$$\mathcal{X}_S \equiv \mathcal{H}\mathcal{X}\mathcal{D}^{-1/2}, \quad \text{donde}$$

$$\mathcal{X} = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1p} \\ x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & & & \\ x_{n1} & x_{n2} & \cdots & x_{np} \end{pmatrix}, \quad \text{matriz de datos}$$

$$\mathcal{H} = \mathbb{I}_n - \frac{1}{n}\mathbb{1}_n\mathbb{1}_n', \quad \text{matriz de centrado}$$

$$\mathcal{D} = \mathrm{DIAG}(S_{X_1X_1}, \ldots, S_{X_pX_p}) = \begin{pmatrix} S_{X_1X_1} & 0 & \cdots & 0 \\ 0 & S_{X_2X_2} & \cdots & 0 \\ \vdots & & & \\ 0 & \cdots & \cdots & S_{X_pX_p} \end{pmatrix}$$

es una matriz diagonal, tal que el i-ésimo elemento en su diagonal es

$$S_{X_iX_i} = \hat{\Sigma}_{ii} = \frac{1}{n}\sum_{j=1}^{n}(x_{ji} - \bar{x}_{\cdot i})^2 = \widehat{VAR}(X_i).$$

$$\mathcal{X}_S = \left(\mathcal{X} - \mathbb{1}_n \bar{x}_{(n)}'\right)\mathcal{D}^{-1/2}$$

$$= \left(\mathcal{X} - \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}(\bar{x}_{\cdot 1}, \ldots, \bar{x}_{\cdot p})\right)\mathcal{D}^{-1/2}$$

$$= \left[\mathcal{X} - \begin{pmatrix} \bar{x}_{\cdot 1} & \cdots & \bar{x}_{\cdot p} \\ \bar{x}_{\cdot 1} & \cdots & \bar{x}_{\cdot p} \\ \vdots & & \vdots \\ \bar{x}_{\cdot 1} & & \bar{x}_{\cdot p} \end{pmatrix}\right]\mathcal{D}^{-1/2}$$

$$= \begin{pmatrix} x_{11} - \bar{x}_{\cdot 1} & x_{12} - \bar{x}_{\cdot 2} & \cdots & x_{1p} - \bar{x}_{\cdot p} \\ x_{21} - \bar{x}_{\cdot 1} & x_{22} - \bar{x}_{\cdot 2} & \cdots & x_{2p} - \bar{x}_{\cdot p} \\ \vdots & \vdots & & \vdots \\ x_{n1} - \bar{x}_{\cdot 1} & x_{n2} - \bar{x}_{\cdot 2} & \cdots & x_{np} - \bar{x}_{\cdot p} \end{pmatrix}\mathcal{D}^{-1/2}$$

$$\mathcal{X}_S = \begin{pmatrix} \frac{(x_{11}-\bar{x}_{\cdot 1})}{S^{1/2}_{x_1x_1}} & \frac{(x_{12}-\bar{x}_{\cdot 2})}{S^{1/2}_{x_2x_2}} & \cdots & \frac{x_{1p}-\bar{x}_{\cdot p}}{S^{1/2}_{x_px_p}} \\ \vdots & \vdots & & \vdots \\ \frac{(x_{n1}-\bar{x}_{\cdot 1})}{S^{1/2}_{x_1x_1}} & \frac{(x_{n2}-\bar{x}_{\cdot 2})}{S^{1/2}_{x_2x_2}} & \cdots & \frac{x_{np}-\bar{x}_{\cdot p}}{S^{1/2}_{x_px_p}} \end{pmatrix}$$

"matriz de datos estandarizados"

$$\bar{x}^S_{(n)} \equiv \frac{1}{n}\mathcal{X}_S' \mathbb{1}_n = \frac{1}{n}\begin{pmatrix} \sum_{i=1}^n (x_{i1}-\bar{x}_{\cdot 1}) \\ \vdots \\ \sum_{i=1}^n (x_{ip}-\bar{x}_{\cdot p}) \end{pmatrix}$$

$$= \begin{pmatrix} \bar{x}_{\cdot 1}-\bar{x}_{\cdot 1} \\ \vdots \\ \bar{x}_{\cdot p}-\bar{x}_{\cdot p} \end{pmatrix} = \underbrace{\begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}}_{p\times 1} \quad \cdots\cdots\cdots \text{(sm)}$$

$$\therefore \quad \hat{\Sigma}^S = \frac{1}{n}\mathcal{X}_S'\mathcal{X}_S - \bar{x}^S_{(n)}\bar{x}^{S\prime}_{(n)} = \frac{1}{n}\mathcal{X}_S'\mathcal{X}_S$$

es la matriz con entrada k,l.

$$\hat{\Sigma}^S_{kl} = \frac{1}{n}\sum_{i=1}^n \frac{(x_{ik}-\bar{x}_{\cdot k})(x_{il}-\bar{x}_{\cdot l})}{\sqrt{S_{x_kx_k}}\sqrt{S_{x_lx_l}}} \quad \left(= \hat{\rho}_{x_kx_l}\right)$$

$\hat{\Sigma}^S = \hat{R}$ ← matriz de correlaciones de $\mathcal{X}$

$\hat{R} = \mathcal{G}_R \mathcal{L}_R \mathcal{G}_R'$ descomposición de Jordan de $\hat{R}$

$\mathbb{L}_{\mathcal{R}} = \mathrm{DIAG}(l_1^{\mathcal{R}}, \ldots, l_p^{\mathcal{R}})$, $l_1^{\mathcal{R}} \geq \cdots \geq l_p^{\mathcal{R}}$ son los valores propios de $\hat{\mathcal{R}}$, con correspondientes vectores propios $g_1^{\mathcal{R}}, \ldots, g_p^{\mathcal{R}}$. Como $tr(\hat{\mathcal{R}}) = \sum_{j=1}^{p} l_j^{\mathcal{R}}$ y la matriz $\hat{\mathcal{R}}$ tiene diagonal dada por $\mathbb{1}_p = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$, entonces

$$\sum_{j=1}^{p} l_j^{\mathcal{R}} = p .$$

Las componentes principales normalizadas son entonces (dado que $\bar{x}_{(n)}^{S} = 0$)

$$\mathcal{Z} = \mathcal{X}_S \, \mathcal{G}_{\mathcal{R}} \quad \cdots \; (\mathrm{CPN})$$

(compárese con la ecuación (CP), página 35).