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SVKM'S INSTITUTE OF TECHNOLOGY, DHULE (5449)
Continuous Assessment-II Examination – May 2025

Course: B. Tech in Computer Engineering — **Semester: IV (SY Comp)**
Subject Name: Probability Theory and Random Processes — **Subject Code: BTBS404**
Max Marks: 20 — **Date: - 30/ 05 /2025** — **Duration: - 1 Hrs.**

Instructions to the Students:

1. Question 1 and 2 are compulsory
2. Assume suitable data, if necessary

CO **Blooms Level** **Marks**

Q-1 Solve any Two

A. In the estimation of regression equations of two variables X and Y the following results were obtained:
$\sum X = 900$, $\sum Y= 700$, n=10, $\sum X^2$=6360, $\sum Y^2$=2860, $\sum XY = 3900$, Where X and Y are deviations from respective means. Obtained the two regression equations.

CO4 3 5

B. The data about the sells & advertisement expenditure of a firm is given below:

	Sales (in Cr.)	Advertisement expenditure (in Cr.)
Means	40	6
S.D.	10	1.5

CO4 3 5

Coefficient correlation is 0.9

(iii) Estimate the likely sales for proposed Advertisement expenditure of 10 crores

(iv) What should be the Advertisement expenditure if the firm proposes a sales target of 60 crores?

C. Find the lines of regression y on x for the following data

x	91	97	108	121	67	124	51	73	111	57
y	71	75	69	97	70	91	39	61	80	47

CO4 3 5

Q-2 Solve any Two

A. Experience has shown that 20% of the manufactured product is of top quality. In one day's production of 400 articles, only 50 are of top quality. Show that either the production of the day chosen was not a representative sample or the hypothesis of 20% was wrong.

CO5 3 5

B. In a large city A, 20% of a random sample of 900 school boys had a slight physical defect. In another large city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant?

CO5 3 5

C. A machinist is expected to make engine parts with axle diameter of 1.75 cm. A random sample of 10 parts shows a mean diameter of 1.85 cm with SD of 0.1 cm. On the basis of this sample, would you say that the work of machinist is inferior? $[t_9(0.05) = 2.26]$

CO5 3 5

*** **End** ***

Q1 (A) $b_{yx} = 0.6132$, $b_{xy} =$

$X = 90,\ Y = 70$

Y on x

$y - \bar{y} = b_{yx}(x - \bar{x})$

$Y = 0.6132x + 14.812$

x on Y

$x - \bar{x} = b_{xy}(y - \bar{y})$

$X = 1.3636y + 5452$

(B) $\bar{x} = 40,\ \sigma_x = 10,\ \bar{y} = 6,\ \sigma_y = 1.5,\ r_{xy} = 0.9$

(i) x (sale) = 64 crores when $y = 10$

(ii) y (expendi) = 8.7 crores when $x = 60$

(C) $$b_{yx} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2} = \frac{3900}{6360} = 0.6132$$

$$\therefore y - 70 = 0.6132(x - 90) = 0.6132x + 125.18$$

$$Y = 0.6132x + 125.18$$

Q2 (A) $H_0: P = 1/5,\ H_1: P \neq 1/5$

$$z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = -3.75$$

LOS: $\alpha = 5\%$ $\therefore Z_\alpha = 1.96$ $\therefore |z| > Z_\alpha$

Ho rejected

(B) $P_1 = P_2$ $\therefore |z| = \dfrac{|P_1 - P_2|}{\sqrt{PQ(1/n_1 + 1/n_2)}} = 0.92,$

LOS: $\alpha = 5\%$ $\therefore Z_\alpha = 1.96$ $\therefore |z| < Z_\alpha$

Ho : Accepted

(C) $H_0: \bar{x} = \mu,\ H_1: \bar{x} \neq \mu$ $\therefore t = (\bar{x} - \mu)/S/\sqrt{n-1} = 3$

$\alpha = 5\%$ $\nu = n - 1 = 9$

t $t_\alpha(0.05) = 2.26$ $\therefore |t| > t_\alpha$

Q.1 [A]

