In[1]:=
```
Quiet@Remove["`*"]
```

In[2]:=
```
(*Prevents Mathematica from using Sec=1/Cos, Csc=1/Sin and Cot=1/Tan*)
$PrePrint = # /. {
      Csc[z_] :> 1/Defer@Sin[z],
      Sec[z_] :> 1/Defer@Cos[z],
      Cot[z_] :> 1/Defer@Tan[z]
     } &;
```

Solve $q_{j,i+1}$ equations

Compared with report, we use notation R1 instead of R_{i+1}, R0 instead of R_i etc.

In[3]:=
```
eqn1 = R1 == Br1 * h + R0;
```

In[4]:=
```
eqn2 = θ1 == Bθ1/R1^2 h + θ0;
```

In[5]:=
```
eqn3 = ϕ1 == Bϕ1/(R1^2 Sin[θ1]^2) h + ϕ0;
```

In[6]:=
```
qsols = Solve[{eqn1, eqn2, eqn3}, {R1, θ1, ϕ1}][[1]] // FullSimplify;
```

... **Solve**: Inconsistent or redundant transcendental equation. After reduction, the bad equation is $B\theta 1\, h + Br1^2\, h^2\, \theta 0 + 2\, Br1\, h\, R0\, \theta 0 + R0^2\, \theta 0 - Br1^2\, h^2\, \text{ArcCsc}[\text{Csc}[\theta 1]] - 2\, Br1\, h\, R0\, \text{ArcCsc}[\text{Csc}[\theta 1]] - R0^2\, \text{ArcCsc}[\text{Csc}[\theta 1]] == 0$.

... **Solve**: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

In[7]:=
```
qsols // TableForm
```

Out[7]//TableForm=

$$R1 \to Br1\, h + R0$$
$$\theta 1 \to \frac{B\theta 1\, h}{(Br1\, h + R0)^2} + \theta 0$$
$$\phi 1 \to \phi 0 + \frac{B\phi 1\, h}{(Br1\, h + R0)^2\, \text{Sin}\left[\frac{B\theta 1\, h}{(Br1\, h + R0)^2} + \theta 0\right]^2}$$

Using Solve gives us incomplete solutions (and as it turns out, incomplete simplifying). We will use Reduce instead, with some number domain assumptions:

In[8]:=
```
qreds = Reduce[{eqn1, eqn2, eqn3}, {R1, θ1, ϕ1}] // FullSimplify;
```

In[9]:=
```
qreds // TableForm
```

Out[9]//TableForm=

$$\begin{aligned}&\left(\text{Im}[Br1]\, \text{Im}[h] \neq \text{Re}[R0 + Br1\, \text{Re}[h]] \;||\right.\\ &\quad \left.\text{Im}[Br1]\, \text{Im}[h] \neq \text{Im}[R0] + \text{Re}\left[R0 + h\, \text{Im}[Br1] + Br1\, \left(\text{Im}[h] + \text{Re}[h]\right)\right]\right) \;\&\&\\ &Br1\, h + R0 == R1 \;\&\&\; \theta 1 == \frac{B\theta 1\, h}{R1^2} + \theta 0 \;\&\&\; \phi 1 == \phi 0 + \frac{B\phi 1\, h}{R1^2\, \text{Sin}[\theta 1]^2}\end{aligned}$$

In[10]:=
```
Refine[qreds, {Br1 ∈ Reals, h ∈ Reals, R0 ∈ Reals}] // FullSimplify // TableForm
```

Out[10]//TableForm=

$$Br1\, h + R0 \neq 0 \;\&\&\; Br1\, h + R0 == R1 \;\&\&\; \theta 1 == \frac{B\theta 1\, h}{R1^2} + \theta 0 \;\&\&\; \phi 1 == \phi 0 + \frac{B\phi 1\, h}{R1^2\, \text{Sin}[\theta 1]^2}$$