+++ title = 'Ideal Gas Law' date = 2024-06-23T18:47:09+08:00 tags = ['kinetic particle theory'] +++

Boyle's law

At constant temperature and number of particles,

$$p \propto V^{-1}$$

where p is the pressure of a gas, and V is the volume of it.

Charles's law

At constant pressure and number of particles,

$$V \propto T$$

where V is the volume of a gas, and T is the temperature of it.

Avogadro's law

At constant pressure and temperature of particles,

$$V \propto N$$

where V is the pressure of a gas, and N is the number of particles.

Ideal gas law

Suppose we have ideal gas with pressure, volume, and temperature of p_1, V_1, and T_1 respectively, with N_1 particles. Firstly, only the pressure and volume is changed:

$$\left(p_1, V_1, N_1, T_1\right) \to \left(p_2, V_2, N_1, T_1\right)$$

By Boyle's law,

$$p_1 V_1 = p_2 V_2$$

Secondly, only the volume and temperature is changed:

$$\left(p_2, V_2, N_1, T_1\right) \to \left(p_2, V_3, N_1, T_2\right)$$

By Charles's law,

$$V_2 T_2 = V_3 T_1$$

Thirdly, only the volume and number of particles is changed:

$$\left(p_2, V_3, N_1, T_2\right) \to \left(p_2, V_4, N_2, T_2\right)$$

By Avogadro's law,

$$V_3 N_2 = V_4 N_1$$

Now since V_4 can be changed to any value, let $V_4 = V_2$. Thus,

$$V_3 N_2 = V_2 N_1$$

$$
\begin{aligned}
p_1V_1 &= p_2V_2 \\
p_1V_1T_2 &= p_2V_2T_2 \\
p_1V_1T_2 &= p_2V_3T_1 \\
p_1V_1N_2T_2 &= p_2V_3N_2T_1 \\
p_1V_1N_2T_2 &= p_2V_2N_1T_1 \\
\frac{p_1V_1}{N_1T_1} &= \frac{p_2V_2}{N_2T_2} \\
\frac{pV}{NT} &= k_B
\end{aligned}
$$

Given an ideal gas,

$$pV = Nk_BT$$

where p, V, and T are the pressure, volume, and temperature of the gas respectively, with N being the number of particles, and
$$\boxed{k_B=\pu{1.38E-23 J K-1}}$$