Temă

(a) $(p \wedge q) \rightarrow p$

p	q	$p \wedge q$	$p \wedge q \rightarrow p$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	1

→ toate 1 => tautologie

$p \rightarrow (p \vee q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
0	0	0	1
0	1	1	1
1	0	1	1
1	1	1	1

b) $(p \wedge (p \rightarrow q)) \rightarrow q$

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$(p \wedge (p \rightarrow q)) \rightarrow q$
0	0	1	0	1
0	1	1	0	1
1	0	0	0	1
1	1	1	1	1

$(\neg q \wedge (\neg p \rightarrow q)) \rightarrow p$

p	q	$\neg q$	$\neg p$	$\neg p \rightarrow q$	$(\neg q \wedge (\neg p \rightarrow q))$	final
0	0	1	1	0	0	1
0	1	0	1	1	0	1
1	0	1	0	1	1	1
1	1	0	0	1	0	1

1

$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg q$

p	q	$p \rightarrow q$	$\neg q$	$\neg q \wedge (p \rightarrow q)$	$(\neg q \wedge (p \rightarrow q) \rightarrow \neg q$
0	0	1	1	1	1
0	1	1	0	0	1
1	0	0	1	0	1
1	1	1	0	0	1

1

c) $((p \rightarrow q) \wedge (\neg p \rightarrow q)) \rightarrow q$

p	q	$p \rightarrow q$	$\neg p \rightarrow q$	$(p \rightarrow q) \wedge (\neg p \rightarrow q)$	final
0	0	1	0	0	1
0	1	1	1	1	1
1	0	0	1	0	1
1	1	1	1	1	1

d) $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$p \rightarrow q \wedge (q \rightarrow r)$	$p \rightarrow r$	final
1	1	1	1	1	1	1	1
1	1	0	1	0	0	0	1
1	0	1	0	1	0	1	1
1	0	0	0	1	0	0	1
0	1	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	0	1	1	1	1	1	1
0	0	0	1	1	1	1	1

1

e) $((p \to q) \wedge (p \to r)) \to (p \to (q \wedge r))$

p	q	r	$p \to q$	$p \to r$	$(p \to q) \wedge (p \to r)$	$q \wedge r$	$p \to (q \wedge r)$	final
0	0	0	1	1	1	0	1	1
0	0	1	1	1	1	0	1	1
0	1	0	1	1	1	0	1	1
0	1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0	1
1	0	1	0	1	0	0	0	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

$((p \to q) \vee (p \to r)) \to (p \to (q \vee r))$

p	q	r	$p \to q$	$p \to r$	$(p \to q) \vee (p \to r)$	$q \vee r$	$p \to (q \vee r)$
0	0	0	1	1	1	0	1
0	0	1	1	1	1	1	1
0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	1	1	1	1
1	1	0	1	0	1	1	1
1	1	1	1	1	1	1	1

f) $((p \to r) \wedge (q \to r)) \to ((p \vee q) \to r)$

p	q	r	$p \to r$	$q \to r$	$(p \to r) \wedge (q \to r)$	$p \vee q$	$(p \vee q) \to r$	final
0	0	0	1	1	1	0	1	1
0	0	1	1	1	1	0	1	1
0	1	0	1	0	0	1	0	1
0	1	1	1	1	1	1	1	1
1	0	0	0	1	0	1	0	1
1	0	1	1	1	1	1	1	1
1	1	0	0	0	0	1	0	1
1	1	1	1	1	1	1	1	1

$((p \to r) \vee (q \to r)) \to ((p \wedge q) \to r)$

p	q	r	$p \to r$	$q \to r$	$(p\to r) \vee (q \to r)$	$p \wedge q$	$(p\wedge q) \to r$	final
0	0	0	1	1	1	0	1	1
0	0	1	1	1	1	0	1	1
0	1	0	1	0	1	0	1	1
0	1	1	1	1	1	0	1	1
1	0	0	0	1	1	0	1	1
1	0	1	1	1	1	0	1	1
1	1	0	0	0	0	1	0	1
1	1	1	1	1	1	1	1	1

P9 a) Să demonstrăm că $(P(M), \Delta)$ - grup comutativ

Fie $A, B \in P(M)$

G1) $A \Delta B = B \Delta A$ (proprietățile scăderii simetrice)

$A \Delta B = (A \setminus B) \cup (B \setminus A) = (B \setminus A) \cup (A \setminus B) = B \Delta A$.

