1: 30 points

Uncertainty modeling

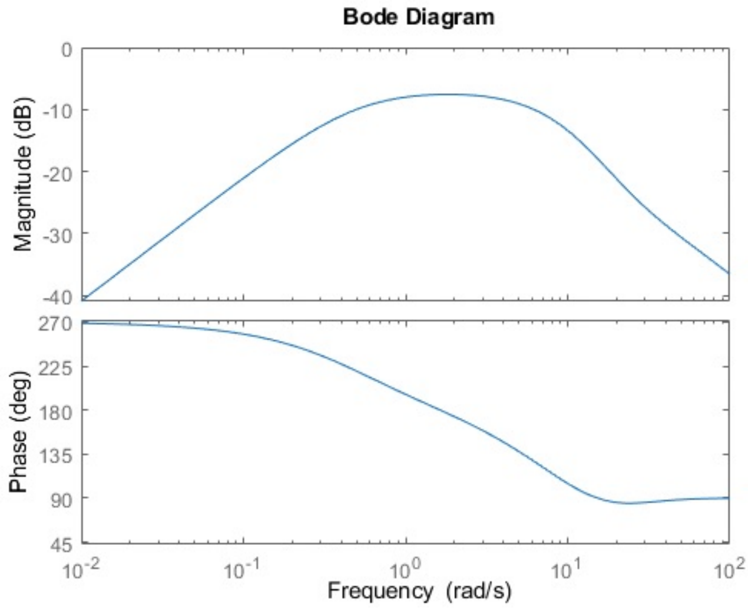
(a) Consider a "true" plant $G(s) = \frac{3e^{-0.1s}}{(2s+1)(0.1s+1)^2}$. Derive and plot the additive uncertainty weight when the nominal model is $G(s) = \frac{3}{2s+1}$.

$$G = G_N + W_a(s)\Delta$$

$$W_a(s)\Delta = G - G_N$$

$$W_a = \frac{3\left(e^{-0.1s} - (0.1s+1)^2\right)}{(2s+1)(0.1s+1)^2}$$

```
%Problem 1a
s = tf('s');
G_true = 3*exp(-0.1*s)/(2*s+1)/(0.1*s+1)^2;
G_n = 3/(2*s+1);
W_a = G_true - G_n
W_a_realizable = tf(pade(W_a,1))
bode(W_a_realizable)
```



(b) Assume we have derived the following detailed model:

$$G_{\text{actual}}(s) = \frac{10(-0.5s+1)}{(6s+1)(0.2s+1)(20s+1)}$$

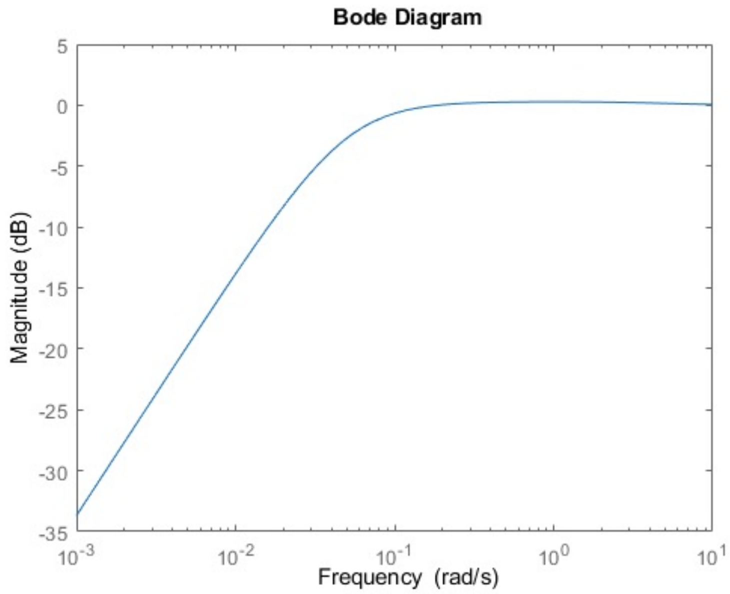
and we want to use the simplified nominal model $G(s) = \frac{10}{6s+1}$ with multiplicative uncertainty. Find an appropriate weighting function $w_I(s)$.

