DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

Mid Semester Examination – May 2025

Course: B. Tech in Computer Engineering **Sem: IV**

Subject Name: Probability Theory and Random Processes **Subject Code: BTBS404**

Max Marks: 20 **Date: - / 05 /2025** **Duration: - 1 Hr.**

Instructions to the Students:
1. Figures to right indicate full marks.
2. Use of nonprogrammable calculator is allowed.
3. Use of Normal Distribution table to find area is allowed.

		CO	Blooms Level	Marks
Q. 1	**Multiple Choice Questions**	CO		6
1 d	A card is drawn from a well shuffled pack of playing cards. What is the probability that it is a red card ? a) $\frac{4}{13}$ b) $\frac{1}{4}$ c) $\frac{1}{52}$ d) $\frac{1}{2}$	CO1	2	
2 b	A die is thrown two times and the sum of numbers on the top faces is noted. What is the probability of this sum being 11 ? a) $\frac{1}{6}$ b) $\frac{1}{18}$ c) $\frac{1}{12}$ d) $\frac{1}{36}$	CO1	2	
3 a	What is the chance that a leap year contains 53 Sundays? a) $\frac{2}{7}$ b) $\frac{4}{7}$ c) $\frac{3}{7}$ d) $\frac{1}{7}$	CO1	2	
4 a	Tickets are numbered from 1 to 100.They are well shuffled and a ticket is drawn at random.What is the probability that the drawn ticket is a multiple of 17? *a*) 0.05 *b*) 0.2 *c*) 0.04 *d*) 0.5	CO1	2	
5 a	All Possible arrangement of the letters of the word "ASSASSINATION" are *a*) $\frac{13!}{3!\,4!\,2!2!}$ *b*) $\frac{13!}{2!\,2!\,2!\,2!}$ *c*) $\frac{13!}{2!\,2!\,2!}$ *d*) 13!	CO1	2	
6 c	If A and B are any two independent events such that P(A)=0.5 and P(AUB)=0.7 ,find P(B) *a*) 0.6 *b*) 0.2 *c*) 0.4 *d*) 0.8	CO1	2	
Q.2	**Solve Any Two of the following.**	CO		6
(A)	Calculate the value of rank correlation coefficient from the following data regarding marks of six students in Statistics and Mathematics in a test.	CO3	3	

Marks in Statistics	40	42	45	35	36	39
Marks in Mathematics	46	43	44	39	40	43

$$\rho = 1 - \frac{\left[6\sum d^2 + \frac{1}{12}(m_i^3 - m_i)\right]}{n^3 - n}$$

$$= 0.77$$

(B)	From the following data calculate Karl Pearson's correlation coefficient. $\bar{x}=18, \bar{y}=19$	CO3	$= 0.9587$

x	6	8	12	15	18	20	24	28	31
y	10	12	15	15	18	25	22	26	28

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$$

(C)	The covariance of two perfectly correlated variables X and Y is 96. Determine σ_x and σ_y, if it is known that variance of X and Y is in the ratio 4:9.	CO3	3

$r = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y} = \frac{144}{\sigma_y^2}$ $\therefore \sigma_x = 8$ and $\sigma_y = 12$

Q. 3	Solve Any One of the following.	CO	8
(A)	The hourly wages of 1,000 workmen are normally distributed around a mean of Rs.70 and with a standard deviation of Rs.5. Estimate the number of workers whose hourly wages will be: i) Between Rs. 69 and Rs. 72 ii) more than Rs. 75 iii) Less than Rs. 63 [(Area between Z=0 and Z=0.4)=0.1554, (Area between Z=0 and Z=0.2)=0.0793, (Area between Z=0 and Z=1)=0.3413 (Area between Z=0 and Z=1.4)=0.4192]	CO2	3

(i) $P(69<x<72) = 0.2347$

(ii) $P(x>75) = 0.1587$

(iii) $P(x<63) = 0.0808$

(B)	Assume that mean height of soldiers to be 68.22 inches with a variance of 10.8 inches. How many soldiers in a regiment of 1,000 would you expect to be i) over six feet tall ii) below 5.5 feet iii) less than 6 feet [(Area between Z=0 and Z=1.15)=0.3749, (Area between Z=0 and Z=0.67)=0.2501]	CO2	3

(i) $P(x>72) = 0.1251$

(ii) $P(x<66) = 0.2499$

(iii) $P(x<72) = 0.8749$

*** End ***

Course Coordinator

Module Coordinator

PAQIC Coordinator

HOD

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Q.1

1.