G2) $A \Delta (B \Delta C) = (A \Delta B) \Delta C$ { Din proprietățile scăderii simetrice

G3) $A \Delta E = E \Delta A = A$

Dar știm că $A \Delta \emptyset = \emptyset \Delta A = A$ $\Big| \Rightarrow E = A$, unde E este elementul neutru al operației "Δ".

G4) Elemente simetrizabile

Presupunem A' simetricul lui A față de operația de "Δ". Să dem că $A' \in P(M)$

$A \Delta A' = A' \Delta A = E \Leftrightarrow A \Delta A' = A' \Delta A = \emptyset$

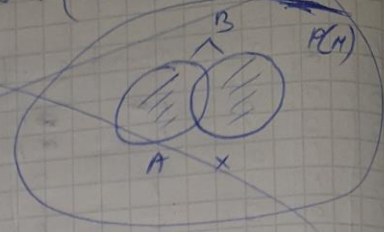
Știm că $A \Delta A = A \Delta A = \emptyset$ $\Big| \Rightarrow A' = A \in P(M)$ } Din ipoteză

$\Rightarrow$ G4 - adev.

Pentru că G1, G2, G3 și G4 sunt adevărate

$\Rightarrow$ $(P(M), \Delta)$ - grup comutativ

~~b) $A \Delta X = B \Rightarrow X = B \setminus A \cup (\neg B \setminus (P(M) \setminus A \setminus X))$~~



$A \Delta X = B \Leftrightarrow (A \Delta A) \Delta X = A \Delta B \Leftrightarrow \boxed{X = A \Delta B}$

Seminar 2.

P7 $f, g : A \to A$

$f \circ g = g \circ f$ - ipoteză

$a \in A$, unic punct fix al f-ției $f \Rightarrow f(a) = a$.

$((f \circ g)(a) = (g \circ f)(a) \Leftrightarrow$

$f(g(a)) = g(f(a)) \Leftrightarrow$

$f(g(a)) = g(a)$

I Pp $g(a) = a \Rightarrow a$ pct fix al funcției g.

$f(g(a)) = g(a) \Leftrightarrow f(a) = a \Leftrightarrow a = a$ evident adev

~~$\Rightarrow$ a pct fix ... satisface condițiile ipotezei.~~

$\Rightarrow$ relația este satisfăcută pentru a pct fix al funcției g.

II Pp $g(a) = c$, $c \neq a$.

$f(g(a)) = g(a) \Rightarrow f(c) = c \to$ fals

pentru că a este unic punct fix al f-ției f.

$\Rightarrow g(a) = a$ sol unică, ~~a ec $f \circ g = g \circ f$~~

$\Rightarrow a$ pct fix al funcției g.

~~P2~~ P16 $\alpha \in S_5$

Varianta 1. $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 5 & 2 & 4 \end{pmatrix}$

$$\alpha^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 4 & 1 & 2 \end{pmatrix}$$

$$\alpha^3 = \alpha^2 \cdot \alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 4 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 5 & 2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 2 & 3 & 1 \end{pmatrix}$$

$$\alpha^4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 5 & 2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 1 & 5 & 3 \end{pmatrix}$$

$$\alpha^5 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 1 & 5 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 5 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} = id$$

$$\Rightarrow \boxed{k = 5}$$

Varianta 2 $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 5 & 2 & 4 \end{pmatrix}$

$$\alpha = (1, 3, 5, 4, 2)$$

$$\alpha^2 = (1,3,5,4,2)(1,3,5,4,2) = (1,5,2,3,4)$$

$$\alpha^3 = (1,5,2,3,4)(1,3,5,4,2) = (1,4,3,2,5)$$

$$\alpha^4 = (1,5,2,3,4)(1,5,2,3,4) = (1,2,4,5,3)$$

$$\alpha^5 = (1,2,3,4,5) \Rightarrow \boxed{k=5}$$

(P18) $(1,2,3), (1,2,4), (1,3,4), (2,3,4)$

(P19) C_n^{ℓ}

(P22) $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 6 & 7 & 1 & 5 & 2 & 8 & 3 & 9 \end{pmatrix}$

