SAHIL

2016UCS0008

16.1-5 Objective— Maximize $\sum_{a_k \in A} v_k$

where A is a set of compatible activities.

Let S_{ij} denote the set of activities that start after activity a_i finishes and that finish before activity a_j starts.

Let us further denote the maximum value

$$\sum_{a_k \in S_{ij}} v_k = dp[i][j]$$

Then, our answer would be $dp[0][n+1]$

We can write a recurrence relation:

$$dp[i][j] = \begin{cases} 0, & \text{if } S_{ij} = \phi \\ \max\limits_{a_k \in S_{ij}} \{ dp[i][k] + dp[k][j] + v_k \}, & \text{if } S_{ij} \neq \phi \end{cases}$$

Here, we chose an activity a_k and then find solutions to smaller subproblems 'i to k' and 'k to j' and maximize over all possibilities.

16.2-3 Given that order of items when sorted by increasing weight is the same as their order when sorted by decreasing value.

eg –

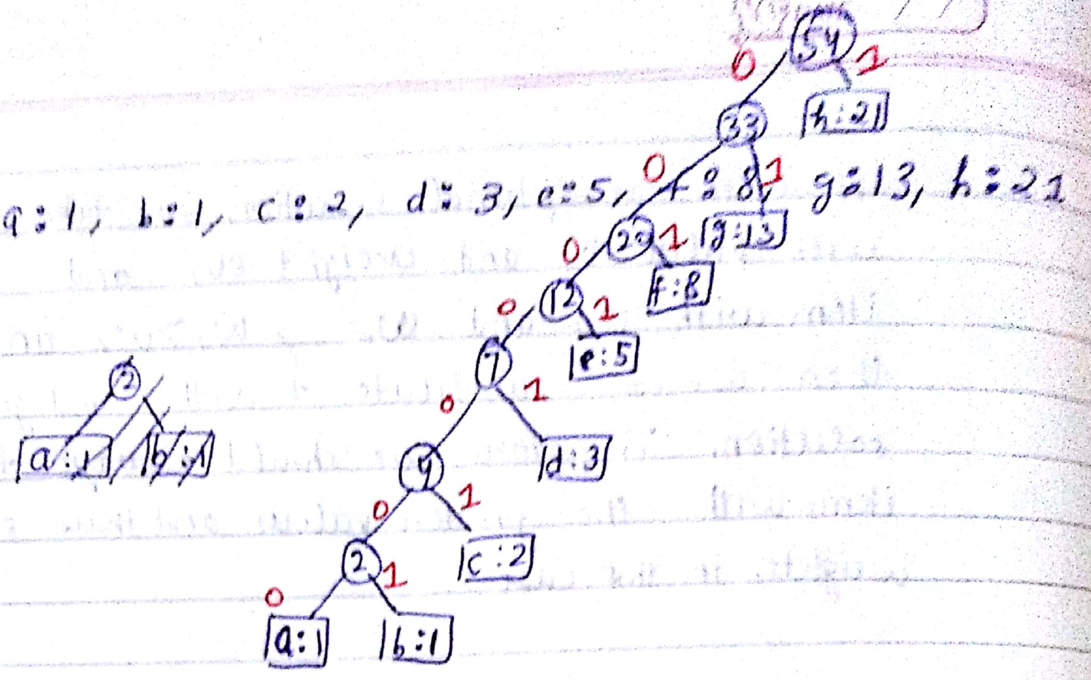
weight	5	6	7	8	10	20
value	12	10	8	4	2	1

Suppose capacity is W = 15

Since a less weight item gives higher value, the greedy strategy is to select items as in the given order until weight of selected items exceeds the capacity W.

Suppose in an optimal solution, we take an item with value v_1 and weight w_1 and drop an item with v_2 and w_2, $w_1 > w_2$ and $v_1 < v_2$ then we can substitute 1 with 2 and get a better solution. Therefore, we should ~~always~~ choose the items with the greatest values, and thus smaller weights, in this case.

16.3-3 a:1, b:1, c:2, d:3, e:5, f:8, g:13, h:21



An optimal Huffman code would thus be
0000000 → a, 0000001 → b, 000001 → c,
00001 → d, 0001 → e, 001 → f, 01 → g, 1 → h

This generalizes to the 1st n fibonacci numbers as the ~~frequencies in the~~ $k < n$th most frequent letter has codeword of $0^{k-1}1$ and the nth most frequent letter has codeword 0^{n-1}.

To see this holds, we prove the recurrence,

$$\sum_{i=0}^{n-1} F(i) = F(n+1) - 1$$

This will show that we should join together the letter with frequency $F(n)$ with the result of joining together the letters with smaller frequencies.

We can show this by induction.

Base step $n=1$ $\quad 0 = F(1) - 1 = 0$ True.

Inductive step – Let this be true for $n = k-1$

$$\Rightarrow \sum_{i=0}^{k-2} F(i) = F(k) - 1$$

We shall show that this holds for $n = k$.

$$\Rightarrow \sum_{i=0}^{k-1} F(i) = \sum_{i=0}^{k-2} F(i) + F(k-1)$$

$= F(k)-1 + F(k-1) = F(k+1)-1$

Hence, proved.