A) → **Given:-**

$\Sigma X = 900$,

$\Sigma Y = 700$,

$n = 10$,

$\Sigma X^2 = 6360$

$\Sigma Y^2 = 2860$

$\Sigma XY = 3900$

$\therefore X$ and Y are deviations from respective means $= (x - \bar{x})$ & $(y - \bar{y})$

→

∴ Given data becomes,

∴ ~~$\Sigma(x-\bar{x}) = 900$,~~

~~$\Sigma(y-\bar{y}) = 700$,~~

$\Sigma(x-\bar{x})^2 = 6360$,

$\Sigma(y-\bar{y})^2 = 2860$

$\Sigma(x-\bar{x})(y-\bar{y}) = 3900$

∴ The regression equation for y on x :-

$$\therefore b_{yx} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$$

$$= \frac{390\cancel{0}}{636\cancel{0}}$$

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$$\therefore \boxed{b_{yx} = \frac{65}{106}}$$

$$\therefore \boxed{b_{yx} = 0.613}$$

$\therefore (y-\bar{y}) = b_{yx}(x-\bar{x})$

$\therefore (y-\bar{y}) = 0.613(x-\bar{x})$

$\therefore (y-700) = 0.613(x-900)$

$\therefore y-700 = 0.613x - 551.7$

$\therefore 0.613x - y = -700 + 551.7$

$$\therefore \boxed{0.613x - y = -148.3}$$

$$\therefore \boxed{y = 0.613x + 148.3}$$

This is eqⁿ for y on x.

∴ The regression equation for x on y:-

$$\therefore b_{xy} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(y-\bar{y})^2}$$

$= \frac{3900}{2860}$

$\therefore \boxed{b_{xy} = 1.363}$

$\therefore (x-\bar{x}) = b_{xy}(y-\bar{y})$

$\therefore (x-900) = 1.363(y-700)$

$\therefore x-900 = 1.363y - 954.1$

$\therefore x = 1.363y - 954.1 + 900$

$\therefore \boxed{x = 1.363y - 54.1}$

This is eqn for x on y.

Q.1 [B]

B) → Given:-

	Sales (in Cr.)	Advertisement expenditure (in Cr)
means	40	6
S.D.	10	1.5

$r = 0.9$,

Let Sales ~~mean~~ be x and advertisement expenditure be y.

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$\therefore \bar{x} = 40, \bar{y} = 6, \sigma_x = 10, \sigma_y = 1.5$

i) Given:- $y = 10, x = ?$

Now,

we have to find x (sales)

$\therefore$ The regression equation of x on y is,

$(x-\bar{x}) = b_{xy}(y-\bar{y})$

→

$\therefore (x-40) = bxy\,(y-6)$

Now,

$\therefore bxy = r\,\frac{\sigma y}{\sigma x}$

$= 0.9 \times \frac{1.5}{10}$

$\therefore \boxed{bxy = 0.135}$

$\therefore$ equation becomes,

$\therefore (x-40) = 0.135\,(10-6)$

$\therefore x-40 = 0.135 \times 4$

$\therefore x-40 = 0.54$

$\therefore \boxed{x = 40.54}$

Given:-

ii) $x = 60$, $y = ?$

Now,

we have to find (y) advertisement expenditure.

$\therefore$ regression eqn of y on x is,

$\therefore (y-\bar{y}) = byx\,(x-\bar{x})$

$\therefore (y-6) = byx\,(60-40)$

$\therefore byx = r\,\frac{\sigma x}{\sigma y}$

$= 0.9 \times \frac{10}{1.5}$

$\therefore \boxed{byx = 6}$

$\therefore$ Equation becomes,

$\therefore (y-6) = 6\,(60-40)$

$\therefore (y-6) = 6 \times 20$

$\therefore y-6 = 120$

$\therefore \boxed{y = 126}$

∴ i) The likely sales for proposed advertisement expenditure of 10 crores is 40.54 Cr.

ii) The advertisement expenditure if firm proposes a sales target of 60 crores is 126 cr.