$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 1 & 7 & 5 & 4 & 2 & 8 & 9 & 3 \end{pmatrix}$

$\alpha = (1,4)(2,6)(3,7,8)(5)(9)$

$\alpha = (1,4)(2,6)(3,8)(3,7)$ ✓

$\beta = (1,6,2)\ (3,7,8,9)\ (4,5)$

$\beta = (1,2)(1,6)\ (3,9)(3,8)(3,7)\ (4,5)$ ✓

(P25) a) $(a,b)(a,b) = (a,b) = id$ ✓

$(a,b)(a,c) = (a,c,b)$ ✓

$(a,b)(c,d) = (a,b)(c,d) \neq (a,d,c)(a,b,c)$ ✗

b)

$(i_1,i_\ell)(i_1,i_{\ell-1}) \cdots (i_1,i_3)(i_1,i_2)$

$= (i_1, i_2, i_3, i_4, i_5, \ldots, i_{\ell-1}, i_\ell)$ – prin inmulțire simplă

c) $(i,j) = (i,i+1)(i+1,i+2) \cdots (j-2,j-1)\ (j-1,j)\ (j-2,j-1) \cdots$

$(i,i+1)$, $\forall\ i<j$

$= (i,j)\ (i+1)(i+2) \cdots (j-1) = (i,j)$

d) $\alpha(i_1, i_2, \dots, i_l)\alpha^{-1} = (\alpha(i_1), \dots$

~~Fie α $(j_1, j_2, \dots, j_l)$???~~

$$\alpha \cdot (i_1, i_2, i_3, \dots, i_l)\alpha^{-1}$$

$$= (\alpha(i_1), \alpha(i_2), \alpha(i_3) \dots \alpha(i_m))$$

încep cu $\alpha(i_1)$. $\alpha^{-1}(\alpha(i_1)) = i_1$; $i_1 \to i_2 \to \alpha(i_2)$

Analog până la $\alpha(i_l)$

~~P26~~

P5 $$\sum_{k=1}^{n} k(k+1)(k+2) = \sum_{k=1}^{n}(k^2+k)(k+2)$$

$$= \sum_{k=1}^{n} k^3 + 2k + 2k^2 + k^2$$

$$= \sum_{k=1}^{n} k^3 + 3k^2 + 2k$$

$$= \sum_{k=1}^{n} k^3 + 3\sum_{k=1}^{n} k^2 + 2\sum_{k=1}^{n} k$$

$$= \frac{n^2(n+1)^2}{4} + \cancel{3} \cdot \frac{[n(n+1)(2n+1)]}{\cancel{6}\,2} + \cancel{2} \cdot \frac{[n(n+1)]}{\cancel{2}}$$

$$= \frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{2} + n(n+1)$$

$$= \frac{n^2(n+1)^2 + 2n(n+1)(2n+1) + 4n(n+1)}{4}$$

$$= \frac{1}{4} \cdot \left[n(n+1)\left[n(n+1) + 4n + 2 + 4\right]\right]$$

$$= \frac{1}{4} \cdot \left[n(n+1)(n^2+5n+6)\right]$$

$$= \frac{n(n+1)(n+2)(n+3)}{4}$$ ✓

(16) $a_1^3 + a_2^3 + \dots + a_m^3 = (a_1 + a_2 + \dots + a_m)^2$ — să dem.

$m = 1: a_1^3 = a_1^2 \Leftrightarrow a_1^2(a_1 - 1) = 0 \Rightarrow$

$\Leftrightarrow a_1^2 = 0 \Rightarrow a_1 = 0 \notin \mathbb{N}^* \rightarrow$ nu convine.