Sol^n d] $\frac{1}{2}$

2.

Sol^n b] $\frac{1}{18}$

3.

Sol^n a] $\frac{2}{7}$

4.

Sol^n a] 0.05

5.

Sol^n a] $\dfrac{13!}{3!\,4!\,2!\,2!}$

6.

Sol^n c] 0.4

Q.2

A]

Soln

$\bar{x} = \frac{237}{6} = 39.5$

$\bar{y} = \frac{255}{6} = 42.5$

X	35	36	39	40	42	45
R_x	1	2	3	4	5	6
Y	39	40	43	43	44	46
R_y	1	2	3.5	3.5	5	6

X	R_x	Y	R_y	$d = R_x - R_y$	d^2
40	4	46	6	-2	4
42	5	43	3.5	1.5	2.25
45	6	44	5	1	1
35	1	39	1	0	0
36	2	40	2	0	0
39	3	43	3.5	0.5	0.25

$\therefore \Sigma d^2 = 6.50$

$\therefore r = 1 - \frac{6\Sigma d^2}{n(n^2-1)}\left[\frac{m_1^2-1}{n!}\right]$

$= \frac{1}{1} - \frac{6(6.50)}{6(36-1)}\left[\frac{(43)^2-1}{2!}\right]$

$= \frac{1}{1} - \frac{39}{210}\left[\frac{1848}{2}\right]$

01

$= \frac{1}{1} - \frac{39}{210}[924]$

$= 1 - 171.6$

$= -170.6$

8]

Sol^n $\bar{x} = \frac{162}{9} = 18$

$\bar{y} = \frac{171}{9} = 19$

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
6	-12	144	10	-9	81	108
8	-10	100	12	-7	49	70
12	-6	36	15	-4	16	24
15	-3	9	15	-4	16	12
18	0	0	18	-1	1	0
20	2	4	25	6	36	12
24	6	36	22	3	9	18
28	10	100	26	7	49	70
31	13	169	28	9	81	117

$\therefore \Sigma (x-\bar{x})^2 = 598$

$\Sigma (y-\bar{y})^2 = 338$

$\Sigma (x-\bar{x})(y-\bar{y}) = 431$

$\therefore r = \frac{\cancel{\text{cov}(x,y)}\ \Sigma (x-\bar{x}) \cdot (y-\bar{y})}{\sqrt{\Sigma (x-\bar{x})^2} \cdot \sqrt{\Sigma (y-\bar{y})^2}}$

$= \frac{431}{\sqrt{598}\sqrt{338}}$

$= \frac{431}{449.5}$

$r = 0.9588$

$\therefore r \approx 0.96$

3

Q.3

A]

Sol^n

$N = 1000$, $\mu = 70$, $\sigma = 5$

① $P(69 \le x \le 72) = P\left(\frac{69-\mu}{\sigma} \le \frac{x-\mu}{\sigma} \le \frac{72-\mu}{\sigma}\right)$

$= P\left(\frac{69-\mu}{\sigma} \le z \le \frac{72-\mu}{\sigma}\right)$

$= P\left(\frac{69-70}{5} \le z \le \frac{72-70}{5}\right)$

$= P(-0.2 \le z \le 0.4)$

= Area bet^n $z_1 = -0.2$ to $z_2 = 0.4$

= (Area bet^n $z=0$ to $z_1 = 0.2$) + (Area bet^n $z=0$ to $z_2 = 0.4$)