$$G = G_N(1 + w_i \Delta)$$

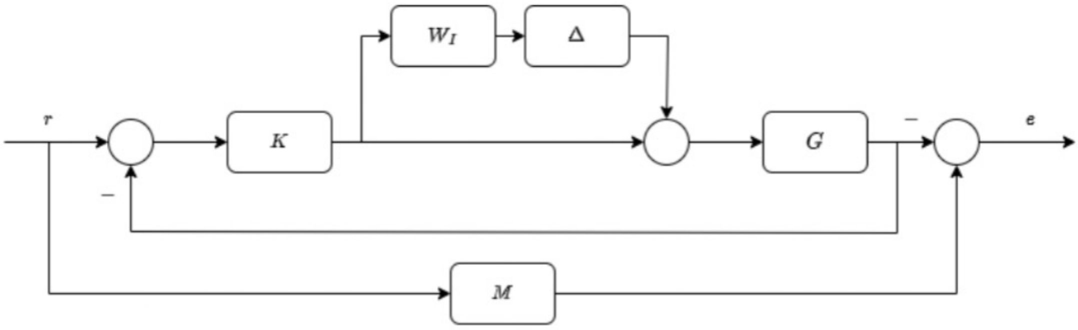
$$w_i \Delta = \frac{G}{G_N} - 1$$

$$w_i \Delta = \frac{-0.5s+1}{(0.2s+1)(20s+1)} - 1$$

```
%Problem 1b
G_actual = 10*(-0.5*s+1)/(6*s+1)/(0.2*s+1)/(20*s+1);
G_n = 10/(6*s+1);
W_i = (G_actual/G_n) - 1
bodemag(W_i)
```



(c) Now using the results from Part b you will design a model matching controller for uncertain plant based on the block diagram below. In this case we would like to match the critically damped model $M = \frac{1}{(s+1)^2}$. Setup and solve the H_∞ optimal control problem and plot the step response of the system G_{actual} against the model M. Did you achieve RS?



$$\begin{bmatrix} y_\Delta \\ z \\ v \end{bmatrix} = P \begin{bmatrix} u_\Delta \\ w \\ u \end{bmatrix}$$

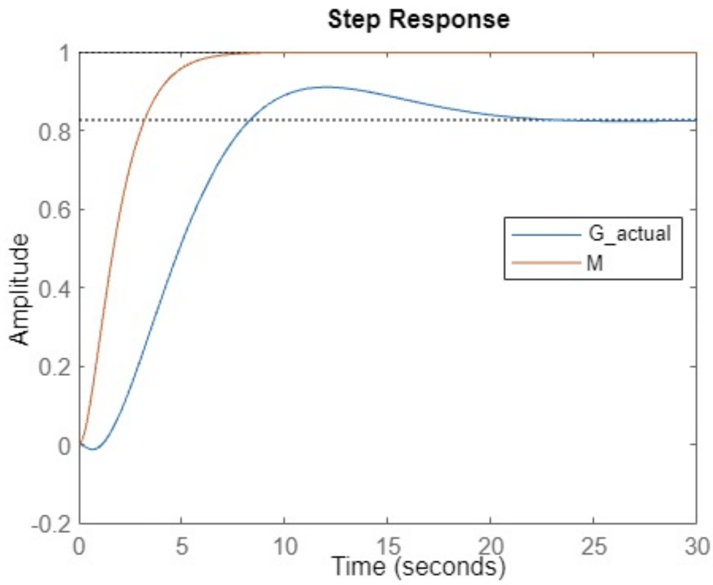
$v = w - G(u + u_\Delta)$

$z = Mw - G(u + u_\Delta)$

$y_\Delta = u w_i$

$$\begin{bmatrix} y_\Delta \\ z \\ v \end{bmatrix} = \begin{bmatrix} 0 & 0 & w_i \\ -G & M & -G \\ -G & 1 & -G \end{bmatrix} \begin{bmatrix} u_\Delta \\ w \\ u \end{bmatrix}$$

```
%Problem 1c
M = 1/(s+1)^2;

P = [0 0 W_i; -G_actual M -G_actual; -G_actual 1 -G_actual];
[K,CL,GAM] = hinfsyn(P,1,1);
G_CL = feedback(G_actual*K,1);

step(G_CL,M)
legend('G_actual','M',location='best')
[stabmarg,~] = robstab(lft(P,K));
mu_RS = 1/stabmarg.LowerBound
```




```
mu_RS = 0
```

As mu_RS is <1, we can see that this system is robustly stable.