$a_1 - 1 = 0 \Rightarrow \boxed{a_1 = 1}$

$m = 2 \Rightarrow a_1^3 + a_2^3 = (a_1 + a_2)^2 \Leftrightarrow a_1^3 + a_2^3 = a_1^2 + 2a_1a_2 + a_2^2$

$a_1^3 - a_1^2 + a_2^3 - a_2^2 - 2a_1a_2 = 0 \Big/ \Rightarrow \cancel{1} - \cancel{1} + a_2^3 - a_2^2 - 2a_2 = 0 \Leftrightarrow$

$a_1 = 1$

$\Leftrightarrow a_2^3 - a_2^2 - 2a_2 = 0 \Rightarrow a_2(a_2^2 - a_2 - 2) = 0 \;/ : a_2 \neq 0$

$a_2^2 - a_2 - 2 = 0$ $\cancel{\Rightarrow a_2^2 - a_2 - 1}$

$\cancel{a_2^2(a_2 - 1) - 1}$

$(a_2 - 2)(a_2 + 1) = 0 \Rightarrow \boxed{a_2 = 2}$

$a_2 + 1 = 0 \Rightarrow a_2 = -1 \notin \mathbb{N}^*$

$m = 3: a_1^3 + a_2^3 + a_3^3 = (a_1 + a_2 + a_3)^2$

$1 + 2^3 + a_3^3 = (1 + 2 + a_3)^2$

$\cancel{9 + a_3^2 = (3 + a_3)^2}$

$\cancel{2 + a_3^3 = 4 + 4a_3 + a_3^2}$

$\cancel{a_3^3 - 4a_3 - a_3^2 = 2}$

$9 + a_3^3 = (3 + a_3)^2$

$\cancel{9} + a_3^3 = \cancel{9} + 6a_3 + a_3^2$

$a_3^3 - 6a_3 - a_3^2 = 0$

$a_3^2 - a_3 - 6 = 0$

$(a_3 - 3)(a_3 + 2) \Rightarrow \boxed{a_3 = 3}$

$P(n): a_n = n, \forall n \in \mathbb{N}^*$

(V) $P(1)$ - adev

$P(1), P(2), \dots P(n) \Rightarrow P(n+1)$

$P(n+1): a_{n+1} = n+1$

$a_1^3 + a_2^3 + \dots + a_n^3 + a_{n+1}^3 = (a_1 + a_2 + \dots + a_n + a_{n+1})^2$

$1^3 + 2^3 + \dots + n^3 + a_{n+1}^3 = (1 + 2 + \dots + n + a_{n+1})^2$

$$\left[\frac{n(n+1)}{2}\right]^2 + a_{n+1} = \left[\frac{n(n+1)}{2} + a_{n+1}\right]^2$$

$$\cancel{\left[\frac{n(n+1)}{2}\right]^2} + a_{n+1}^3 = \cancel{\left[\frac{n(n+1)}{2}\right]^2} + \cancel{2} \cdot \frac{n(n+1)}{\cancel{2}} \cdot a_{n+1} + a_{n+1}^2$$

$a_{n+1}^3 = n(n+1) a_{n+1} + a_{n+1}^2 \quad / : a_{n+1} \neq 0$

$a_{n+1}^2 - n(n+1) - a_{n+1} = 0$

$a_{n+1}^2 - a_{n+1} - n(n+1) = 0$

$\cancel{(a_{n+1})(n+1)}$

$[a_{n+1} - (n+1)][a_{n+1} + n] = 0 \Rightarrow \boxed{a_{n+1} = n+1}$ (V)

$\Rightarrow P(n+1)$ (A)

(P11) $n = 1$: $\begin{vmatrix} 0 & 1 \\ 1 & a_1 \end{vmatrix} = -1 = -a_1 \cdot \frac{1}{a_1}$ (V)

$n = 2$: $\begin{vmatrix} 0 & 1 & 1 \\ 1 & a_1 & 0 \\ 1 & 0 & a_2 \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 1 \\ a_1 & 0 \end{vmatrix} + a_2 \cdot \begin{vmatrix} 0 & 1 \\ 1 & a_1 \end{vmatrix}$

