24-677 Modern Control Theory
Project 1

Exercise 1:

1. Linearized State Equations for Arbitrary Point

A =

$$\begin{pmatrix} 0 & \dot{\psi} & 0 & \dot{y} & 0 & 0 \\ \frac{2\,\mathrm{Ca}\left(\frac{\dot{y}-\mathrm{lr}\,\dot{\psi}}{\dot{x}^2}+\frac{\cos(\delta)\,(\dot{y}+\mathrm{lf}\,\dot{\psi})}{\dot{x}^2}\right)}{m}-\dot{\psi} & -\frac{2\,\mathrm{Ca}\,(\cos(\delta)+1)}{m\,\dot{x}} & 0 & \frac{2\,\mathrm{Ca}\left(\frac{\mathrm{lr}}{\dot{x}}-\frac{\mathrm{lf}\,\cos(\delta)}{\dot{x}}\right)}{m}-\dot{x} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{2\,\mathrm{Ca}\,(\dot{\psi}\,\mathrm{lf}^2+\dot{y}\,\mathrm{lf}+\dot{\psi}\,\mathrm{lr}^2-\dot{y}\,\mathrm{lr})}{\mathrm{Iz}\,\dot{x}^2} & -\frac{2\,\mathrm{Ca}\,(\mathrm{lf}-\mathrm{lr})}{\mathrm{Iz}\,\dot{x}} & 0 & -\frac{2\,\mathrm{Ca}\,(\mathrm{lf}^2+\mathrm{lr}^2)}{\mathrm{Iz}\,\dot{x}} & 0 & 0 \\ \cos(\psi) & -\sin(\psi) & -\dot{y}\cos(\psi)-\dot{x}\sin(\psi) & 0 & 0 & 0 \\ \sin(\psi) & \cos(\psi) & \dot{x}\cos(\psi)-\dot{y}\sin(\psi) & 0 & 0 & 0 \end{pmatrix}$$

B =

$$\begin{pmatrix} 0 & \frac{1}{m} \\ \frac{2\,\mathrm{Ca}\,(\cos(\delta)-\delta\sin(\delta))}{m} & 0 \\ 0 & 0 \\ \frac{2\,\mathrm{Ca}\,\mathrm{lf}}{\mathrm{Iz}} & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

With Terms and Values Substituted In

A =

$$\begin{pmatrix} 0 & \dot{\psi} & 0 & \dot{y} & 0 & 0 \\ \frac{40\cos(\delta)\left(\frac{59\,\dot{\psi}}{50}+\dot{y}\right)}{\dot{x}^2}-\frac{40\left(\frac{41\,\dot{\psi}}{50}-\dot{y}\right)}{\dot{x}^2}-\dot{\psi} & -\frac{40\,(\cos(\delta)+1)}{\dot{x}} & 0 & -\frac{5\,\dot{x}^2+236\cos(\delta)-164}{5\,\dot{x}} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{64\,(2581\,\dot{\psi}+450\,\dot{y})}{6009\,\dot{x}^2} & -\frac{9600}{2003\,\dot{x}} & 0 & -\frac{165184}{6009\,\dot{x}} & 0 & 0 \\ \cos(\psi) & -\sin(\psi) & -\dot{y}\cos(\psi)-\dot{x}\sin(\psi) & 0 & 0 & 0 \\ \sin(\psi) & \cos(\psi) & \dot{x}\cos(\psi)-\dot{y}\sin(\psi) & 0 & 0 & 0 \end{pmatrix}$$

B =

$$\begin{pmatrix} 0 & \frac{1}{1000} \\ 40\cos(\delta)-40\,\delta\sin(\delta) & 0 \\ 0 & 0 \\ \frac{94400}{6009} & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

MATLAB Code

```
syms F_in x y delta psi x_dot y_dot psi_dot X Y s m Ca f g lf lr Iz delT

m = 1000;
f = 0.025;
g = 9.81;
Ca = 20000;
lf = 1.18;
lr = 0.82;
Iz = 3004.5;
delT = 0.032;

x_ddot = psi_dot * y_dot + 1 / m * (F_in - f*m*g)
y_ddot = (-psi_dot) * x_dot + ((2 * Ca) / m) * (cos(delta)*(delta - (y_dot + lf * psi_dot)/x_dot) - (y_dot - lr * psi_dot)/ x_dot)


psi_dot = psi_dot;
psi_ddot = (2 * lf * Ca / Iz) * (delta -(y_dot + lf * psi_dot)/ x_dot) - (2*lr*Ca)/Iz*(- (y_dot - lr * psi_dot) /x_dot)

X_dot = x_dot * cos(psi) - y_dot * sin(psi)
Y_dot = x_dot * sin(psi) + y_dot * cos(psi)

state_dot = [x_ddot; y_ddot; psi_dot; psi_ddot; X_dot; Y_dot];
state = [x_dot;y_dot;psi;psi_dot;X;Y]

A = simplify(jacobian(state_dot, state))
u = [1 / m * F_in; 2 * Ca / m * (cos(delta)*(delta) - (y_dot + lf * psi_dot) / x_dot); 0; (2 * lf * Ca/ Iz) * (delta); 0; 0];

B = jacobian(u,[delta;F_in])
C = [1 0 0 0 0 0;
     0 0 1 0 0 0];
D = [0 0;0 0];
```