Q.2 [A]

A) → Given:-

$n = 400, \boxed{p} = \frac{5\cancel{0}}{40\cancel{0}} = \frac{1}{8} = \boxed{0.125}$

$\therefore \boxed{P} = \frac{2\cancel{0}}{10\cancel{0}} = \boxed{0.2}$

$\therefore \boxed{Q} = 1 - P = 1 - 0.2 = \boxed{0.8}$

Here,
p = probability of sample,
P = probability of population.

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i) Null Hypothesis: $H_0 : p = P$
Alternate Hypothesis: $H_1 : p \neq P$

ii) test statistics:-

$$\therefore |z| = \left| \frac{p - P}{\sqrt{\frac{PQ}{n}}} \right|$$

$$= \left| \frac{0.125 - 0.2}{\sqrt{\frac{0.2 \times 0.8}{400}}} \right|$$

→

$$= \left| \frac{-0.075}{\sqrt{\frac{0.16}{400}}} \right|$$

$$= \left| \frac{-0.075 \times 20}{0.4} \right|$$

$$\boxed{|z| = 3.75}$$

iii) level of significance $\Rightarrow \alpha = 5\%$.

iv) Critical value:-

For $\alpha = 5\%$,

$$\boxed{Z\alpha = 1.96}$$

v) Decision:-

As $Zcal = 3.75 > Z\alpha = 1.96$

∴ The null Hypothesis is rejected.

∴ The either production of the day chosen was not a representative sample or the hypothesis of 20% was wrong.

Q.2 [B]

→ Given:- city A

$n_1 = 900$

$p_1 = \frac{20}{100}$

$\boxed{p_1 = 0.2}$

city B

$n_2 = 1600$

$p_2 = \frac{18.5}{100}$

$\boxed{p_2 = 0.185}$

Now,

(START WRITING FROM HERE ONLY)

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$= \frac{900 \times 0.2 + 1600 \times 0.185}{900 + 1600}$$

$$= \frac{180 + 296}{2500}$$

$$= \frac{476}{2500}$$

$$\therefore \boxed{P = 0.1904}$$

$$\therefore Q = 1 - P = 1 - 0.1904$$

$$\therefore \boxed{Q = 0.8096}$$

Where,

p_1 = probability of sample from city A

p_2 = probability of sample from city B.

P = probability of population.

Now,

i) Null Hypothesis: $H_0 : p_1 = p_2$

Alternative Hypothesis: $H_1 : p_1 \neq p_2$

ii) Test statistics:-

→

$$\therefore |Z| = \left| \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \right|$$

$$= \left| \frac{0.2 - 0.185}{\sqrt{0.1904 \times 0.8096 \times \left(\frac{1}{900} + \frac{1}{1600}\right)}} \right|$$

$$= \left| \frac{0.015}{\sqrt{\frac{0.1904 \times 0.8096 \times 2500}{900 \times 1600}}} \right|$$

$$= \left| \frac{0.015}{\sqrt{2.676 \times 10^{-4}}} \right|$$

$$\therefore |Z| = \left| \frac{0.015}{1.635 \times 10^{-2}} \right| = \left| \frac{0.015 \times 100}{1.635} \right|$$

$$\therefore \boxed{|Z| = 0.917}$$

iii) Level of significance $\Rightarrow \alpha = 5\%$.

iv) Critical Value:-

for LOS $\alpha = 5\%$,

$\boxed{Z\alpha = 1.96}$

v) Decision :- $Z_{cal} = 0.917 < Z\alpha = 1.96$

$\therefore$ The Null Hypothesis is accepted.

$\therefore$ There is no significant difference between proportions of two samples from city A and city B.

Q.2 [c]

c) → **Given:-**

$\bar{x} = 1.75$ cm, $n = 10$, $\mu = 1.85$cm,
$s = 0.1$ cm, $t_9 (0.05) = 2.26$

Now,

i) Null Hypothesis: $H_0 : \bar{x} = \mu$
Alternate Hypothesis : $H_1 : \bar{x} < \mu$

ii) test statistics for small sample test :-

$$\therefore |t| = \left| \frac{\bar{x} - \mu}{s/\sqrt{n-1}} \right|$$

$$= \left| \frac{1.75 - 1.85}{\frac{0.1}{\sqrt{10-1}}} \right|$$

$$= \left| \frac{-0.1}{\frac{0.1}{\sqrt{9}}} \right|$$

$$= \left| \frac{-\cancel{0.1} \times \sqrt{9}}{\cancel{0.1}} \right|$$

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$$\therefore \boxed{|t| = 3}$$

iii) Level of significance :- $\alpha = 5\%$.