$\Rightarrow -1$

$= -a_1 - a_2 = -(a_1 + a_2)$

$= -a_1 a_2 \left(\frac{a_1}{a_1 a_2} + \frac{a_2}{a_1 a_2}\right)$

$= -a_1 a_2 \left(\frac{1}{a_2} + \frac{1}{a_1}\right) = -a_1 a_2 \cdot \left(\frac{1}{a_1} + \frac{1}{a_2}\right)$

P

$n=3$: $\begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & a_1 & 0 & 0 \\ 1 & 0 & a_2 & 0 \\ 1 & 0 & 0 & a_3 \end{vmatrix} = 1 \cdot \begin{vmatrix} 0 & 1 & 1 \\ 1 & a_1 & 0 \\ 1 & 0 & a_2 \end{vmatrix} + a_3 \cdot \underbrace{\begin{vmatrix} 0 & 1 & 1 \\ 1 & a_1 & 0 \\ 1 & 0 & a_2 \end{vmatrix}}_{n=2}$ ①

$\begin{vmatrix} 1 & 1 & 1 \\ a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ \cancel{0} & \cancel{0} & \cancel{a_3} \end{vmatrix} = a_2 \cdot \begin{vmatrix} 1 & 1 \\ a_1 & 0 \end{vmatrix} = -a_1 a_2$

① $= -a_1 a_2 + a_3 \cdot (-a_1 a_2) \cdot \left(\frac{1}{a_1} + \frac{1}{a_2}\right)$

$= -a_1 a_2 - a_1 a_2 a_3 \left(\frac{1}{a_1} + \frac{1}{a_2}\right)$

$= -a_1 a_2 a_3 \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right)$

$P(n) = \begin{vmatrix} 0 & 1 & 1 & \dots & 1 & 1 \\ 1 & a_1 & 0 & \dots & 0 & 0 \\ 1 & 0 & a_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 0 & 0 & \dots & a_{n-1} & 0 \\ 1 & 0 & 0 & \dots & 0 & a_n \end{vmatrix} = -a_1 a_2 \dots a_n \left(\frac{1}{a_1} + \frac{1}{a_2} + \dots \frac{1}{a_n}\right), \forall n \in$ ~~$\forall n \in \mathbb{N}^*$~~

(V) - P(1) adev

$P(1), P(2), \dots P(3) \Rightarrow P(n+1)$

~~(V) - P(1) adev~~

$P(n+1)$: $\begin{vmatrix} 0 & 1 & 1 & \dots & 1 & 1 \\ 1 & a_1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 0 & 0 & \dots & a_n & 0 \\ 1 & 0 & 0 & \dots & 0 & a_{n+1} \end{vmatrix} = 1 \cdot \underbrace{\begin{vmatrix} 1 & 1 & \dots & 1 & 1 \\ a_1 & 0 & \dots & 0 & 0 \\ 0 & a_2 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & a_n & 0 \end{vmatrix}}_{\overset{not}{=} A} + a_{n+1} \underbrace{\begin{vmatrix} 0 & 1 & \dots & 1 & 1 \\ 1 & a_1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 0 & \dots & a_{n-1} & 0 \\ 1 & 0 & \dots & 0 & a_n \end{vmatrix}}_{P(n)}$

$\begin{vmatrix} 1 & 1 & \dots & 1 & 1 \\ a_1 & 0 & \dots & 0 & 0 \\ 0 & a_2 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & a_n & 0 \end{vmatrix} = a_n \cdot \begin{vmatrix} 1 & 1 & \dots & 1 & 1 \\ a_1 & 0 & \dots & 0 & 0 \\ 0 & a_2 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & a_{n-1} & 0 \end{vmatrix} = a \cdot (a_{n-1}) \cdot \begin{vmatrix} 1 & 1 & \dots \\ a_1 & 0 & \dots \\ 0 & a_2 & \dots \\ \vdots & \vdots & \ddots \\ 0 & 0 & \dots \end{vmatrix}$

