1 (a) $\{\neg, \wedge\}$

Fie F, G două prop atomice

I) $\neg F \equiv \neg F$ ✓

II) $F \vee G = \neg\neg(F \vee G) = \neg(\neg F \wedge \neg G)$

III) $F \wedge G$ ✓

IV) $F \Rightarrow G \equiv \neg\neg(F \Rightarrow G) = \neg(F \wedge (\neg G))$

V) $F \leftrightarrow G \equiv (F \Rightarrow G) \wedge (G \Rightarrow F)$

$\neg(F \wedge (\neg G)) \wedge \neg(G \wedge (\neg F))$ ✓

(b) $\{\neg, \vee\}$

I) $\neg F \equiv \neg F$ ✓

II) $F \vee G \equiv F \vee G$ ✓

III) $F \wedge G \equiv \neg\neg(F \wedge G) \equiv \neg(\neg F \vee \neg G)$ ✓

IV) $F \Rightarrow G \equiv (\neg F \vee G)$

V) $F \leftrightarrow G \equiv (F \Rightarrow G) \wedge (G \Rightarrow F) \equiv (\neg F \vee G) \wedge (\neg G \vee F)$ ✓

c) ???

d) $\{\uparrow\}$

(I) $\neg F \sim F \uparrow F \overset{def}{=} \neg(F \wedge F)$

(II) $F \wedge G \sim \neg(\neg(F \wedge G)) \sim \neg(F \uparrow G) \overset{(I)}{\sim} (F \uparrow G) \uparrow (F \uparrow G)$

(III) $F \vee G \sim (\neg F) \uparrow (\neg G) = (F \uparrow F) \uparrow (G \uparrow G)$

(IV) $F \Rightarrow G = \neg F \vee G \overset{(I)}{\sim} (F \uparrow F) \vee G \overset{(3)}{\sim} ((F \uparrow F) \uparrow (F \uparrow F)) \uparrow (G \uparrow G)$ ~~$((F \uparrow F) \uparrow (G \uparrow G)) \uparrow ($~~

$F \Leftrightarrow G \sim F \Rightarrow G \wedge G \Rightarrow F$

$\sim \underbrace{((F \uparrow F) \uparrow (F \uparrow F))}_{A_1} \uparrow (G \uparrow G) \wedge \underbrace{((G \uparrow G) \uparrow (G \uparrow G))}_{A_2} \uparrow (F \uparrow F)$

$\sim (A_1 \uparrow A_2) \uparrow (A_1 \uparrow A_2)$

e) $\{\nabla\}$

(I) $\neg F \sim \neg(F \vee F) \overset{def}{\sim} F \nabla F$

(II) $F \wedge G = \neg(\neg(F \wedge G)) \sim \neg(\neg F \vee \neg G) \sim$

$= \neg \neg(F \nabla F) \overset{(I)}{\sim} (F \nabla F) \nabla (G \nabla G)$

(III) $F \vee G = \neg(\neg(F \vee G)) = \neg(F \nabla G) = (F \nabla G) \nabla (F \nabla G)$

(IV) $F \Rightarrow G \sim \neg F \vee G \overset{(II)}{\sim} (\neg F \nabla G) \nabla (\neg F \nabla G) \sim$

$(F \nabla F \nabla G) \nabla (F \nabla F \nabla G)$

(V) $F \Leftrightarrow G \sim F \Rightarrow G \wedge G \Rightarrow F \overset{IV}{\sim} ((F \nabla F \nabla G) \nabla (F \nabla F \nabla G)) \wedge ((G \nabla G \nabla F) \nabla (G \nabla G \nabla F))$

2. prim:

$M = ((\neg A \wedge B \wedge \neg C) \vee (\neg A \wedge B \wedge C)) \vee (A \wedge \neg B \wedge C) \vee (A \wedge B \wedge C)$

$= ((\neg A \wedge B) \wedge \underbrace{(\neg C \vee C)}_{T}) \vee (A \wedge \neg B \wedge C) \vee (A \wedge B \wedge C)$