$= 0.0793 + 0.1554$

$= 0.2347$

No. of workers whose hourly wages bet^n Rs. 69 and Rs. 72

$= N \times 0.2347$

$= 1000 \times 0.2347$

$= 234.7$

≈ 235

② $P(x \ge 75) = P\left(\frac{x-\mu}{\sigma} \ge \frac{75-\mu}{\sigma}\right)$

$= P\left(z \ge \frac{75-70}{5}\right)$

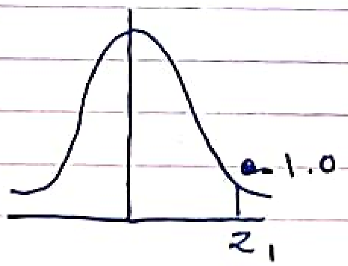
07

$= P(z \ge 1)$

$= 0.5 - 0.3413$

$= 0.5 - 0.3413$

$= 0.1587$



No. of workers whose hourly wages more than Rs. 75

$= N \times 0.1587$

$= 1000 \times 0.1587$

$= 158.7 \approx 159$

③ $P(X \leq 63) = P\left(\frac{x-\mu}{\sigma} < \frac{63-\mu}{\sigma}\right)$

$= P\left(z \leq \frac{63-70}{5}\right)$

$= P\left(z < \frac{-7}{5}\right)$

$= P(z \leq -1.4)$

$=$ Area betn $z=0$ and $z=1.4$

$= 0.4192$

No. of workers hourly wages less Rs 63 $= N \times 0.4192$

$= 1000 \times 0.4192$

$= 419.2$

$= 420$

Q.2

c]

Sol^n Given: var(x) : Var(Y) = 4 : 9

$\therefore \frac{\sigma_x}{\sigma_y} = \frac{\sqrt{var(x)}}{\sqrt{var(y)}}$

$= \frac{\sqrt{4}}{\sqrt{9}}$

$\sigma_x = \frac{2}{3}\sigma_y$

$\because cov(x,y) = 96$, $r = 1$ (perfectly)

$\because r = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y}$

$= \frac{96}{\frac{2}{3}\sigma_y \cdot \sigma_y}$

$= \frac{96 \times 3}{2}(\sigma_y)^2$

$1 = \frac{144}{1}(\sigma_y)^2$

$\frac{1}{1}(\sigma_y)^2 = 144$

$\sigma_y = 12$

$\therefore \frac{\sigma_x}{\sigma_y} = \frac{2}{3}$

$\sigma_x = \frac{2}{3} \times 12$

$= 8$

(Sign of Evaluator)

(START WRITING FROM HERE ONLY)

Q.1

1) $\longrightarrow$ d) $\frac{1}{2}$

2) $\longrightarrow$ b) $\frac{1}{18}$

3) $\longrightarrow$ a) $\frac{2}{7}$

4) $\longrightarrow$ a) 0.05

5) $\longrightarrow$ a) $\frac{13!}{3!\,4!\,2!\,2!}$

6) $\longrightarrow$ c) 0.4

Q.2

(A) →

Marks in statistics	40	42	45	35	36	39
Marks in Mathematics	46	43	44	39	40	43

Let the rank in statistics be R_x and in mathematics be R_y

∴ Rank calculation table:

Let marks in statistics be x and in mathematics be y.

x	35	36	39	40	42	45
R_x	1	2	3	4	5	6
Y	39	40	43	43	44	46
R_y	1	2	3.5	3.5	5	6

∴ Calculation table is given as:

x	R_x	y	R_y	$d = R_x - R_y$	d^2
40	4	46	6	-2	4
42	5	43	3.5	1.5	2.25
45	6	44	5	1	1
35	1	39	1	0	0
36	2	40	2	0	0
39	3	43	3.5	-0.5	0.25

$\therefore \Sigma d^2 = 7.5$

Here ranks are repeated.
i.e. 3.5 repeated 2 times.