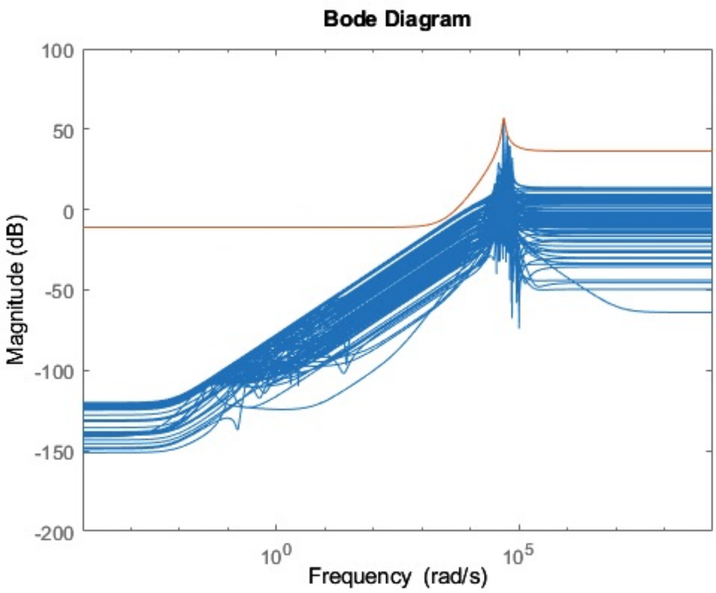
2: 30 points

Disk Drive Control Application

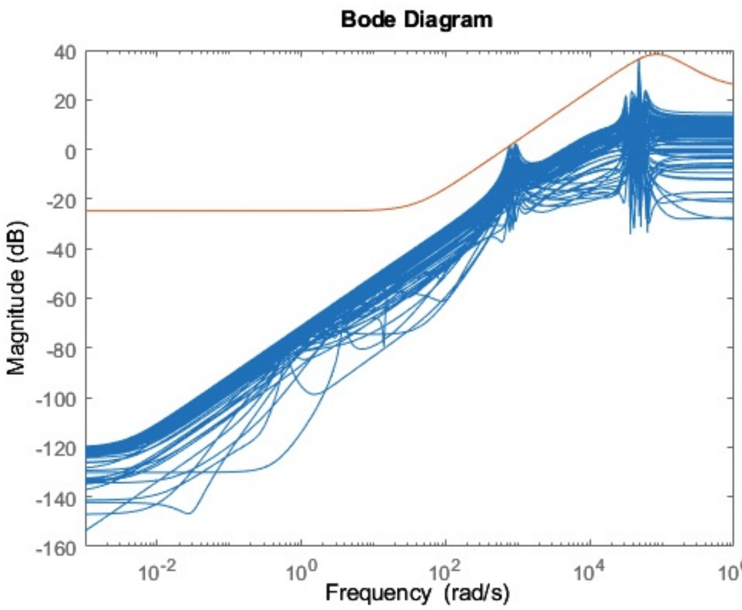
The file HDDModel_DS_Uncertain.m contains a dual-stage HDD model that includes uncertainty from various sources.

(a) The file contains 2 uncertain models - VCM and PZT. Use Matlab to fit a 2nd order multiplicative uncertainty weight that best approximates the uncertainty for each model. Report the final weight for each, and plot $\frac{G_P - G}{G}$ for various perturbed plants G_p vs. the uncertainty weight for each plant.

```
%Problem 2a
HDDModel_DS_Uncertain;

PZT_samples = usample(PZT,100);
[PZT_p,PZT_info] = ucover(PZT_samples,PZT.NominalValue,2);
W_pzt = tf(PZT_info.W1)
bodemag((PZT_samples-PZT.NominalValue)/PZT.NominalValue,W_pzt);

VCM_samples = usample(VCM,100);
[VCM_p,VCM_info] = ucover(VCM_samples,VCM.NominalValue,2);
W_vcm = tf(VCM_info.W1)
bodemag((VCM_samples-VCM.NominalValue)/VCM.NominalValue,W_vcm);
```