$= \dots = a_n (a_{n-1}) \cdot (a_n - 2) \cdot \dots \cdot \begin{vmatrix} 1 & 1 \\ a_1 & 0 \end{vmatrix} \Rightarrow$

$A = a_n \cdot (a_{n-1}) \cdot (a_{n-2}) \cdot \dots (-a_1)$

$A = -a_n (a_{n-1})(a_{n-2}) \cdot \dots a_1$

$P(n+1) = -a_n \cdot (a_{n-1}) \cdot (a_{n-2}) \dots \cdot a_1 + (a_{n+1})(-a_1 a_2 \dots a_n)\left(\frac{1}{a_1} + \frac{1}{a_2} + \dots \frac{1}{a_n}\right)$

$= -a_1 a_2 \dots a_n - a_1 a_2 a_3 \dots a_n a_{n+1}\left(\frac{1}{a_1} + \frac{1}{a_2} + \dots \frac{1}{a_n}\right)$

$-a_1 a_2 \dots a_{n+1}\left(\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{a_1 a_2 \dots a_n}{a_1 a_2 \dots a_n a_{n+1}}\right)$

$= -a_1 a_2 \dots a_{n+1}\left(\frac{1}{a_1} + \frac{1}{a_2} + \dots \frac{1}{a_{n+1}}\right)$ - adev ✓

~~P14~~

Seminar 1.

P10 Arătați că $(P(M), \Delta, \cap)$ este inel comutativ

I $(P(M), \Delta)$ - grup comutativ - demonstrat la P9.

II Să dem. că $P(M, \cap)$ este monoid comutativ

$\forall M_1, M_2 \in P(M)$

① $M_1 \cap M_2 = M_2 \cap M_1 \rightarrow$ prop "$\cap$" $\rightarrow$ comutativitate

② $M_1 \cap (M_2 \cap M_3) = (M_1 \cap M_2) \cap M_3$ - proprietatea "$\cap$" $\rightarrow$ asociativitate $\Rightarrow$

③ $\exists x \in P(M)$ a.î. $M_1 \cap x = M_1 \Rightarrow x = M_1 / M_1 = \emptyset \in P(M) \Rightarrow \exists$ el. neutru.

$\Leftrightarrow$ monoid comutativ

III $M_1 \cap (M_2 \Delta M_3)$ de comparat cu $(M_1 \cap M_2) \Delta (M_2 \cap M_3)$

$M_1 \cap (M_2 \Delta M_3) = M_1 \cap (M_2 \setminus M_3 \cup M_3 \setminus M_2) = M_1 \cap (M_2 \setminus M_3) \cup M_1 \cap (M_3 \setminus M_2)$

$= (M_1 \cap M_2) \setminus (M_1 \cap M_3) \cup (M_1 \cap M_3) \setminus (M_1 \cap M_2)$

$= (M_1 \cap M_2) \Delta (M_2 \cap M_3)$ ✓ $\Rightarrow \cap$ asociativă în raport cu Δ

I, II, III $\Rightarrow (P(M), \Delta, \cap)$ - inel comutativ)

~~(P18) A; ⊂ B, ∀i ∈ I ⟸~~

(P19) 3 elem: $A=\{a,b,c\}$.

Există 1 niciună relație de echivalență cu 3 clase

- $\{a\}, \{b\}, \{c\}$

- ∃ 3 rel de echiv cu 2 clase
- $\{a\}$ cu $\{b,c\}$, $\{b\}$ cu $\{a,c\}$ și $\{c\}$ cu $\{a,b\}$

