Hohmann Transfer to Mars

These calculations are adapted from a homework assignment in Caltech course Ae105 Aerospace Engineering in fall 2013.

In[∘]:=
```
Quiet@Remove["`*"]
```

Constants

Standard gravitational parameters, μ

In[∘]:=
```
μSun = UnitConvert[ M☉ G , "km^3/s^2"]
```

Out[∘]= $1.327\times 10^{11}\ \mathrm{km^3/s^2}$

In[∘]:=
```
μEarth = UnitConvert[ M⊕ G , "km^3/s^2"]
```

Out[∘]= $3.986\times 10^{5}\ \mathrm{km^3/s^2}$

In[∘]:=
```
μMars = UnitConvert[ M♂ G , "km^3/s^2"]
```

Out[∘]= $4.283\times 10^{4}\ \mathrm{km^3/s^2}$

Planet radii, R

In[∘]:=
```
REarth = Earth PLANET [ average radius ]
```

Out[∘]= 6371.008 km

In[∘]:=
```
RMars = Mars PLANET [ average radius ]
```

Out[∘]= 3389.5 km

Orbital distances

In[∘]:=
```
aEarth = UnitConvert[ Earth PLANET [ semimajor axis ], "km"]
```

Out[∘]= 1.49597887×10^{8} km

In[∘]:=
```
aMars = UnitConvert[ Mars PLANET [ semimajor axis ], "km"]
```

Out[∘]= 2.27936637×10^{8} km

In[∘]:=
```
aHohmann = UnitConvert[(aEarth + aMars)/2, "km"]
```

Out[∘]= 1.88767262×10^{8} km

In[∘]:= `rPark = REarth + 160 km`

Out[∘]= 6531.008 km

In[∘]:= `rArrival = RMars + 125 km`

Out[∘]= 3514.5 km

Transfer time to Mars

In[∘]:= `UnitConvert[`$\pi\sqrt{\frac{\text{aHohmann}^3}{\mu\text{Sun}}}$`, "days"]`

Out[∘]= 258.9 days

In[∘]:= `UnitConvert[%, "months"]`

Out[∘]= 8.511 mo

Departure speed on Hohmann-transfer

Departure on Hohmann-transfer

v needed at periapsis of Hohmann transfer orbit with respect to sun

Insert in Vis Viva equation:

In[∘]:= $\text{vHohmannP} = \sqrt{\mu\text{Sun}\left(\frac{2}{\text{aEarth}} - \frac{1}{\text{aHohmann}}\right)}$

Out[∘]= 32.73 km/s

$V_{\infty,\oplus}$: Hyperbolic excess velocity (velocity in addition to earth escape velocity, plus sign because it’s a speed increase) in the geocentric frame not considering earth-sat interaction - which equivalently is the speed of the satellite after it’s outside of earth’s sphere of influence on the Hohmann transfer orbit in the heliocentric frame.

In other words, $V_{\infty,\oplus}$is the speed that is missing heliocentrially at perihelion of the Hohmann transfer orbit if we had the same speed as Earth. We set this equal to the speed in excess of an outgoing hyperbolic geocentric orbit.

Earth speed with respect to the sun:

In[•]:= $\text{vEarth} = \sqrt{\frac{\mu\text{Sun}}{\text{aEarth}}}$

Out[•]= 29.78 km/s

In[•]:= vInfEarth = vHohmannP - vEarth

Out[•]= 2.94 km/s

This is the speed that is needed at $r = \infty$ when modelled as a hyperbolic trajectory in the Earth system. In the following we will call it $v^{+}_{\infty,\oplus}$ or $v^2_{\infty,\oplus}$ when it's squared.

We first use the Vis Viva equation in the geocentric system on a hyperbolic trajectory, i.e. $a_{\text{hyp}} < 0$, which gives us the square speed $v^2_{r,\oplus}$ at an arbitrary r:

$$v^2_{r,\oplus} = \mu_\oplus \left(\frac{2}{r} - \frac{1}{a_{\text{hyp}}}\right)$$

We are interested in finding the necessary speed of the hyperbolic trajectory at the distance $R_\oplus + h$:

$$v^2_{p,\oplus} = \mu_\oplus \left(\frac{2}{R_\oplus + h} - \frac{1}{a_{\text{hyp}}}\right)$$

The last term can be determined by setting $r = \infty$ which gives us the hyperbolic excess velocity (velocity in addition to escape velocity infinitely far away):

$$v^2_{\infty,\oplus} = \mu_\oplus \left(\frac{2}{\infty} - \frac{1}{a_{\text{hyp}}}\right) = -\frac{\mu_\oplus}{a_{\text{hyp}}}$$

Remember that a is negative for a hyperbola. We know $v^2_{\infty,\oplus}$ since we just calculated this as $v_{\text{Hohmann},p} - v_{\oplus/\odot}$ as 2.94 km/s. Inserted back into $v^2_{p,\oplus}$ we get:

$$v^2_{p,\oplus} = 2\,\frac{\mu_\oplus}{R_\oplus + h} + v^2_{\infty,\oplus}$$

We recognize the first term as the squared escape velocity from Earth at altitude h so we can also write:

$$v^2_{p,\oplus} = v^2_{\text{esc}} + v^2_{\infty,\oplus}$$

Let us compare this result with the naive approach of saying $v_{p,\oplus} = v_{\text{esc}} + v_{\infty,\oplus}$. This is obviously a higher needed velocity departure velocity at hyperbolic perigee $v_{p,\oplus}$ and it is due to the assumption that you first escape the earth, then perhaps later apply the extra $v_{\infty,\oplus}$ to get to that speed. However when you burn all at once at perigee, you take advantage of the Oberth effect, i.e. that applied delta-v give more kinetic energy when falling at high speed close to a body.