iv) Critical Value :- for $\alpha = 5\%$, Here, for one tailed test, we consider, $2\alpha = 10\%$ and $\nu = \text{dof} = n-1 = 10-1 = 9$

$\therefore \boxed{Z\alpha = 2.26}$ ..... (given) t

v) Decision :- $3 > 2.26$

∴ The Null hypothesis is rejected.

∴ The work of machinist is not inferior according to above.

Q.1 [c]

c) → Given:-

x	91	97	108	121	67	124	51	73	111	57
y	71	75	69	97	70	91	39	61	80	47

Now,

~~x~~ ~~y~~

$$\therefore \bar{x} = \frac{91+97+108+121+67+124+51+73+111+57}{10}$$

$$= \frac{900}{10}$$

$$\therefore \boxed{\bar{x} = 90}$$

$$\therefore \bar{y} = \frac{71+75+69+97+70+91+39+61+80+47}{10}$$

$$= \frac{700}{10}$$

$$\therefore \boxed{\bar{y} = 70}$$

↪

Now,

x	$(x-\bar{x})$ $(x-90)$	y	$(y-\bar{y})$ $(y-70)$	$(x-\bar{x})(y-\bar{y})$	$(x-\bar{x})^2$	$(y-\bar{y})^2$
91	1	71	1	1	1	1
97	7	75	5	35	49	25
108	18	69	-1	-18	324	1
121	31	97	27	837	961	729
67	-23	70	0	0	529	0
124	34	91	21	714	1156	441
51	-39	39	-31	1209	1521	961
73	-17	61	-9	153	289	81
111	21	80	10	210	441	100
57	-33	47	-23	759	1089	529
				3900	6360	

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$\therefore$ line of regression of y on x

$\therefore (y-\bar{y}) = b_{yx}(x-\bar{x})$

$$\therefore b_{yx} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$$

$\therefore byx = \frac{3900}{6360}$

$\therefore \boxed{byx = 0.613}$

$\therefore (y-\bar{y}) = 0.613(x-\bar{x})$

$\therefore (y-70) = 0.613(x-90)$

$\therefore (y-70) = 0.613x - 55.17$

$\therefore y = 0.613x - 55.17 + 70$

$\therefore \boxed{y = 0.613x + 14.83}$

∴ This is line of reg. of y on x.

—— x —— x —— x ——

[Q-1]

B)

	Sales (in cr)	Adv
Means	40	6
S.D	10	1.5

Let sales be x and Advertisement expenditure be Y.

$\therefore \quad \bar{X} = 40, \quad \bar{Y} = 6$

$\sigma_x = 10, \quad \sigma_y = 1.5$

$r = 0.9$

(i) $Y = 10$

Find eqⁿ of regression of x on Y.

$$(x-\bar{x}) = \cancel{b}\, r.\frac{\sigma x}{\sigma y}(y-\bar{y})$$

$$(x-40) = 0.9 \times \frac{10}{1.5}(10-6)$$

$$x - 40 = 24$$

$$\boxed{x = 64}$$

Sales are 64.

(ii) $x = 60$.

S

Y on x.

$$(Y-\bar{Y}) = r\frac{\sigma y}{\sigma x}(x-\bar{x})$$

$$(Y-6) = 0.9\left(\frac{1.5}{10}\right)(20)$$

$$Y - 6 = 2.7$$

$$\boxed{Y = 8.7}$$

Advertisement expenditure are $8.7 \approx 9$

A) $\bar{x} = 900$, $\bar{Y} = 700$, $n = 10$

$\Sigma(x-\bar{x})^2 = 6360$, $\Sigma(Y-\bar{Y})^2 = 2860$

$\Sigma(x-\bar{x})(Y-\bar{Y}) = 3900$

(i) Regression eqⁿ for x on Y.