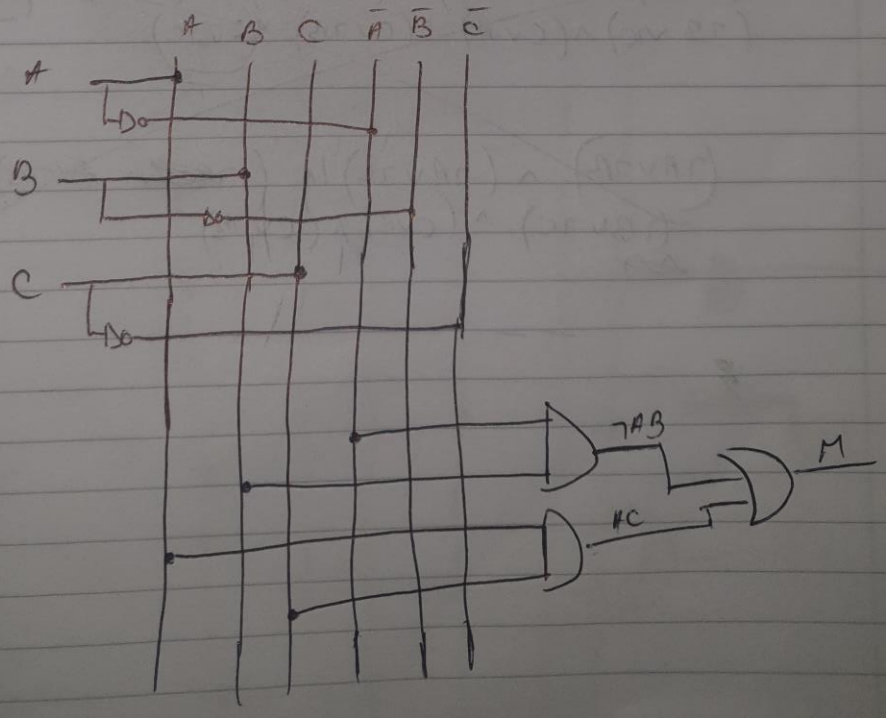
$(\neg A \wedge B) \vee (A \wedge \neg B \wedge C) \vee (A \wedge B \wedge C)$

$(\neg A \wedge B) \vee ((A \wedge C) \wedge \neg B) \vee (A \wedge C \wedge B))$

$(\neg A \wedge B) \vee ((A \wedge C) \wedge \underbrace{(\neg B \vee B)}_{T})$

$(\neg A \wedge B) \vee (A \wedge C)$ ①

a)



petrat perfect:

$(\neg A \wedge \neg B \wedge C) \vee (A \wedge \neg B \wedge \neg C)$

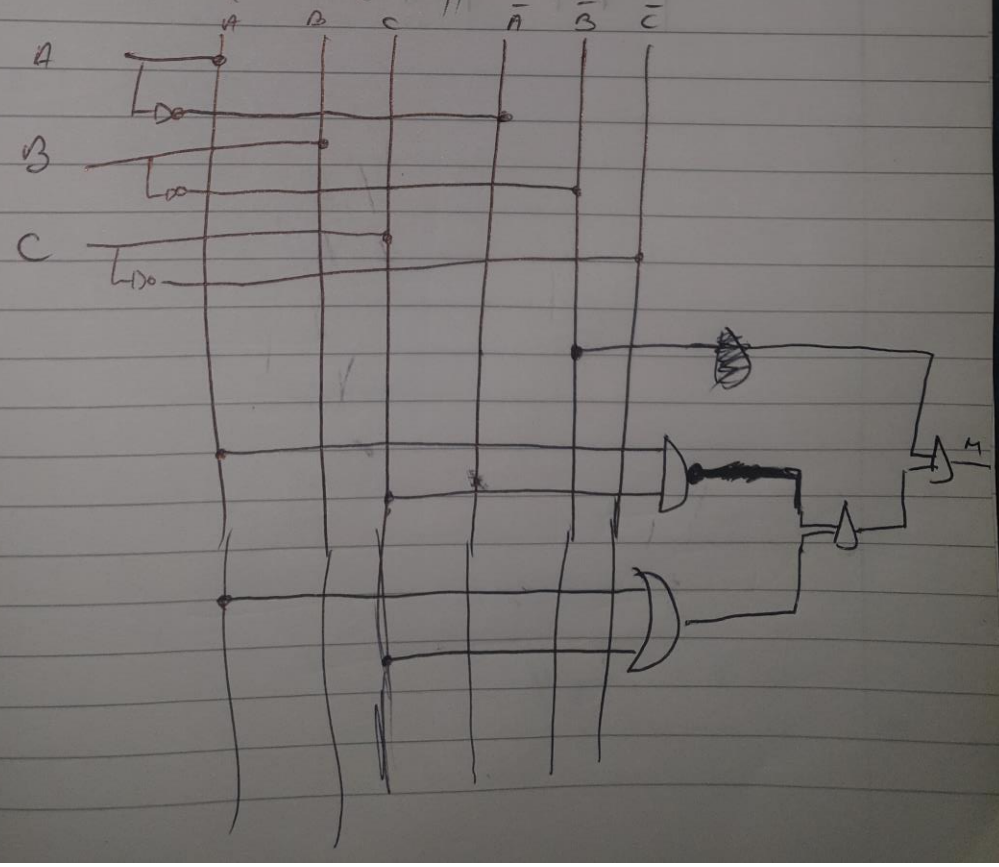
$= (\neg B) \wedge ((\neg A \wedge C) \vee (A \wedge \neg C))$

$= \neg B \wedge ((\underbrace{\neg A \vee A}_{T}) \wedge (\neg A \vee \neg C) \wedge (C \vee A) \wedge (\underbrace{C \vee \neg C}_{T}))$

