos\theta_{k,\frac{1}{2}} + \sin\theta_{\frac{1}{2}}\sin\theta_{k,\frac{1}{2}}\cos(\varphi_{\frac{1}{2}}-\varphi_{k,\frac{1}{2}})\right)}}\right]$$

$\Leftrightarrow$

$$\boxed{B_{\theta,1} = B_{\theta,0} + \frac{h}{2}\left[\frac{B_{\phi,0}^2 + B_{\phi,1}^2}{R_{\frac{1}{2}}^2\sin^2(\theta_{\frac{1}{2}})\tan(\theta_{\frac{1}{2}})} + 2\frac{\beta_{\frac{1}{2}}}{\lambda_{\frac{1}{2}}}\right]} \quad (5)$$

$$B_{\phi,1} = B_{\phi,0} - \frac{h}{2}\left[-2\cdot\sum_k \frac{\eta_k}{-2}\frac{-2R_{\frac{1}{2}}R_{k,\frac{1}{2}}\left(-\sin\theta_{\frac{1}{2}}\sin\theta_{k,\frac{1}{2}}\sin(\varphi_{\frac{1}{2}}-\varphi_{k,\frac{1}{2}})\right)}{\sqrt{R_{\frac{1}{2}}^2 + R_{k,\frac{1}{2}}^2 - 2R_{\frac{1}{2}}R_{k,\frac{1}{2}}\left(\cos\theta_{\frac{1}{2}}\cos\theta_{k,\frac{1}{2}} + \sin\theta_{\frac{1}{2}}\sin\theta_{k,\frac{1}{2}}\cos(\varphi_{\frac{1}{2}}-\varphi_{k,\frac{1}{2}})\right)}}\right]$$

$\Leftrightarrow$

$$B_{\phi,1} = B_{\phi,0} + \frac{h}{2}\left[2\cdot\sum_k \frac{\eta_k\left[-R_{\frac{1}{2}}R_{k,\frac{1}{2}}\sin\theta_{\frac{1}{2}}\sin\theta_{k,\frac{1}{2}}\sin(\varphi_{\frac{1}{2}}-\varphi_{k,\frac{1}{2}})\right]}{\sqrt{R_{\frac{1}{2}}^2 + R_{k,\frac{1}{2}}^2 - 2R_{\frac{1}{2}}R_{k,\frac{1}{2}}\left(\cos\theta_{\frac{1}{2}}\cos\theta_{k,\frac{1}{2}} + \sin\theta_{\frac{1}{2}}\sin\theta_{k,\frac{1}{2}}\cos(\varphi_{\frac{1}{2}}-\varphi_{k,\frac{1}{2}})\right)}}\right]$$

$\Leftrightarrow$

$$\boxed{B_{\phi,1} = B_{\phi,0} + \frac{h}{2}\left[2\frac{\gamma_{\frac{1}{2}}}{\lambda_{\frac{1}{2}}}\right]} \quad (6)$$

$$\mathcal{H}_m = \frac{B_R^2}{2} + \frac{B_\theta^2}{2R^2} + \frac{B_\phi^2}{2R^2 \sin^2\theta} - \sum_k \eta_k \frac{1}{\sqrt{R^2 + R_k^2 - 2RR_k(\cos\theta \cos\theta_k + \sin\theta \sin\theta_k \cos(\varphi - \varphi_k))}}$$

$$q_1 = q_{\frac{1}{2}} + \frac{h}{2} \frac{\partial}{\partial p} H(p_1, q_{\frac{1}{2}})$$

$$\boxed{R_1 = R_{\frac{1}{2}} + \frac{h}{2}(B_{R,1})} \quad (7)$$

$$\boxed{\theta_1 = \theta_{\frac{1}{2}} + \frac{h}{2}\left(\frac{B_{\theta,1}}{R_{\frac{1}{2}}^2}\right)} \quad (8)$$

$$\boxed{\phi_1 = \phi_{\frac{1}{2}} + \frac{h}{2}\left(\frac{B_{\phi,1}}{R_{\frac{1}{2}}^2 \sin^2(\theta_{\frac{1}{2}})}\right)} \quad (9)$$

Run in order:

(1)
(2)
(3)
(6)
(5)
(4)
(7)
(8)
(9)