

## Assignment 6- Parameter Estimation

$$1 \rightarrow f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$X_1, X_2, \dots, X_n$  } Sample of size  $n$

$$L(X_1, X_2, X_3, \dots, X_n) = f(x_1) \cdot f(x_2) \cdot f(x_3) \dots f(x_n) \\ = \left( \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}} \right) \cdot \left( \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}} \right) \dots$$

Taking  $\ln$  both sides

$$\ln(L) = -\frac{n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^n \left( -\frac{(x_i - \mu)^2}{2\sigma^2} \right) \quad \text{--- (1)}$$

Taking derivative w.r.t  $\mu$  (of eq (1))

$$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^n \frac{-(2(x_i - \mu))}{2\sigma^2} = 0$$

$$= \sum_{i=1}^n (x_i - \mu) = 0$$

$$= n\bar{x} - n\mu = 0$$

$$\bar{x} = \mu$$

Hence  $\theta_1 = \bar{x}$  i.e. sample mean

Taking derivative w.r.t  $\sigma^2$  (of eq (1))

$$\frac{\partial \ln(L)}{\partial \sigma^2} = -\frac{n}{2\sigma^2} + \sum_{i=1}^n \frac{-(x_i - \mu)^2}{2(\sigma^2)^2} = 0$$

$$-n + \sum_{i=1}^n \frac{-(x_i - \mu)^2}{\sigma^2} = 0$$

$$n = \sum_{i=1}^n \frac{(x_i - \mu)^2}{\sigma^2}$$

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2$$

$$\text{Hence } \theta_2 = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2$$



2 → Binomial distribution  $\rightarrow {}^nC_{x_i} \theta^{x_i} (1-\theta)^{n-x_i}$

$$L = \prod_{i=1}^n {}^nC_{x_i} \theta^{x_i} (1-\theta)^{n-x_i}$$

log on both sides

$$\log L = \sum_{i=1}^n (\log ({}^nC_{x_i}) + \log \theta^{x_i} + \log (1-\theta)^{n-x_i})$$

$$\log L = \sum_{i=1}^n \log ({}^nC_{x_i}) + \log \theta \sum_{i=1}^n x_i + \log (1-\theta) \sum_{i=1}^n (n-x_i)$$

Differentiate wrt  $\theta$

$$\frac{d \log(L)}{d\theta} = 0$$

$$\frac{1}{\theta} \sum x_i - \frac{1}{1-\theta} \sum (n-x_i) = 0$$

$$\frac{1}{\theta} \sum x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta} \sum x_i = 0$$

$$\frac{1}{\theta(1-\theta)} \sum x_i = \frac{n^2}{1-\theta}$$

$$\frac{\sum x_i}{\theta} = n^2$$

$$\boxed{\theta = \frac{\sum x_i}{n^2}} \rightarrow \underline{\underline{\theta}}$$