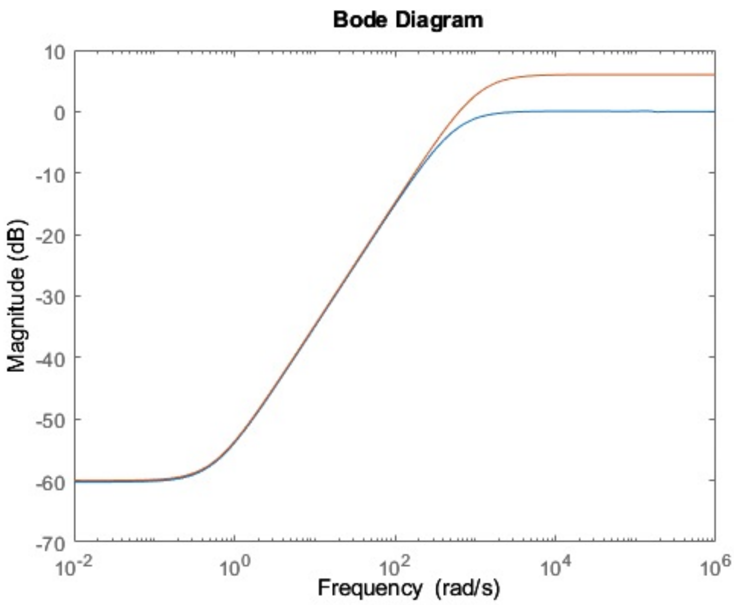

```
W_pzt =

  61.03 s^2 + 3.717e05 s + 5.883e08
  ---------------------------------
       s^2 + 4192 s + 2.222e09

Continuous-time transfer function.
```




```
W_vcm =

  17.48 s^2 + 1.068e07 s + 4.009e08
  ---------------------------------
    s^2 + 1.301e05 s + 6.962e09

Continuous-time transfer function.
```

(b) Perform single stage robust controller design for the VCM plant using *mixsyn*. Maximize the crossover frequency such that the low frequency disturbances are rejected by a factor of 1000, the sensitivity peak is below 2, and $\gamma < 1$. A first order performance weight is fine. Compute $\| \begin{bmatrix} W_P S \\ W_T T \end{bmatrix} \|_\infty$ for your final design and plot the Bode magnitude plot of the uncertain sensitivity function vs. the performance weight.

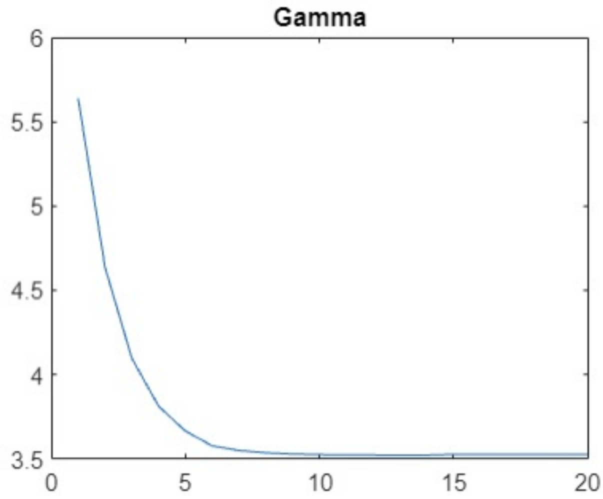
```
%Problem 2b
%Used class exxample for robust control
wh = 1500;
wl = 0;
w_try = wh;
w_new = 1/2*(wh+wl);
while(abs(w_new - w_try)>.001)
    w_try = w_new;
    Wp = makeweight(1000, w_try, 1/2);
    [K,CL,GAM] = mixsyn(VCM.NominalValue, Wp, [], W_vcm);
    if GAM<1
    wl = w_try;
    else
    wh = w_try; %
    end
    w_new = 1/2*(wh+wl);
end
S = 1/(1 + VCM.NominalValue*K);
bodemag(S,1/Wp)
```

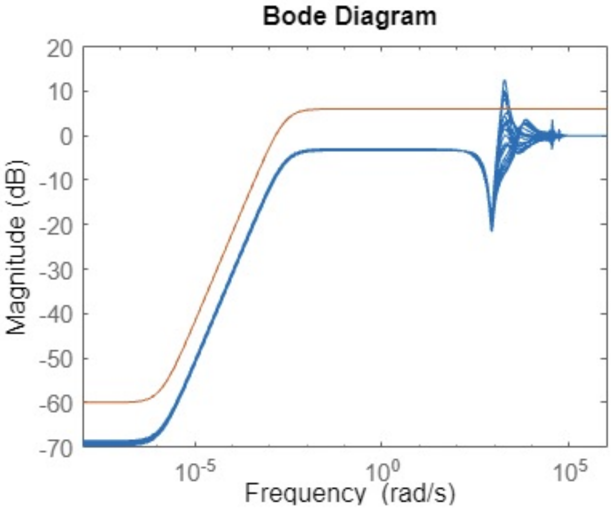


(c) Perform dual stage robust controller design for the dual stage system $G = [VCM \quad PZT]$. Use the same performance criteria from part b, and again maximize the crossover frequency such that $\gamma < 3.5$. For each step of your iteration, capture γ. Plot the value of γ vs. iteration count and plot the Bode magnitude plot of the uncertain sensitivity function vs. the performance weight for the final design. Does your final design satisfy robust performance?

