1. Newton's 2nd

$$F_s = \sum F = \overset{\text{Sun}}{F_\odot} + \overset{\text{Earth}}{F_\oplus} + \overset{\text{Mars}}{F_{\text{♂}}} = m_s a_s$$

$$\Leftrightarrow \frac{F_s}{m_s} = a_s = \boxed{\frac{d^2\bar{r}}{dt^2} = G\left(\frac{M_\odot}{r_\odot^2}\hat{r}_\odot + \frac{M_\oplus}{r_\oplus^2}\hat{r}_\oplus + \frac{M_{\text{♂}}}{r_{\text{♂}}^2}\hat{r}_{\text{♂}}\right)}$$

2 Hamilton's Equations

Step 0: Lagrangian L $\quad L = T - V$

Step 1: Generalized momenta $p_i(\underline{q}, \underline{\dot{q}}, t) = \frac{\partial L}{\partial \dot{q}_i}$

Step 2: Legendre transform $\dot{q}_i = \dot{q}_i(\underline{q}, \underline{p}, t)$

Step 3: Hamiltonian $H(\underline{q}, \underline{p}, t) = \sum_{i=1}^{n} p_i \dot{q}_i - L$

Step 4: Hamilton's equations of motion

$$\dot{q}_i = \frac{\partial H}{\partial p_i}$$

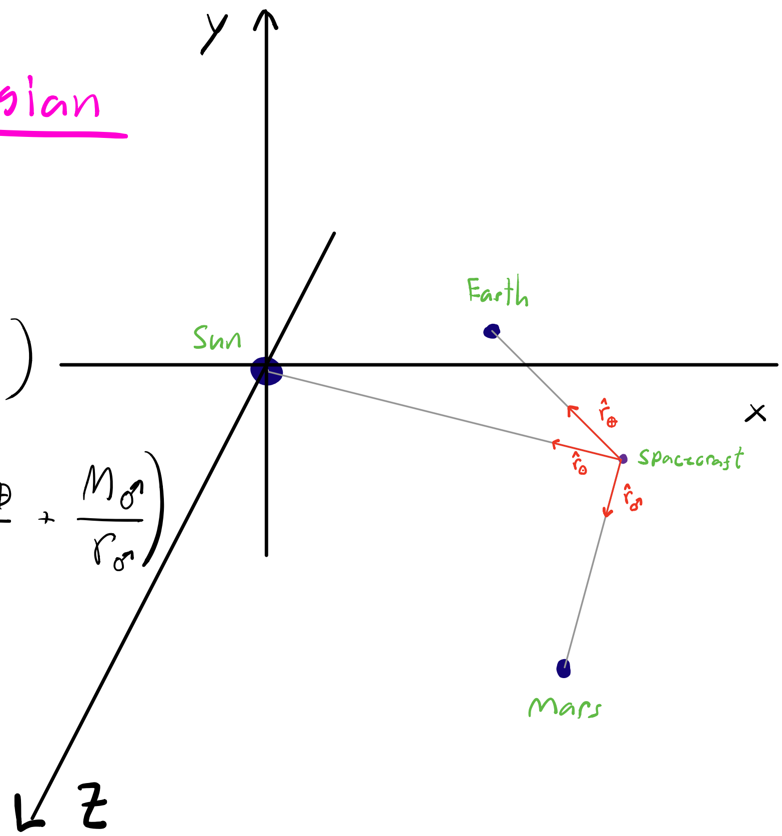
$$\dot{p}_i = -\frac{\partial H}{\partial q_i}$$

Step 0 Lagrangian

$$L = T - V$$

$$T = \frac{1}{2} m_s \left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right)$$

$$V = -G m_s \left(\frac{M_\odot}{r_\odot} + \frac{M_\oplus}{r_\oplus} + \frac{M_{♂}}{r_{♂}} \right)$$



$q_x = x$	$\odot$ Sun
$q_y = y$	$\oplus$ Earth
$q_z = z$	$♂$ Mars

$$\Longrightarrow L = m_s \left[\frac{1}{2} \left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2 \right) + G \left(\frac{M_\odot}{r_\odot} + \frac{M_\oplus}{r_\oplus} + \frac{M_{♂}}{r_{♂}} \right) \right]$$

Step 1 p_i

$$p_i = \frac{\partial L}{\partial \dot{q}_i} = \frac{1}{2} m_s \cdot 2 \dot{q}_i = m_s \dot{q}_i$$

Step 2 Legendre Transform

$$\dot{q}_i = \frac{p_i}{m_s}$$

Step 3 Hamiltonian

$$H = \sum_i p_i \dot{q}_i - L$$

$$= \frac{p_x^2 + p_y^2 + p_z^2}{m_s} - m_s \Bigg[\frac{1}{2}\left(\frac{p_x^2}{m_s^2} + \frac{p_y^2}{m_s^2} + \frac{p_z^2}{m_s^2}\right) + G\left\{\frac{m_\odot}{\sqrt{x-x_\odot)^2+(y-y_\odot)^2+(z-z_\odot)^2}} + \frac{m_\oplus}{\sqrt{x-x_\oplus)^2+(y-y_\oplus)^2+(z-z_\oplus)^2}} - \frac{m_{\text{♂}}}{\sqrt{x-x_{\text{♂}})^2+(y-y_{\text{♂}})^2+(z-z_{\text{♂}})^2}}\right\} \Bigg]$$

Step 4 Hamilton's Equations

$$\dot{q}_i = \frac{\partial H}{\partial p_i} = \frac{2p_i}{m_s} - \frac{p_i}{m_s} = \frac{p_i}{m_s}$$

$$\dot{p}_i = -\frac{\partial H}{\partial q_i} = -G m_s \left[\frac{m_\odot (q_i - q_{i,\odot})}{\left[(q_x - q_{x\odot})^2 + (q_y - q_{y\odot})^2 + (q_z - q_{z\odot})^2\right]^{\frac{3}{2}}} + \frac{m_\oplus (q_i - q_{i\oplus})}{\left[(q_x - q_{x\oplus})^2 + (q_y - q_{y\oplus})^2 + (q_z - q_{z\oplus})^2\right]^{\frac{3}{2}}} + \frac{m_{\text{♂}} (q_i - q_{i,\text{♂}})}{\left[(q_x - q_{x\text{♂}})^2 + (q_y - q_{y\text{♂}})^2 + (q_z - q_{z\text{♂}})^2\right]^{\frac{3}{2}}} \right]$$