∃ 1 rel de echiv cu 1 clasă

$\{1,2,3\}$

Total 5

4 elem: $A=\{a,b,c,d\}$

cu 4 clase: $\{a\}, \{b\}, \{c\}, \{d\}$ – 1

cu 3 clase: $\{a\}$ cu $\{b,c,d\}$; $\{b\}$ cu $\{a,c,d\}$

$\{c\}$ cu $\{a,b,d\}$; d cu $\{a,b,c\}$ – 4

cu 2 clase: $\{a,b\}$ cu $\{c,d\}$; $\{a,c\}$ cu $\{b,d\}$

$\{a,d\}$ cu $\{b,c\}$ – 3

cu 1 clasă $\{a,b,c,d\}$ – 1

Total 9

(P22) $(1,2), (1,3) \dots (1,n)$ – $n-1$

$(2,3), (2,4) \dots (2,n)$ – $n-2$

$(3,4), (3,5) \dots (3,n)$ – $n-3$

$\dots$

$(n-1, n) \rightarrow$ – 1

$\Rightarrow$ nu Total $= (n-1)(n-2)(n-3)\cdot\ldots\cdot 1 = (n-1)!$

(P22) $\underbrace{a < b < \dots < n}$ => nr de posibilități = numărul

de permutări posibile = $n!$

(P23) a, b, c – 3 elem

luate câte 3: ord strica (+6)

luate câte 2:

$a<b$ $b<a$ $c<a$

$a<c$ $b<c$ $c<b$ } (+3)

luate câte 1:

$a<b$	$b<c$	$a<c$	$b<a$	$c<b$	$c<a$
c	a	b	c	a	b

} (+6)

$\boxed{\text{Total } 9+6=15}$

~~P9 b) $A \Delta X = B \iff (A \Delta A) \Delta X = A \Delta B$~~

~~$\iff \boxed{X = A \Delta B}$~~

(P12) a) $A \subseteq B \iff A \cup B = B$

$A_i \subseteq B$, $\forall i \in I \iff A_i \cup B = B$, $\forall i \in I$ =>

$\bigcup_{i \in I} (A_i \cup B) = B \iff \left(\bigcup_{i \in I} A_i\right) \cup B = B \iff \bigcup_{i \in I} A_i \subseteq B$

b) $A \cap B \subseteq A$, $A \cap B \subseteq B$, $\forall A, B \subseteq T$

$B \subseteq \bigcap_{i \in I} A_i \subseteq A_i$, $\forall i \in I$ => $B \subseteq A_i$, $\forall i \in I$

$B \subseteq A_i$, $(\forall)\ i \in I$

fie $b \in B$ => $b \in A_i$, $\forall i \in I$ => $b \in \bigcap_{i \in I} A_i$

=> $B \subseteq \bigcap_{i \in I} A_i$

c) fie $a \in \bigcup_{i \in I} A_i$ => $\exists\ i_0 \in I$ a.î. $a \in A_{i_0}$ } $a \in B$

$A_i \subseteq B$, $\forall i \in I$ => $A_{i_0} \subseteq B$

=> $\bigcup A_i \subseteq B$.

$A \subseteq A \cup B$, $B \subseteq A \cup B$ $\quad \xrightarrow{\bigcup_{i\in I} A_i \subseteq B} A_i \subseteq B, \forall i \in I$.

$A_i \subseteq \bigcup_{i\in I} A_i \quad , \forall i \in I$)

$\bigcup_{i\in I} \subseteq B \Leftrightarrow \left(\bigcup_{i\in I} A_i\right) \cup B = B \Leftrightarrow \bigcup_{i\in I} (A_i \cup B) = B \Leftrightarrow$

$\Leftrightarrow A_i \cup B \subseteq B, \forall i \in I$

$\left.\begin{array}{l} A_i \cup B \subseteq B \\ A_i \subseteq A_i \cup B \end{array}\right\} \Rightarrow A_i \subseteq B, \forall i \in I$

P31 S_5:

$(i,j) \to$ impară $\quad (-1)^{5-4} = -1$

$(i,j,k) \to$ pară $\to (-1)^{5-3} = 1$

$(i,j)(k,l) \to$ pară $\to (-1)^{5-3} = 1$

$(i,j)(k,l,m) \to$ impară $\to (-1)^{5-2} = -1$

$(i,j,k,l,m) \to$ pară $\to (-1)^{5-1} = 1$

id $\to$ pară

$\Rightarrow$ permutările pare (cele care formează A_5) sunt: id, (i,j,k), $(i,j)(k,l)$, (i,j,k,l,m)