$\therefore m_1 = 2$

$\therefore n = 6$

$\therefore$ rank correlation coefficient

$$\therefore r = 1 - \frac{6\left[\Sigma d^2 + m_1(m_1^2 - 1)\right]}{n(n^2-1)}$$

$$\therefore r = 1 - \frac{6\,[7.5 + 2(4-1)]}{6(36-1)}$$

$$\therefore r = 1 - \frac{6\,[7.5+6]}{6 \times 35}$$

$$\therefore r = 1 - \frac{6 \times 13.5}{210}$$

$$\therefore r = \frac{210 - 81}{210}$$

$$\therefore r = \frac{129}{210}$$

$$\therefore \boxed{r = 0.6142}$$

$\therefore$ Rank Correlation coefficient $r = 0.6142$

Q.2

(B) →

x	6	8	12	15	18	20	24	28	31
y	10	12	15	15	18	25	22	26	28

Here $\bar{x} = 18$
$\bar{y} = 19$

Calculation Table is given as:

x	$(x-\bar{x})$	$(x-\bar{x})^2$	y	$(y-\bar{y})$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
6	-12	144	10	-9	81	171
8	-10	100	12	-7	49	70
12	-6	36	15	-4	16	24
15	-3	9	15	-4	16	12
18	0	0	18	-1	1	0
20	2	4	25	6	36	12
24	6	36	22	3	9	18
28	10	100	26	7	49	70
31	13	169	28	9	81	117
		$\Sigma(x-\bar{x})^2 = 598$			$\Sigma(y-\bar{y})^2 = 338$	$\Sigma(x-\bar{x})(y-\bar{y}) = 494$

2

∴ Karl Pearson's Correlation coefficient

$$\therefore r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$$

$$\therefore r = \frac{494}{\sqrt{598}\sqrt{338}}$$

$$\therefore r = \frac{494}{24.45 \times 18.38} = \frac{494}{449.391}$$

$$\therefore \boxed{r = 1.09}$$

Q.2

(C) $\rightarrow$ Given - $\text{COV}(X,Y) = 96$, $r = 1$

$$\frac{\text{var}(X)}{\text{var}(Y)} = \frac{4}{9}$$

$$\therefore \frac{\sigma_x^2}{\sigma_y^2} = \cancel{\frac{2}{3}} \frac{4}{9}$$

$$\therefore \frac{\sigma_x}{\sigma_y} = \frac{2}{3}$$

$$\therefore \sigma_x = \frac{2}{3}\sigma_y \quad \text{——} \quad \text{I}$$

$$\therefore r = \frac{\text{COV}(X,Y)}{\sigma_x \sigma_y}$$

$$\therefore 1 = \frac{96}{2/3\sigma_y \cdot \sigma_y}$$

$$\therefore \sigma_y^2 = \frac{\overset{48}{\cancel{96}}}{\cancel{2}} \times 3$$

$$\therefore \sigma_y^2 = 144$$

$$\therefore \sigma_y = \sqrt{144}$$

$$\therefore \boxed{\sigma_y = 12}$$

3

from eqⁿ I,

$$\sigma_x = \frac{2}{3}\sigma_y$$

$$\therefore \sigma_x = \frac{2}{\cancel{3}} \times \overset{4}{\cancel{12}}$$

$$\therefore \boxed{\sigma_x = 8}$$

(START WRITING FROM HERE ONLY)

Q.3

(A) $\longrightarrow$ Given : $N=1000$, $\mu=70$, $\sigma=5$

i) $P(69 \leq x \leq 72) = P\left(\frac{69-\mu}{\sigma} \leq \frac{x-\mu}{\sigma} \leq \frac{72-\mu}{\sigma}\right)$

$= P\left(\frac{69-70}{5} \leq z \leq \frac{72-70}{5}\right)$

$= P(-0.2 \leq z \leq 0.4)$

$=$ (Area betn $z=0$ to $z=0.2$) + (Area betn $z=0$ to $z=0.4$)

$= 0.0793 + 0.1554$

$= 0.2347$

$\therefore$ Probability of no. of workers whose hourly wages are betn 69 to 72 $= 0.2347$

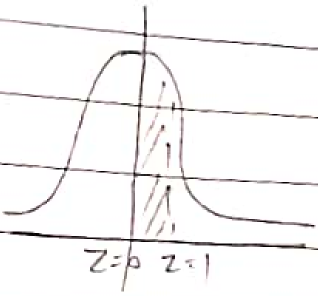
$\therefore$ No. of workers whose hourly wages are betn 69 to 72 $= 0.2347 \times 1000$