The final design fails to satisfy robust performance as GAM > 1

```
%Problem 2c
G_nom = [VCM.NominalValue PZT.NominalValue];
G_mult = [VCM_p PZT_p];
G = [VCM PZT];
G_v = VCM.NominalValue;
G_p = PZT.NominalValue;

wh = 1500;
wl = 0;
w_try = wh;
w_new = 1/2*(wh+wl);
gam_arr = [];
while(abs(w_new-w_try)>.001)
    systemnames = 'G_v G_p W_vcm W_pzt Wp';
    inputvar = '[ud1;ud2;d;u1;u2]';
    outputvar = '[W_vcm;W_pzt;Wp;-G_v-G_p-d]';
    input_to_G_v = '[u1+ud1]';
    input_to_G_p = '[u2+ud2]';
    input_to_W_vcm = '[u1]';
    input_to_W_pzt = '[u2]';
    input_to_Wp = '[G_v + G_p + d]';
    w_try = w_new;
    Wp = makeweight(1000,w_try,1/2);
    P = sysic;
    [K,CL,GAM] = hinfsyn(P,1,2);
    gam_arr = [gam_arr GAM];
    if GAM<3.5
    wl = w_try;
    else
    wh = w_try;
    end
    w_new = 1/2*(wh+wl);
end
plot(1:length(gam_arr),gam_arr)
title("Gamma")
bodemag(1/(1+G_mult*K),1/Wp)
```





3: 20 points

Aircraft Control Application

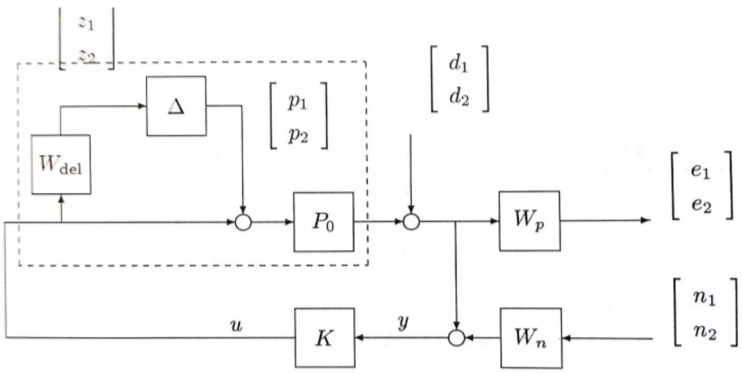
The nominal plant model for a highly maneuverable aircraft is given by

$$A = \begin{bmatrix} -0.0226 & -36.6 & -18.9 & -32.1 \\ 0 & -1.9 & 0.983 & 0 \\ 0.0123 & -11.7 & -2.63 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ -0.414 & 0 \\ -77.8 & 22.4 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 57.3 & 0 & 0 \\ 0 & 0 & 0 & 57.3 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

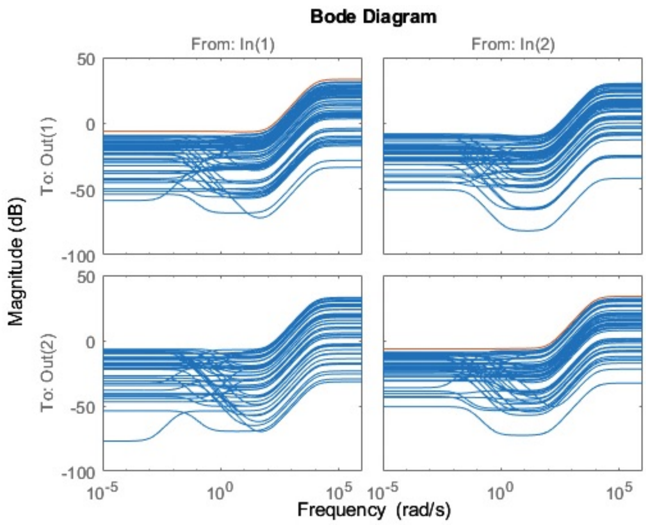
Consider the block diagram below with

$$W_p = \begin{bmatrix} \frac{s+3}{s+0.03} & 0 \\ 0 & \frac{0.5(s+3)}{s+0.03} \end{bmatrix} \quad W_n = \begin{bmatrix} \frac{2(s+1.28)}{s+320} & 0 \\ 0 & \frac{2(s+1.28)}{s+320} \end{bmatrix}$$



