+++ title = '$\int_a^b \sqrt{1-x^2}\,\mathrm{d}x$' date = 2024-06-23T20:08:24+08:00 tags = ['integrals'] +++

Some Formulae

$$\cos 2\theta + i\sin 2\theta$$

$$= e^{i(2\theta)}$$

$$= \left(e^{i\theta}\right)^2$$

$$= (\cos\theta + i\sin\theta)^2$$

$$= (\cos\theta)^2 + 2(\cos\theta)(i\sin\theta) + (i\sin\theta)^2$$

$$= \cos^2\theta + 2i\sin\theta\cos\theta - \sin^2\theta$$

$$= \left(\cos^2\theta - \sin^2\theta\right) + i(2\sin\theta\cos\theta)$$

$$\begin{cases} \cos 2\theta = \cos^2\theta - \sin^2\theta \\ \sin 2\theta = 2\sin\theta\cos\theta \\ \end{cases}$$

$$\begin{aligned} \cos 2\theta &= \cos^2\theta - \sin^2\theta \\ \cos 2\theta &= \cos^2\theta - \left(1 - \cos^2\theta\right) \\ \cos 2\theta &= \cos^2\theta - 1 + \cos^2\theta \\ \cos 2\theta &= 2\cos^2\theta - 1 \\ 2\cos^2\theta &= \cos 2\theta + 1 \\ \cos^2\theta &= \frac{\cos 2\theta + 1}{2} \end{aligned}$$

The Integral

$$\int_a^b \sqrt{1-x^2}\,\mathrm{d}x$$

$$= \int_\alpha^\beta \sqrt{1-(\sin\theta)^2}\cdot \cos\theta\,\mathrm{d}\theta$$

$$= \int_\alpha^\beta \sqrt{\sin^2\theta + \cos^2\theta - \sin^2\theta}\cdot \cos\theta\,\mathrm{d}\theta$$

$$= \int_\alpha^\beta \sqrt{\cos^2\theta}\cdot \cos\theta\,\mathrm{d}\theta$$

$$= \int_\alpha^\beta \cos\theta\cos\theta\,\mathrm{d}\theta$$

$$= \int_\alpha^\beta \cos^2\theta\,\mathrm{d}\theta$$

$$= \int_\alpha^\beta \frac{\cos 2\theta + 1}{2}\,\mathrm{d}\theta$$

$$= \left.\frac{\frac{1}{2}\sin 2\theta + \theta}{2}\right|_\alpha^\beta$$

$$= \frac{1}{2}\left(\sin\theta\cos\theta + \theta\right)\Big|_\alpha^\beta$$

$$= \frac{1}{2}\left(\sin\beta\cos\beta + \beta - \sin\alpha\cos\alpha + \alpha\right)$$

$$= \frac{1}{2}\left(b\sqrt{1-b^2} + \sin^{-1} b - a\sqrt{1-a^2} + \sin^{-1} a\right)$$