$= 234.7$

≈ 235 //

ii) $P(x > 75) = P\left(\frac{x-\mu}{\sigma} > \frac{75-\mu}{\sigma}\right)$

$= P\left(z > \frac{75-70}{5}\right)$



$= P(Z > 1)$

$= 0.5 - P(0 \le Z \le 1)$

$= 0.5 - (\text{Area betn } Z=0 \text{ to } Z=1)$

$= 0.5 - 0.3413$

$= 0.1587$

$\therefore$ Probability of No. of workers whose hourly wages is more than $75 = \underline{\underline{0.1587}}$

$\therefore$ No. of workers whose hourly wages is more than $75 = 0.1587 \times 1000$

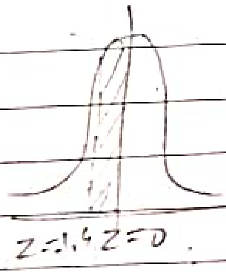
$= 158.7$

≈ 159 //

iii) $P(x < 63) = P\left(\frac{x-\mu}{\sigma} < \frac{63-\mu}{\sigma}\right)$

$= P\left(Z < \frac{63-70}{5}\right)$

$= P(Z < -1.4)$



$= \cancel{P}\, 0.5 - P(0 < Z < -1.4)$

$= 0.5 - (\text{Area betn } Z=0 \text{ to } Z=1.4)$

$= 0.5 - 0.4192$

$= 0.0808$

$\therefore$ Probability of No. ~~w~~ of workers whose hourly wages is less than $63 = \underline{\underline{0.0808}}$

$\therefore$ No. of workers whose hourly wages is less than $63 = 0.\cancel{1}0808 \times 1000$

$= 80.8$

≈ 81 //

(START WRITING FROM HERE ONLY)

Q.1. M.C.Q's

1. $\Rightarrow$ d] $\frac{1}{2}$

2. $\Rightarrow$ c] $\frac{1}{12}$

3. $\Rightarrow$ a] $\cancel{\frac{1}{2}}\ \frac{2}{7}$

4. $\Rightarrow$ a] 0.05

5. ⇒ a] $\frac{13!}{3!\,4!\,2!\,2!}$

6. ⇒ c] 0·4.

Q.2. Solve any two :

A]

⇒ Let, the marks in Statistics = x
, the marks in mathematics = y.

Here, $n = 6$.

x	40	42	45	35	36	39
y	46	43	44	39	40	43

Now, ~~arrangindg~~ arrange in ascending order.

x	35	36	39	40	42	45
R_x	1	2	3	4	5	6
y	39	40	43	43	44	46
R_y	1	2	3.5	3.5	5	6

3

x	R_x	y	R_y	$d = R_x - R_y$	d^2
40	4	46	6	−2	4
42	5	43	3·5	1·5	2·25
45	6	44	5	1	1
35	1	39	1	0	0
36	2	40	2	0	0
39	3	43	3·5	0·5	2·5

Sum of $d^2 = \Sigma d^2 = 9.75$

Here, in y 43 is repeated 2 times, $m_1 = 2$.

Correlation coeff $= r = 1 - \dfrac{6\left[\Sigma d^2 + \frac{m_1(m_1^2-1)}{12}\right]}{n(n^2-1)}$

$\therefore r = 1 - \dfrac{6\left[9.75 + \frac{2(2^2-1)}{12}\right]}{6(6^2-1)}$

$\therefore r = 1 - \dfrac{10.25}{35}$

$\therefore r = 1 - 0.2929$

$\therefore r = 7.071 \times 10^{-1}$

$\therefore \boxed{r = 0.7071}$

$\therefore$ The value of rank correlation coefficient is 0.7071.

c]

$\Rightarrow$ Given

$cov(X, Y) = 96$.

$\sigma_x = ?$, $\sigma_y = ?$, $r = 1$.