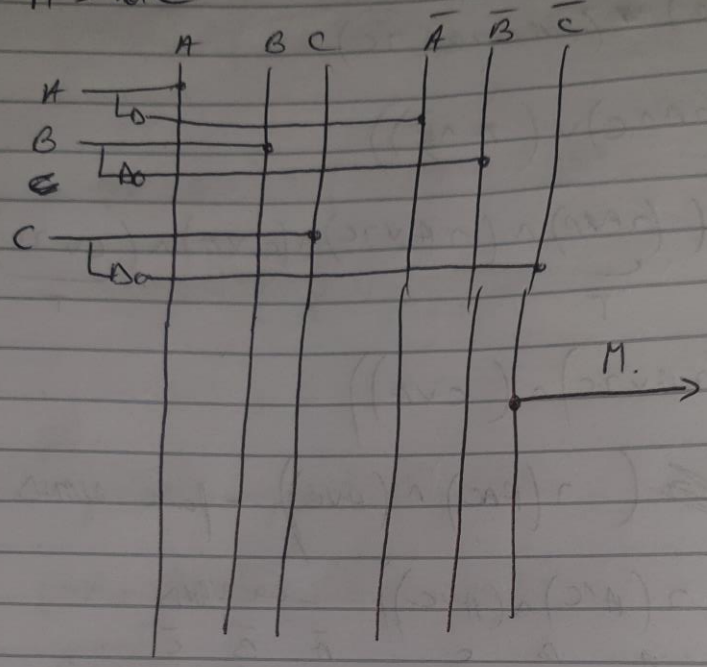
$= \neg B \wedge ((\neg A \vee \neg C) \wedge (C \vee A))$

$= \neg B \wedge (\neg(A \wedge C) \wedge (A \vee C))$ - forma NAND

$= \neg B \wedge (\neg(\neg A \wedge C) \wedge (A \vee C))$ - cu NAND



M = not C



$(A \wedge B \wedge \bar{T}) \vee (A \wedge \neg B \wedge \neg T) \vee (\neg A \wedge \neg B \wedge \bar{T}) = S$

$(A \wedge B \wedge T) \vee (A \wedge B \wedge \neg T) \vee (A \wedge \neg B \wedge T) \vee (\neg A \wedge B \wedge T) = R$

$S = (A \wedge B \wedge \bar{T}) \vee (A \wedge \neg B \wedge \neg T) \vee (\neg A \wedge B \wedge \neg T) \vee (\neg A \wedge \neg B \wedge \bar{T})$

$((A \wedge T) \wedge \underbrace{(B \vee \neg B)}_{T}) \vee$ ~~$(\neg A \wedge (B \wedge T))$~~ $(\neg A \wedge B \wedge T) \vee (\neg A \wedge \neg B \wedge \bar{T})$

$(A \wedge \bar{T}) \vee (\neg A \wedge B \wedge \neg T) \vee (\neg A \wedge \neg B \wedge \bar{T})$

$(A \wedge T) \vee (\neg (A \vee T) \wedge B) \vee (\neg A \wedge \neg B \wedge \bar{T})$

$(\underbrace{(A \wedge \bar{T}) \vee (\neg (A \wedge T))}_{T}) \wedge ((A \wedge T) \vee B) \vee (\neg A \wedge \neg B \wedge \bar{T})$

$((A \wedge T) \vee B) \vee (\neg A \wedge \neg B \wedge \bar{T}) = S$

~~$(A \vee B \wedge \bar{T}) \vee (\neg A \wedge \neg B \wedge \bar{T})$~~

$R = ((A \wedge B \wedge T) \vee (A \wedge B \wedge \neg T)) \vee (A \wedge \neg B \wedge \bar{T}) \vee (\neg A \wedge B \wedge \bar{T})$ ~~$= R$~~

$= ((A \wedge B) \wedge (\underbrace{\bar{T} \vee \neg T}_{T})) \vee (T \wedge ((A \wedge \neg B) \vee (\neg A \wedge B)))$

$= (A \wedge B) \vee (T \wedge (\underbrace{(A \vee \neg A)}_{T} \wedge (A \vee B) \wedge (\neg B \vee A) \wedge \underbrace{(\neg B \vee B)}_{T}))$

$= (A \wedge B) \vee (T \wedge ((A \vee B) \wedge (\neg B \vee A)))$

$R = (A \wedge B) \vee (\bar{T} \wedge ((A \vee B) \wedge \neg (B \wedge A)))$

$$R = (A \wedge B) \vee (T \wedge ((A \vee B) \wedge \neg(B \wedge A)))$$

$$((A \wedge B) \vee T) \wedge ((A \wedge B) \vee (A \vee B)) \wedge \underbrace{((A \wedge B) \vee \neg(A \wedge B))}_{T}$$

$$R = (A \wedge B \vee \bar{T}) \wedge ((A \wedge B) \vee (A \vee B))$$

$$S = ((A \wedge \bar{T}) \vee B) \vee (\neg A \wedge \neg B \wedge T)$$