3 cicluri: $\frac{A_5^3}{3} = \frac{5!}{2! \cdot 3} = 20$

~~3 cicluri~~ $(i,j,k) = (k,i,j) = (j,k,i)$

5 cicluri: $\frac{5!}{5} = 4! = 24$

$sgn(\sigma) = (-1)^{n-k}$

produse de 2 transpoziții disjuncte: $\frac{C_5^2 \cdot C_3^2}{2} =$

$=10$

(11) $(0,1) \sim [0,1) \sim (0,1] \sim [0,1]$

$f:(0,1) \to [0,1)$

$f(x) = \frac{1}{1-0} x - \frac{0}{1-0} = x$ bijectivă

$g:(0,1] \to [0,1]$

$g(x) = \frac{x}{1-0} - \frac{0}{1-0} = x$ - bijectivă

$\left.\begin{array}{l} f(x) = x \\ g(x) = x \end{array}\right\}$ $f(x) = g(x) \Rightarrow$ bij f^{+} $[0,1] \sim (0,1)$

$1 \Rightarrow (0,1) \sim [0,1) \sim (0,1] \sim [0,1]$

b) $f:(a,b) \to (0,1)$

Să dem $(a,b) \sim (0,1)$, $\forall$ a $b \in \mathbb{R}$, $a \leq b$

$f(x) = \frac{1}{b-a} \cdot x - \frac{a}{b-a}$ - bijectivă

$A(a,0)$

$B(b,1)$

~~$y = \frac{x}{b-a} - \frac{a}{b-a} = f(x)$~~

~~$y = mx + n$~~

~~$\begin{cases} 0 = ax + n \\ 1 = bx + n \end{cases} \Rightarrow$~~

$y = mx + n$

$$\begin{cases} 0 = am + n \\ 1 = bm + n \end{cases} \iff \begin{cases} n = -am \\ 1 = bm - am \end{cases} \iff \begin{cases} n = -a \cdot \frac{1}{b-a} \\ m = \frac{1}{b-a} \end{cases}$$

$$\iff \begin{cases} n = \frac{a}{a-b} \\ m = \frac{1}{b-a} \end{cases}$$

$y = f(x) = \frac{x}{b-a} + \frac{a}{a-b} \Rightarrow$ bijectivă

c) $\mathbb{R} \sim (0,1)$

$f: \mathbb{R} \to (0,1)$

fie $a = \lim_{x \to \infty} x \in \mathbb{R}$

$$f(x) = \begin{cases} \frac{x}{2a}, & x > 0 \\ 0.5, & x = 0 \\ -\frac{x}{a}, & x < 0 \end{cases}$$

(P8) A, B - mulțimi finite

$card(A) = card(B)$

$f: A \to B$

Se consideră afirmația: f este injectivă dacă și numai dacă f este injectivă

Ne putem da seama ușor că domeniul și codomeniul trebuie să aibă același număr de elemente pentru ca afirmația să fie adevărată

① Pentru ca funcția să fie ~~[illegible]~~ injectivă, fiecare element din A trebuie să aibă exact un coresp în B.

(c) Pentru ca funcția să fie surjectivă trebuie ca fiecare element din B să aibă măcar un corespondent în A.

Presupunem că cele 2 mulțimi A și B sunt infinite. Pentru două mulțimi infinite nu se poate afirma că au același număr de elemente, astfel afirmația considerată nefiind adevărată.

(P26) a) $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 4 & 1 & 5 & 2 \end{pmatrix} = (1,3,4)(2,6)$

$S_6 = \{1,2,3,4,5,6\}$

(1,2)	(1,6)	(2,6)	(4,5)
(1,3)	(2,3)	(3,4)	(4,6)
(1,4)	(2,4)	(3,5)	(5,6)
(1,5)	(2,5)	(3,6)	

$\to$ 15 permutări $\dfrac{(n)(n-1)}{2}$

~~(P14) x ∈ M → $\frac{x}{x+1}$ → $\frac{1}{x+1}$ ∈ A~~