“Note that this means that a relatively small extra delta-v above that needed to accelerate to the escape speed results in a relatively large speed at infinity. For example, at a place where escape speed is 11.2 km/s, the addition of 0.4 km/s yields a hyperbolic excess speed of 3.02 km/s” [1].

$V_{P/\text{Earth}}$: speed needed to escape Earth on a hyperbolic trajectory seen from Earth, resulting in the correct speed for the Hohmann transfer orbit at

perihelion far away from Earth (i.e. we now also consider escape velocity needed from parking orbit)

In[•]:= vPEarth = $\sqrt{\frac{2 * \mu\text{Earth}}{\text{rPark}} + \text{vInfEarth}^{\wedge}2}$

Out[•]= 11.43 km/s

vDepart: same as $V_{P/\text{Earth}}$ except we now subtract the speed we already have from the parking orbit (i.e. Δv to leave Earth on the hyperbolic orbit, resulting in the correct speed far away for a Hohmann transfer orbit at perihelion, from a circular earth orbit of radius rPark with respect to the circular satellite orbit reference frame)

In[•]:= vSatInitial = $\sqrt{\frac{\mu\text{Earth}}{\text{rPark}}}$ (*km/s*)

Out[•]= 7.812 km/s

In[•]:= ΔvDepart = vPEarth − vSatInitial (*km/s*)

Out[•]= 3.62 km/s

So hyperbolic excess velocity V_∞^+ = 2.94 km/*s*. (extra speed needed heliocentrically)

$V_{P/\text{Earth}}$ = 11.43 km/*s* needed to depart earth at altitude h. (speed needed geocentrically)

3.62 km/s needed to depart earth from circular orbit at altitude h. (extra speed needed parking-orbit-centrically)

Arrival from Hohmann transfer orbit

vHohmannA: Velocity at the end of Hohmann transfer orbit with respect to sun

Velocity at the end of Hohmann-transfer with respect to the sun:

In[•]:= vHohmannA = $\sqrt{\mu\text{Sun}\left(\frac{2}{\text{aMars}} - \frac{1}{\text{aHohmann}}\right)}$

Out[•]= 21.480 km/s

In[•]:= vMars = $\sqrt{\frac{\mu\text{Sun}}{\text{aMars}}}$

Out[•]= 24.13 km/s

$V_{\infty,♂}$: Δv change in velocity needed to stay with mars in it’s orbit with respect to sun - i.e. incoming speed with respect to mars not considering mars-

satellite interaction

In[ ]:= vInfMars = vMars - vHohmannA

Out[ ]= 2.65 km/s

$V_{P/\text{Mars}}$: incoming v with respect to mars coming in a hyperbolic orbit to arrival altitude, considering mars-satellite interaction

In[ ]:= $\text{vPMars} = \sqrt{\frac{2 * \mu\text{Mars}}{\text{rArrival}} + \text{vInfMars}^2}$

Out[ ]= 5.60 km/s

The first term is the velocity gained from the Mars-satellite interaction, i.e. the escape velocity from r = rArrival but equivalently the velocity gained from the gravitational attraction of Mars, coming from zero speed infinitely far away. vInfMars is the excess hyperbolic velocity, i.e. the velocity needed in excess to the escape velocity, equal to the difference of incoming speed of the elliptical Hohmann transfer orbit seen from Mars.

Note that it doesn't matter if you come with extra or not enough speed compared to mars in it's orbit with respect to the sun, the difference gets squared, which makes sense.

v needed with respect to mars to be in circular orbit of radius rArrival

In[ ]:= $\text{vSatFinal} = \sqrt{\frac{\mu\text{Mars}}{\text{rArrival}}}$

Out[ ]= 3.491 km/s

vArrive: Δv needed to arrive in circular orbit around mars with respect to spacecraft reference frame

In[ ]:= vArrive = vSatFinal - vPMars

Out[ ]= -2.11 km/s

So we have to speed down by 2.11 km/s when we get to mars.

So V_{∞}^{-} = 2.65 km/s.

$V_{P/\text{Mars}}$ = 5.60 km/s.

A Δv of -2.11 km/s needed to arrive at Mars.

1 **https://en.wikipedia.org/wiki/Hyperbolic_trajectory#Velocity**

For good illustrations of hyperbolic orbits, see:

http://hopsblog-hop.blogspot.com/2013/03/what-heck-is-vinf.html

https://www.researchgate.net/figure/Departure-orbit-geometry-Source-Curtis-p-443-7-Hence-the-velocity-of-the_fig3 _ 323256893