Variance of X & Y is in ration 4:9

Solⁿ :-

$\therefore \dfrac{V_x}{V_y} = \dfrac{4}{9}$

$\therefore$ By square root on both side

$\therefore \dfrac{\sqrt{V_x}}{\sqrt{V_y}} = \dfrac{2}{3}$

$\therefore \sigma_x = \sqrt{V_x}$, $\sigma_y = \sqrt{V_y}$

$\therefore \frac{\sigma_x}{\sigma_y} = \frac{2}{3} \rightarrow \sigma_x = \frac{2}{3}\sigma_y$ — ①

By karl's pearson's formula,

$\therefore r = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y}$

$\therefore 1 = \frac{96}{\frac{2}{3}\sigma_y \cdot \sigma_y}$ — from ①

$\therefore \sigma_y^2 = 1.44 \times 10^2$

$\therefore [\sigma_y = 12]$

from ①.

$\therefore \sigma_x = \frac{2}{3} \times 12$

$\therefore [\sigma_x = 8]$

$\therefore \boxed{\sigma_x = 8 \text{ and } \sigma_y = 12}$

Q. 3. Solve any one

A] $\Rightarrow$ Given

N = 1000 workers,

Mean = μ = 70.

S.D = σ = 5.

By Normal Distribution,

i] Between Rs. 69 and Rs. 72

$\therefore$ ~~$P(69 \le x < 72)$~~ $= P\left(\frac{69-70}{5} < \frac{x-\mu}{\sigma}\right.$

$\therefore P(69 < x < 72) = P\left(\frac{69-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{72-\mu}{\sigma}\right)$

$= P\left(\frac{69-70}{5} < z < \frac{72-70}{5}\right)$

$= P(-0.2 < z < 0.4)$

$= P(-0.2 < z < 0) + P(0 < z < 0.4)$

~~= Area betn z =~~

$= P(0 < z < 0.2) +$ Area bet^n $z = 0$ and $z = 0.4$

$=$ Area bet^n $z = 0$ and $z = 0.2$ $+ 0.1554$

$\therefore P(69 < x < 72) = 0.0793 + 0.1554$

$\therefore \boxed{P(69 < x < 72) = 0.2347}$

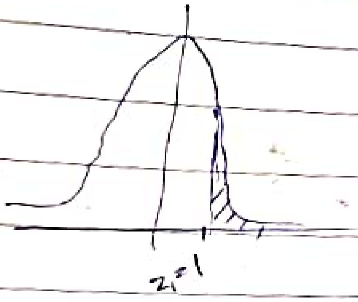
$\therefore$ The number of works whose hourly wages will be between Rs.69 and Rs 72 is 1000×0.2347
$= 234.7 \simeq 235$ workers.

ii] more than Rs 75

$P(x > 75) = P\left(\frac{x-\mu}{\sigma} > \frac{75-\mu}{\sigma}\right)$

$= P\left(z > \frac{75-70}{5}\right)$

$= P(z > 1)$



$= 0.5 - P(0 < z < 1)$

$= 0.5 -$ Area bet.^{n} $z = 0$ and $z = 1$

$= 0.5 - 0.3413$

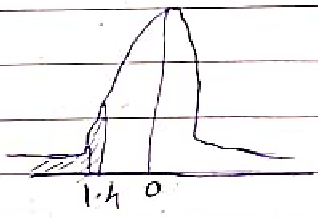
$= 1.587 \times 10^{-1}$

$\therefore P(x > 75) = 0.1587$

∴ The number of workers whose hourly wages will be more than Rs.75 is 1000×0.1587
$= 158.7 = 159$ workers.

iii] P(less than Rs.63

$\therefore P(x < 63) = P\left(\frac{x-\mu}{\sigma} < \frac{63-\mu}{\sigma}\right)$



$= P\left(z < \frac{63-70}{5}\right)$

$= P(z < -1.4)$

8

$\cancel{= 0.5 - P(-1.4 <}$

$= 0.5 - P(0 < z < 1.4)$

$= 0.5 -$ Area bet.^{n} $z=0$ and $z=1.4$

$= 0.5 - 0.4192$

$\therefore \left[P(x<63) = 0.08080\right]$

(START WRITING FROM HERE ONLY)

$\therefore$ The hourly wages of less than Rs.63 workers numbers is $0.08080 \times 1000 = 80.80$
$\simeq 81$ workers.