2. Linearized Model about Operating Point

Operating Point Calculation and Updated Matrixes

A_op =

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{40}{3} & 0 & -\frac{42}{5} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -\frac{1600}{2003} & 0 & -\frac{82592}{18027} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 6 & 0 & 0 & 0 \end{pmatrix}$$

F_in_vector =

$$\begin{pmatrix} \frac{981}{4} \\ \frac{981}{4} \end{pmatrix}$$

B_op =

$$\begin{pmatrix} 0 & \frac{1}{1000} \\ 40 & 0 \\ 0 & 0 \\ \frac{94400}{6009} & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

delta_vector =

$$\begin{pmatrix} 0 \\ \frac{\pi}{2} \end{pmatrix}$$

MATLAB Code

```
x_dot = 6;
y_dot = 0;
psi_dot = y_dot;
psi = y_dot;

syms F_in delta
eqn = [F_in/1000 + psi_dot*y_dot - 981/4000 == 0, (40*((41*psi_dot)/50 - y_dot))/x_dot - psi_dot*x_dot + 40*cos(delta)*(delta - ((59*psi_dot)/50 + y_dot)/x_dot) == 0];
S = solve(eqn);
F_in_vector = S.F_in
delta_vector = S.delta

delta = 0;

%Plug into A and B Matrix
A_op = simplify(subs(A))
B_op = simplify(subs(B))
C_op = simplify(subs(C));
D_op = simplify(subs(D));
```

3. Transfer Functions

```
G_s =
```

$$\begin{pmatrix} 0 & \frac{1}{1000\,s} \\ \frac{4800\,(177\,s+2000)}{s\,(54081\,s^2+968856\,s+2940800)} & 0 \end{pmatrix}$$

```
z_one =

  0×1 empty double column vector
p_one = 0
z_two = -11.2994
p_two = 3×1
          0
   -14.0426
    -3.8724
```

MATLAB Code

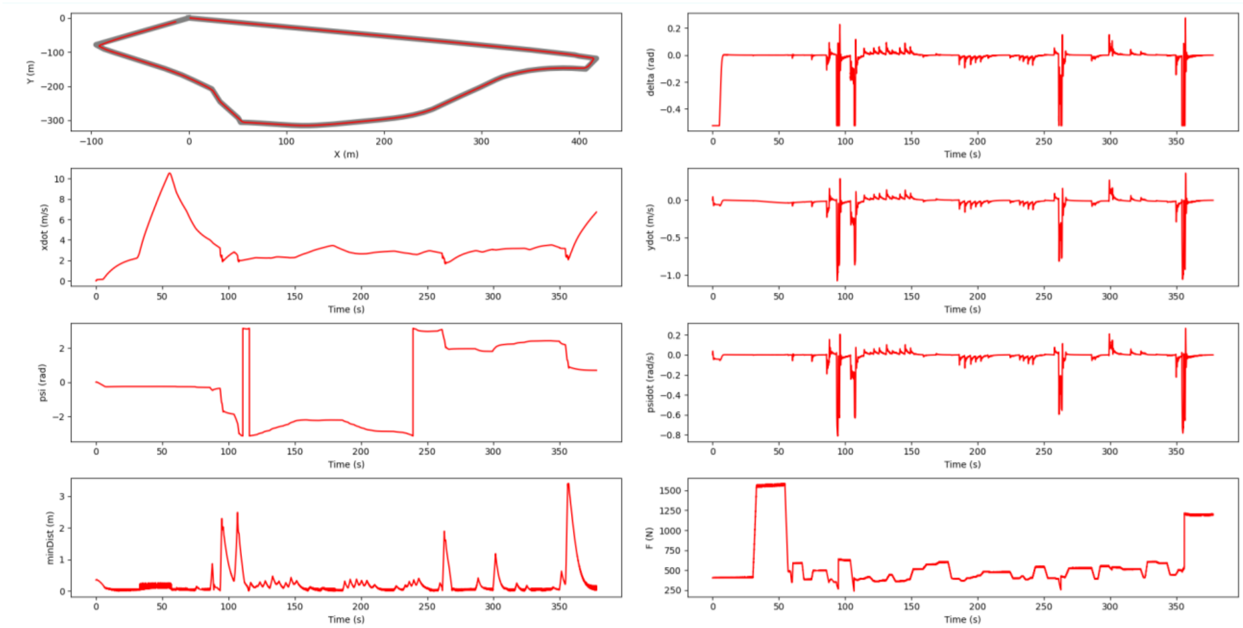
```
I = eye(length(A_op));
G_s = simplify(C_op *inv((s*I-A_op))* B_op + D_op)

%TF 1
num_one = [1];
denom_one = [1000 0];
[z_one,p_one] = tf2zp(num_one,denom_one)

%TF 2
num_two = [849600 9600000];
denom_two = [54081 968856 2940800 0];
[z_two,p_two] = tf2zp(num_two,denom_two)
```

TF 1: Zeros: N/A Poles: s = 0

TF 2: Zeros: -11.04 Poles: s = 0, -14.04, -3.87

Exercise 2:

Program Graphs




```
Evaluating...
Score for completing the loop: 30.0/30.0
Score for average distance: 30.0/30.0
Score for maximum distance: 30.0/30.0
Your time is  377.6
Your total score is : 100.0/100.0
total steps:  377600
maxMinDist:  3.4004615817176393
avgMinDist:  0.2506376103305572
```