(a) The file *responses.mat* gives a vector of responses for the system. Fit a multiplicative uncertainty weight W_{del} to the response. Create a Bode magnitude plot that shows the quality of your fit.

```
%Problem 3a
clear;clc
load("responses.mat");
s = tf('s');
A = [-0.0226 -36.6 -18.9 -32.1;0 -1.9 0.983 0;0.0123 -11.7 -2.63 0;0 0 1 0];
B = [0 0;-0.414 0;-77.8 22.4;0 0];
C = [0 57.3 0 0;0 0 0 57.3];
D = [0 0;0 0];

W_p = [(s+3)/(s+0.03) 0; 0 0.5*(s+3)/(s+0.03)];
W_n = 2*(s+1.28)/(s+320)*eye(2);
G = ss(A,B,C,D);

[G_uncertain,G_info] = ucover(Gp_samples, G, [2,2]);
W_del = G_info.W1;
actual = (G_uncertain - G) / G;
bodemag(inv(G)*Gp_samples-eye(2), W_del)
```



(b) Design an H_∞ optimal controller considering the uncertainty. Plot the Bode magnitude of the sensitivity function for 10 samples of the uncertain plant. Do you meet robust performance specs? What about robust stability?

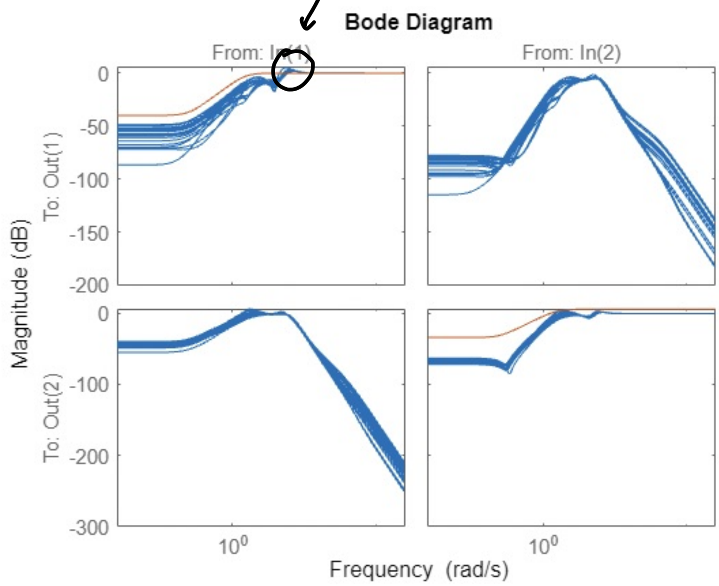
System is robustly stable

System does not have robust performance

```
%Problem 3b
W_del = G_info.W1;
systemnames = 'G W_p W_n W_del';
inputvar = '[p{2};n{2};d{2};u{2}]';
outputvar = '[W_del;W_p;W_n-G-d]';
input_to_G = '[u+p]';
input_to_W_p = '[d+G]';
input_to_W_n = '[n]';
input_to_W_del = '[u]';
cleanupsysic = 'yes';
P = sysic;
[K,CL,GAM] = hinfsyn(P,2,2);
GAM

S = eye(2)-feedback(G_uncertain*K,eye(2));
bodemag(S,inv(W_p))
[stabmarg,~,~,info] = robuststab(S)
```

```
GAM = 2.4100
```

fails RP




```
stabmarg = struct with fields:
               LowerBound: 1.1295
               UpperBound: 1.1295
    DestabilizingFrequency: 73.9725

DESTABUNC = struct with fields:
    Gp_samples_InputMultDelta: [2×2 ss]

REPORT = 6×114 char array
    'System is robustly stable for the modeled uncertainty.
    ' -- It can tolerate up to 113% of the modeled uncertainty.
    ' -- There is a destabilizing perturbation amounting to 113% of the modeled uncertai
    ' -- This perturbation causes an instability at the frequency 74 rad/seconds.
    ' -- Sensitivity with respect to each uncertain element is:
    '      100% for Gp_samples_InputMultDelta. Increasing Gp_samples_InputMultDelta by 2

INFO = struct with fields:
          Sensitivity: [1×1 struct]
            Frequency: [140×1 double]
    BadUncertainValues: [140×1 struct]
            MussvBnds: [1×2 frd]
            MussvInfo: [1×1 struct]
```