$$(x-\bar{x}) = bxy(Y-\bar{Y})$$

$$bxy = \frac{(x-\bar{x})(y-\bar{y})}{\Sigma(Y-\bar{Y})^2}$$

$$= \frac{3900}{2860}$$

$$bxy = 1.36$$

$$X - 900 = 1.36\,(Y - 700)$$

$$X - 900 = 1.36Y - 952$$

$$X = 1.36\,Y - 952 + 900$$

$$\boxed{X = 1.36\,Y - 52}$$

Regression equation for Y on x

$$(Y - \bar{Y}) = b_{yx}\,(x - \bar{x})$$

$$\therefore \quad b_{yx} = \frac{(x-\bar{x})(Y-\bar{Y})}{\Sigma(x-\bar{x})^2}$$

$$= \frac{3900}{6360}$$

$$\boxed{b_{yx} = 0.61}$$

$$(Y - 700) = 0.61\,(x - 900)$$

$$Y - 700 = 0.61x - 549$$

$$\boxed{Y = 0.61x + 151}$$

c)

x	$(x-\bar{x})$	$(x-\bar{x})^2$	y	$(y-\bar{y})$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
91	1	1	71	1	1	1
97	7	49	75	5	25	35
108	18	324	69	-1	1	-18
121	31	961	97	27	729	351
67	-23	529	70	0	0	0
124	34	1156	91	21	441	714
51	-39	1521	39	-31	961 ~~81~~	1209
73	-17	289	61	-9	81	153
111	21	441	80	10	100	210
57	-33	1089	47	-23	529	759

$$\bar{X} = 90 \quad , \quad \bar{Y} = 70$$

$$\Sigma(x - \bar{x})^2 = 6360$$

$$\Sigma(Y - \bar{Y})^2 = 2868$$

$$\Sigma(X - \bar{X})(Y - \bar{Y}) = 3414$$

line of regression of Y on x

$$(y-\bar{y}) = b_{yx}(x-\bar{x})$$

$$\because b_{yx} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$$

$$= \frac{3414}{6360}$$

$$\boxed{b_{yx} = 0.53}$$

$$(y-70) = 0.53(x-90)$$

$$y-70 = 0.53x - 47.7$$

$$y = 0.53x + 22.3$$

The eq^n of regression for Y on x is

$$\boxed{y = 0.53x + 22.3}$$

[Q-2]

4) $n_1 = 400$

$\mu = 50$

$\bar{x} = 0.20.$

$$|z| = \frac{\bar{x} - \mu}{n}$$

$$= \frac{0.20 - 50}{400}$$

$$\boxed{|z| = 0.12}$$

i) $H_0 = \mu$ (Null hypothesis)

$H_1 \neq \mu$ (Alternative hypothesis)

ii) Level of significance at 5%.

$\therefore$ critical value = 1.96.

iii) Decision :

$Z_{cal} > Z_\alpha$ --- ~~A~~ Rejected.

$Z_\alpha > Z_{cal}$ .. Accepted.

③

Here,

As $0.12 < 1.96$

$\therefore$ H_0 is occepted.

$\therefore$ In one day's production of 400 articles, only 50 are of top quality.

B)

~~$\bar{x} = \frac{20}{100} = 0.2$~~

Let students of city A be x

~~city of~~

students of city B be x_2.

$\therefore \bar{x}_1 = \frac{20}{100} = 0.2$

$\bar{x}_2 = \frac{18.5}{100} = 0.185$

$n_1 = 900$

$n_2 = 1600$

$$p = \frac{0.2\times900 + 0.185\times1600}{900+1600}$$

$$= \frac{180+296}{2500}$$

$$= 0.19$$

$Q = 1-p$

$Q = 1-0.19$

$Q = 0.81$

$$|z| = \frac{0.5-0.19}{\cancel{\sqrt{\frac{0.19\times0.81}{2500}}}}$$

$|z| = 39.74 \approx 40.$

1) Null Hypothesis : $H_0 = \mu$

Alternate hypothesis : $H_0 \neq \mu$

11) Level of significance : 5%.

Critical value = 1.96

$Z_{cal} > Z_\alpha$ ∴

Ho is rejected.